EXPOSURE MODELS IN REINSURANCE

November 18th, 2016
Exposure Models in Reinsurance : Table of Contents

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- Transition Methods
- Types of Exposure Curves
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Motivation

Exposure Models in Reinsurance
Motivation

• Pricing of a Non-Proportional per risk reinsurance programs
  ▪ it should not be based only on historical claims experience
  ▪ current exposure should also be considered
  ▪ shift in business – movements in the portfolio volumes
  ▪ in case of insufficient claim history it is indispensable

• Historically, the need for a fast and accurate pricing process

• Aim: distribution of premium between primary insurer and reinsurer for each band/risk

<table>
<thead>
<tr>
<th>Risk Profile Name</th>
<th>Band SI/PML</th>
<th>Nr. Of Risks</th>
<th>Total SI/PML</th>
<th>Premium</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fire - Property Small Risks</td>
<td>0</td>
<td>500 000</td>
<td>81 847</td>
<td>4 073 604 954</td>
</tr>
<tr>
<td>Fire - Property Small Risks</td>
<td>500 000</td>
<td>1 000 000</td>
<td>1 566</td>
<td>1 118 702 402</td>
</tr>
<tr>
<td>Fire - Property Small Risks</td>
<td>1 000 000</td>
<td>2 500 000</td>
<td>1 246</td>
<td>1 934 995 601</td>
</tr>
<tr>
<td>Fire - Property Small Risks</td>
<td>2 500 000</td>
<td>5 000 000</td>
<td>291</td>
<td>946 439 987</td>
</tr>
<tr>
<td>Fire - Property Small Risks</td>
<td>5 000 000</td>
<td>10 000 000</td>
<td>73</td>
<td>484 292 205</td>
</tr>
<tr>
<td>Fire - Property Small Risks</td>
<td>10 000 000</td>
<td>20 000 000</td>
<td>39</td>
<td>527 940 298</td>
</tr>
<tr>
<td>Fire - Property Small Risks</td>
<td>20 000 000</td>
<td>30 000 000</td>
<td>16</td>
<td>410 949 649</td>
</tr>
</tbody>
</table>
Motivation

Exposure Pricing

• it uses Risk Profiles with the current available portfolio information
  ▪ it contains homogeneous risk types
  ▪ all risks of the same size (Sum Insured, Probable Maximum Loss, Estimated Maximum Loss) are grouped together in Risk Bands
  ▪ Total Exposure (SI, PML, EML), Total Premium as well as Number of Risks in each band are known

• Application of a single claim distribution per risk band
  ▪ Problem is that claim distribution is not known

➢ application of Exposure Curves
Motivation

Exposure Curves

- allow direct sharing of risk premium between insurer and reinsurer
- reinsurance risk premium is a function of the deductible
- are usually in a tabular form
- constructed from claim history of large homogeneous portfolios
Exposure Models in Reinsurance

Construction and Interpretation of Exposure Curves
Construction and Interpretation of Exposure Curves

Severity Distribution of Claims

• Empirical Distribution Function

Claim Set
Average = 5064
Construction and Interpretation of Exposure Curves

Severity Distribution of Claims

- Empirical Distribution Function with Sum Insured
Construction and Interpretation of Exposure Curves

Degree of Damage

- Ratio of Claim Severity and Sum Insured

Observations

Degree of Damage

0% 10% 20% 30% 40% 50% 60% 70% 80% 90% 100%
Construction and Interpretation of Exposure Curves

Degree of Damage Distribution

- Empirical Distribution Function

Degree of Damage

Cumulative Probability

Average = 33.3%
Construction and Interpretation of Exposure Curves

Degree of Damage Distribution $F$

- Empirical Distribution Function

$$F(x) - 	ext{Degree of Damage Distribution Function}$$

$$X \sim F \Rightarrow \mathbb{E}[X] = 33.3\%$$
Construction and Interpretation of Exposure Curves

Degree of Damage Distribution

- Transition to the Exposure Curve

\[
G(d) = \frac{\int_0^d (1 - F(y)) \, dy}{\int_0^1 (1 - F(y)) \, dy} = \frac{1}{\mathbb{E}[X]} \int_0^d (1 - F(y)) \, dy
\]

\[
G(20\%) = \frac{16.3\%}{33.3\%} = 48.9\%
\]
Construction and Interpretation of Exposure Curves

Exposure Curve G

Interpretation:
With a deductible of 20% of the SI / PML, the indemnity is reduced by 51.1% of the risk premiums.
Construction and Interpretation of Exposure Curves

Properties of the Exposure Curve $G$

By definition it holds that:

- $G(0) = 0$
- $G(1) = 1$

Because

- $G'(d) = \frac{1 - F(d)}{E[X]} \geq 0$
- and $G''(d) = -\frac{F'(d)}{E[X]} = -\frac{f(d)}{E[X]} \leq 0$

$G$ is increasing and concave in $[0, 1]$
Construction and Interpretation of Exposure Curves

**Example 1: Total damages only**
- Portfolio A produces only total damages
- Then it is obvious that \( G(d) = d \)
Construction and Interpretation of Exposure Curves

Example 2
- Portfolio B produces
  - 10% of claims that are total damages,
  - 50% of claims are 60% partial damages,
  - and 40% of claims are 25% partial damages
- Then
  \[ G(d) = \frac{1}{50\%} \cdot \begin{cases} d, & 0\% \leq d \leq 25\% \\ 10\% + 60\% \cdot d, & 25\% \leq d \leq 60\% \\ 10\% + 36\% + 10\% \cdot d, & 60\% \leq d \leq 100\% \end{cases} \]
Example 3

• Portfolio C produces
  - 10% of claims that are total damages,
  - 40% of claims are 80% partial damages,
  - 30% of claims are 40% partial damages,
  - and 20% of claims are 10% partial damages

• Then
  \[ G(d) = \frac{1}{56\%} \cdot \begin{cases} 
    d, & 0% \leq d \leq 10\% \\
    2\% + 80\% \cdot d, & 10\% \leq d \leq 40\% \\
    2\% + 12\% + 50\% \cdot d, & 40\% \leq d \leq 80\% \\
    2\% + 12\% + 32\% + 10\% \cdot d, & 80\% \leq d \leq 100\% 
  \end{cases} \]
Example 4
- Portfolio D produces
  - 10% of claims that are total damages,
  - 10% of claims are 30% partial damages,
  - 30% of claims are 20% partial damages,
  - and 50% of claims are 10% partial damages
Exposure Models in Reinsurance

Transition Methods
Transition Methods
From Exposure Curve to Degree of Damage Distribution

- Total Damage Probability \( G'(1) \)

- It can be derived: \( G'(d) = \frac{1-F(d)}{\mathbb{E}[X]} \)

- to remember: \( F(d) = 1 - G'(d) \cdot \mathbb{E}[X] \)

\[
\mathbb{E}[X] = \frac{1}{G'(0)}
\]

\[
F(d) = \begin{cases} 
1 - \frac{G'(d)}{G'(0)}, & 0 \leq d < 1 \\
1, & d = 1 
\end{cases}
\]
Transition Methods
From Exposure Curve to Claim Frequency and Severity Distribution

Sum Insured /PML

Exposure Curve G

Reduction of the Expected Claim [in%]

Deductible [in%]

Risk Premium

Claims Severity

Claims Frequency
Transition Methods

From Exposure Curve to Claim Frequency

- Example

**Sum Insured = 1.5M €**

**Expected Claim Severity = 0.5 M €**

**Risk Premium**

\[ P = 25 \, 000 \, € \]

**Expected Claim Frequency = 5%**

\[ \frac{P}{SI \cdot E(X)} \]

Illustration of the average claims frequency depending on the claims severity [in% of sum insured]
Transition Methods
From Exposure Curve to Claim Severity Distribution

The graph shows a cumulative probability distribution of claim severities across different claim sets. The x-axis represents the sum insured, while the y-axis shows the cumulative probability. The inset diagram illustrates the cumulative distribution function (CDF) of claim severities normalized by the sum insured, $SI_k$. The policy with $SI_k = 40,000 €$ is highlighted, indicating its significance in the distribution.
## Transition Methods

### Overview - Claim Frequency and Severity Distribution

<table>
<thead>
<tr>
<th>Policies / Bands</th>
<th>SI / PML</th>
<th>Premium</th>
<th>Claim Frequency (at x)</th>
<th>Claim Severity</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>$SI_1$</td>
<td>$P_1$</td>
<td>$\lambda_1(x) = \frac{P_1}{SI_1} \cdot G' \left( \frac{x}{SI_1} \right)$</td>
<td>$F \left( \frac{x}{SI_1} \right)$</td>
</tr>
<tr>
<td>#2</td>
<td>$SI_2$</td>
<td>$P_2$</td>
<td>$\lambda_2(x) = \frac{P_2}{SI_2} \cdot G' \left( \frac{x}{SI_2} \right)$</td>
<td>$F \left( \frac{x}{SI_2} \right)$</td>
</tr>
<tr>
<td>(\ldots)</td>
<td>(\ldots)</td>
<td>(\ldots)</td>
<td>(\ldots)</td>
<td>(\ldots)</td>
</tr>
<tr>
<td>#k</td>
<td>$SI_k$</td>
<td>$P_k$</td>
<td>$\lambda_k(x) = \frac{P_k}{SI_k} \cdot G' \left( \frac{x}{SI_k} \right)$</td>
<td>$F \left( \frac{x}{SI_k} \right)$</td>
</tr>
<tr>
<td>(\ldots)</td>
<td>(\ldots)</td>
<td>(\ldots)</td>
<td>(\ldots)</td>
<td>(\ldots)</td>
</tr>
<tr>
<td>#n</td>
<td>$SI_n$</td>
<td>$P_n$</td>
<td>$\lambda_n(x) = \frac{P_n}{n} \cdot G' \left( \frac{x}{SI_n} \right)$</td>
<td>$F \left( \frac{x}{SI_n} \right)$</td>
</tr>
</tbody>
</table>

"Collective Model (f.g.u.)"  
{Poisson$(\lambda(0))$, $Y(x)$}  

\[
\lambda(x) = \sum_{k=1}^{n} \lambda_k(x) \\
Y(x) = 1 - \frac{\lambda(x)}{\lambda(0)}
\]
Exposure Models in Reinsurance

Types of Exposure Curves
Types of Exposure Curves

- **Lloyds Curves**
  - Does not vary by amount of insurance or occupancy class
  - Underlying unknown (marine losses? WWII Fires?)

- **Salzmann (Personal Property)**
  - Based on actual Homeowners data (INA, 1960)
  - Varies by Construction/Protection Class
  - Building losses only and Fire losses only
  - Salzmann recommends not using them, only meant as an example

- **Reinsurer Curves (Munich, Skandia, etc.)**

- **Ludwig Curves (Personal and Commercial)**
  - Based on actual **Homeowners** and Commercial data, (based on relatively small portfolio of Hartford Insurance Group)
  - Includes all property coverages and perils (also 1989 hurricane Hugo losses)
  - Old data: 1984 – 1988
Types of Exposure Curves

• ISO’s PSOLD (Insurance Services Office)
  - Recent Data – updated every 2 years
  - Varies by amount of insurance, occupancy class, state, coverage, and peril
  - Continuous Distribution (no need for Interpolation)
  - Based on ISO data only
  - US specific (see White [2005])

• Swiss Re curves
  - also called Gasser curves (developed by Peter Gasser)
  - widely used by European reinsurers

• MBBEFD curves
  - new parametrisation of all curves above
Types of Exposure Curves

Curve Selection

- Whether a lot of total losses occur, or partial and small losses are the rule, depends on various factors
- The decisive factors are (see Guggisberg [2004])
  - **Perils covered**
    - fire causes more damage to an individual building than a windstorm
    - while gas explosion can completely destroy a house, lightning strikes generally causes only partial damage
    - earthquakes cause minor to devastating damage to buildings
  - **Class of risk**
    - gunpowder factories are more likely to suffer total losses than food processing plants

<table>
<thead>
<tr>
<th>Class of Risk</th>
<th>Average Degree of Damage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residential Building</td>
<td>1.9%</td>
</tr>
<tr>
<td>Administration Building</td>
<td>0.5%</td>
</tr>
<tr>
<td>Farm Building</td>
<td>4.9%</td>
</tr>
<tr>
<td>Industrial Building</td>
<td>4.4%</td>
</tr>
</tbody>
</table>
Types of Exposure Curves

Curve Selection

- **Size of risk**
  - fire often causes only partial damage to a large building, whereas small buildings are more likely to suffer total destruction in the event of fire in terms of Sum Insured or PML
  - the larger a risk, the smaller the PML usually is as a percentage of the SI

- **Fire protection measures**
  - has a considerable influence on the shape of loss distribution function
  - make it possible to stop fires at an earlier stage – total overall loss is smaller and the share of minor losses increases

**Summary:**

<table>
<thead>
<tr>
<th>Peril/Type</th>
<th>Curve tends towards the diagonal</th>
<th>Curve runs in the middle area</th>
<th>Curve runs in the outer area</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fire</td>
<td>Risk with poor fire protection</td>
<td>Risks with average fire protection</td>
<td>Risks with above average fire protection</td>
</tr>
<tr>
<td>Personal lines</td>
<td>Commercial lines</td>
<td>Industrial lines</td>
<td>Administrative building</td>
</tr>
<tr>
<td>Farm building</td>
<td>Industrial building</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Windstorm</td>
<td>Radio tower</td>
<td>Office building</td>
<td></td>
</tr>
<tr>
<td>Hurricane</td>
<td>Radio tower</td>
<td>Office building</td>
<td></td>
</tr>
</tbody>
</table>
Exposure Models in Reinsurance

MBBEFD
Distributions
MBBEFD Distributions

Background

- In general **exposure curves** are given in tabular form
- Problems:
  - limited number of curves available
  - piecewise linear functions
  - do not catch slight changes in reinsurance program
  - only conditionally suitable for the calculation of the number of claims
- Aim:
  - replace table values with function
    - For exposure curves this means that piecewise linear function becomes a continuous function
MBBEFD Distributions

Background

• the abbreviation stands for Maxwell-Boltzmann, Bose-Einstein, Fermi-Dirac

• curves from Physics used in the field of Statistical Mechanics

• suitable for damage curve modeling in **property insurance** (see Bernegger [1997])
  ▪ Continuous distributions
  ▪ described by two parameters: $b \geq 0$ and $g \geq 1$
  ▪ Swiss Re Y-Exposure Curves with a single parameter $c$ are the special case of MBBEFD curves

• MBBEFD curves are common in Europe
  ▪ less common in North America
MBBEFD Distributions

MBBEFD Exposure Curves

- Exposure Curve for normalized retention $m \in [0; 1]$ is defined as

$$G_{b,g}(m) = \begin{cases} 
    m, & g = 1 \land b = 0 \\
    \frac{\ln[1+(g-1)m]}{\ln[g]}, & b = 1 \land g > 1 \\
    \frac{1-b^m}{1-b}, & bg = 1 \land g > 1 \\
    \frac{\ln\left[(g-1)b+(1-gb)b^m\right]}{\ln[gb]}, & b > 0 \land b \neq 1 \land bg \neq 1 \land g > 1
\end{cases}$$

- case $bg < 1$ corresponds to MB, $bg = 1$ to BE and $bg > 1$ to FD distribution

- Interpretation:
  - $g = \frac{1}{\text{Probability of Total Loss}} = \frac{G'(0)}{G'(1)}$
  - b has no direct interpretation
corresponding degree of damage random variable $X$ defined on interval $[0;1]$ has CDF

$$F_{b,g}(x) = \begin{cases} 
1, & x = 1 \\
0, & x < 1 \land (g = 1 \lor b = 0) \\
1 - \frac{1}{1 + (g - 1)x}, & x < 1 \land b = 1 \land g > 1 \\
1 - b^x, & x < 1 \land bg = 1 \land g > 1 \\
1 - \frac{1 - b}{(g - 1) b^{1-x} + (1 - gb)}, & x < 1 \land b > 0 \land b \neq 1 \land bg \neq 1 \land g > 1 
\end{cases}$$
Because of the finite probability \( \frac{1}{g} \) for a total loss, the density function \( f(x) = F'(x) \) is defined only on the interval \([0; 1)\):

\[
f_{b,g}(x) = \begin{cases} 
0, & g = 1 \land b = 0 \\
\frac{(g - 1)}{(1 + (g - 1)x)^2}, & b = 1 \land g > 1 \\
-\ln[b]b^x, & bg = 1 \land g > 1 \\
\frac{(b - 1)(g - 1)\ln[b]b^{1-x}}{((g - 1)b^{1-x} + (1 - gb))^2}, & b > 0 \land b \neq 1 \land bg \neq 1 \land g > 1
\end{cases}
\]
MBBEFD Mean Degree of Damage

\[
\mathbb{E}(X) = \begin{cases} 
1, & g = 1 \lor b = 0 \\
\frac{\ln[g]}{g - 1}, & b = 1 \land g > 1 \\
\frac{b - 1}{\ln[b]} = \frac{g - 1}{\ln[g]g'}, & bg = 1 \land g > 1 \\
\frac{\ln[gb] (1 - b)}{\ln[b](1 - gb)}, & b > 0 \land b \neq 1 \land bg \neq 1 \land g > 1 
\end{cases}
\]

VIGRe
MBBEFD Distributions

Parameters Estimation

- **Method of Moments**
  
  \[ g = \frac{1}{Probability \ of \ Total \ Loss} = \frac{G'(0)}{G'(1)} \]

  - \( b \) can be derived iteratively from equation \( \mathbb{E}[X] = \frac{\ln(gb)(1-b)}{\ln(b)(1-gb)} \)

- **Mean Least Squares** (R package: see Dutang et al. [2016])
Swiss Re Exposure Curves

- Swiss Re $Y_c$ Exposure Curves are very common among non-proportional underwriters

- Parameter $c = 0, 1.5, 2, 3, 4$ denotes the concavity of the curve
  - $c = 0$ is the total loss (diagonal)
  - The higher $c$ the curve becomes more concave

- $c$ is a single parameter for defining the MBBEFD parameters $b$ and $g$:
  
  $$b_c = b(c) = \exp[3.1 - 0.15 (1 + c)c]$$
  
  $$g_c = g(c) = \exp[(0.78 + 0.12c)c]$$

MBBEFD Distributions

Swiss Re Exposure Curves

<table>
<thead>
<tr>
<th>Risk Group</th>
<th>Building Sum Insured from</th>
<th>Building Sum Insured to</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Personal lines</td>
<td>200 000 CHF</td>
<td>400 000 CHF</td>
</tr>
<tr>
<td>2. Commercial lines (small scale)</td>
<td>400 000 CHF</td>
<td>1 000 000 CHF</td>
</tr>
<tr>
<td>3. Commercial lines (medium scale)</td>
<td>1 000 000 CHF</td>
<td>2 000 000 CHF</td>
</tr>
<tr>
<td>4. Industrial lines and large commercial</td>
<td>over 2 000 000 CHF</td>
<td>-</td>
</tr>
</tbody>
</table>

![Swiss Re Exposure Curve Diagram]
Swiss Re Exposure Curves

- Big industrial companies insure their risks with **captive**
- Many small losses are not longer passed on to the market and so do not appear in the statistics
  - Therefore the major and total losses have greater impact
  - Exposure Curves for captive business tend more towards diagonal than those based on the entire claims
- Swiss Re developed three **captive exposure curves**
  - Fire
  - Business interruption
  - Fire and business interruption combined
- Can be used on qualitatively comparable portfolios made of policies with high deductibles
- Have designation $Y_6$
  - This naming says nothing about shape
  - Curves lie between Gasser curves $Y_3$ and $Y_4$
MBBEFD Distributions

Swiss Re Exposure Curves

• there are three more Exposure Curves for Oil and Petrochemicals (OPC)
  ▪ fire – runs in the area of $Y_2$
  ▪ business interruption – runs between diagonal and $Y_1$
  ▪ fire and business interruption combined – lies between $Y_1$ and $Y_2$

• all three have a high proportion of major losses typical for OPC

• original deductibles in OPC are usually high
  ➢ major losses are of greater importance
  ➢ Exposure Curves for OPC business tend more towards diagonal
MBBEFD Distributions

Swiss Re Exposure Curves

<table>
<thead>
<tr>
<th>Exposure Curve</th>
<th>Parameter c</th>
<th>b</th>
<th>g</th>
<th>p</th>
<th>$\mathbb{E}[X]$</th>
<th>Scope of application</th>
<th>Basis</th>
<th>Size of Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>Swiss 1</td>
<td>1.5</td>
<td>12.65</td>
<td>4.22</td>
<td>23.69%</td>
<td>34.86%</td>
<td>Personal lines</td>
<td>SI</td>
<td>&lt;400 000 CHF</td>
</tr>
<tr>
<td>Swiss 2</td>
<td>2.0</td>
<td>9.03</td>
<td>7.69</td>
<td>13%</td>
<td>22.09%</td>
<td>Commercial lines (small scale)</td>
<td>SI</td>
<td>&lt;1 000 000 CHF</td>
</tr>
<tr>
<td>Swiss 3</td>
<td>3.0</td>
<td>3.67</td>
<td>30.57</td>
<td>3.27%</td>
<td>8.72%</td>
<td>Commercial lines (medium scale)</td>
<td>SI</td>
<td>&lt;2 000 000 CHF</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.1</td>
<td>3.29</td>
<td>2.81%</td>
<td>7.89</td>
<td>Captive BI</td>
<td>PML</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.4</td>
<td>2.35</td>
<td>1.76%</td>
<td>5.84%</td>
<td>Captive Fire &amp; BI combined</td>
<td>PML</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.8</td>
<td>1.44</td>
<td>0.91%</td>
<td>3.89%</td>
<td>Captive Fire</td>
<td>PML</td>
<td></td>
</tr>
<tr>
<td>Swiss 4</td>
<td>4</td>
<td>1.11</td>
<td>154.5</td>
<td>0.65%</td>
<td>3.19%</td>
<td>Industrial lines &amp; large commercial</td>
<td>PML</td>
<td>&gt;2 000 000</td>
</tr>
<tr>
<td>Lloyd’s</td>
<td>5</td>
<td>0.25</td>
<td>992.3</td>
<td>0.10%</td>
<td>1.22%</td>
<td>Industry</td>
<td>Top location</td>
<td></td>
</tr>
<tr>
<td>Up to 8</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>Large scale Industry/ Multinational Companies</td>
<td>PML</td>
<td></td>
</tr>
</tbody>
</table>

$p = \text{Probability of Total Loss}$

source: Guggisberg [2004]
MBBEFD Distributions

Swiss Re Exposure Curves

source: Guggisberg [2004]
MBBEFD Distributions

Notes

• MBBEFD distributions are suitable only for property insurance

• These exposure curves are not sensitive to inflation
  ▪ Maximum Loss is assumed to be equal to Sum Insured or to Probable Maximum Loss

• This makes it necessary to check the exposure curves only at relatively long intervals

• Limitation of exposure curves is that these curves were estimated on the market portfolios, so do not have to be accurate and give reasonable results on analyzed portfolio.
  ➢ 1. Validate on loss profile of the company
  ➢ 2. Validation on working layers (amount of losses to the layer implied by the curve)
Exposure Models in Reinsurance

Pricing of Reinsurance Contracts with Exposure Curves
Pricing of Reinsurance Contracts with Exposure Curves

Components of the Reinsurance Price

- Profit Margin
- Cost of Capital
- Internal and External costs
- RI Risk Premium

- Depends on the company (and broker)
- Depends solely on the underlying business
Illustration - premium distribution by Exposure Curve

- original policy with SI = 1.5 M € with risk premium P = 25 000 €
- XL contract 0.9 M € xs 0.3M €
- in terms of SI: XL contract 60% xs 20%

The non-proportional coverage of the risk costs 47.05% of the original premium, i.e. RI Premium = 11 762.5 €
Pricing of Reinsurance Contracts with Exposure Curves

**Risk Profile**

- Each portfolio contains risks of different size and quality with different cover
- Division of portfolio into sub-segments with a homogeneous risk structure

<table>
<thead>
<tr>
<th>Risk Profile Name</th>
<th>Band SI/PML</th>
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<td>Fire - Property Small Risks</td>
<td>500 000</td>
<td>1 000 000</td>
<td>1 566</td>
<td>1 118 702 402</td>
</tr>
<tr>
<td>Fire - Property Large Risks</td>
<td>2 000 000</td>
<td>5 000 000</td>
<td>783</td>
<td>2 237 404 803</td>
</tr>
<tr>
<td>Fire - Industrial Risks</td>
<td>0</td>
<td>8 000 000</td>
<td>179</td>
<td>1 018 401 239</td>
</tr>
<tr>
<td>Fire - Industrial Risks</td>
<td>8 000 000</td>
<td>25 000 000</td>
<td>392</td>
<td>4 474 809 607</td>
</tr>
<tr>
<td>Fire - Industrial Risks</td>
<td>25 000 000</td>
<td>90 000 000</td>
<td>312</td>
<td>23 703 696 107</td>
</tr>
<tr>
<td>Fire - Industrial Risks</td>
<td>90 000 000</td>
<td>80 000 000</td>
<td>73</td>
<td>5 915 249 922</td>
</tr>
<tr>
<td>Fire - Industrial Risks</td>
<td>80 000 000</td>
<td>160 000 000</td>
<td>18</td>
<td>2 343 974 273</td>
</tr>
<tr>
<td>Fire - Industrial Risks</td>
<td>160 000 000</td>
<td>320 000 000</td>
<td>10</td>
<td>2 111 761 192</td>
</tr>
<tr>
<td>Fire - Industrial Risks</td>
<td>320 000 000</td>
<td>480 000 000</td>
<td>4</td>
<td>1 643 798 597</td>
</tr>
</tbody>
</table>

- Modelling with different exposure curves
Pricing of Reinsurance Contracts with Exposure Curves

**Modelling Steps**

1. Calculation of the **average Sum Insured** per band \( k = 1, \ldots, n \)
   \[
   \overline{SI}^k = \frac{\sum_{i=1}^{N^k} SI_i}{N^k},
   \]
   where \( N^k \) is number of risks in the \( k^{th} \) band

2. Calculation of the **normalized retention** per band as percentage of SI/PML
   \[
   \overline{m}^k = \min\left(\frac{R}{\overline{SI}^k}, 1\right)
   \]

3. Selection of the appropriate **exposure curve** for each band

4. Calculation of the **value of exposure curve** function \( G(\overline{m}^k) \) for each \( k^{th} \) band

5. Calculation of **mean gross loss** per band
   \[
   \mathbb{E}[Y^k] = P^k \cdot LR^k = \mathbb{E}[Y^k_{Ced}] + \mathbb{E}[Y^k_{Re}],
   \]
   where \( P^k \) is the gross premium and \( LR^k \) is gross Loss Ratio for \( k^{th} \) band

\[\text{VIG}^{Re}\]
Pricing of Reinsurance Contracts with Exposure Curves

Modelling Steps

6. Calculation of reinsurer’s mean ceded loss per band
\[ \mathbb{E}[Y^k_{Re}] = (1 - G(\bar{m}^k)) \cdot \mathbb{E}[Y^k] \]

7. Calculation of the mean aggregated loss into layer
   i. in case of one layer with unlimited capacity for all risk profiles it can be expressed as
   \[ \mathbb{E}[Y_{Re}] = \sum_{k=1}^{n} \mathbb{E}[Y^k_{Re}] \]
   ii. the case of more (L) layers with the corresponding retentions denoted as \((l)R, l = 1, \ldots, L\), reinsurer’s mean ceded loss per \(k\)-th band and \(l\)-th layer can be expressed as
   \[ \mathbb{E}[(l)Y^k_{Re}] = \begin{cases} \left( G\left((l+1)\bar{m}^k\right) - G\left((l)\bar{m}^k\right) \right) \cdot \mathbb{E}[Y^k], & l < L \\ \left( 1 - G\left((l)\bar{m}^k\right) \right) \cdot \mathbb{E}[Y^k], & l = L \end{cases} \]
   and reinsurer’s mean ceded loss in \(l\)-th layer as
   \[ \mathbb{E}[(l)Y_{Re}] = \sum_{k=1}^{n} \mathbb{E}[(l)Y^k_{Re}] \]
### Pricing of Reinsurance Contracts with Exposure Curves

#### Example – Quotation of XL 2M € xs 0.5M €

**Step 1**

<table>
<thead>
<tr>
<th>Band SI/PML</th>
<th>Total SI/PML</th>
<th>Nr. Of Risks</th>
<th>Premium</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100 000</td>
<td>3 895 341 592</td>
<td>46 425</td>
</tr>
<tr>
<td>100 000</td>
<td>200 000</td>
<td>2 237 330 404</td>
<td>13 994</td>
</tr>
<tr>
<td>200 000</td>
<td>300 000</td>
<td>1 910 346 260</td>
<td>7 483</td>
</tr>
<tr>
<td>300 000</td>
<td>500 000</td>
<td>1 316 269 834</td>
<td>4 014</td>
</tr>
<tr>
<td>500 000</td>
<td>750 000</td>
<td>1 146 935 002</td>
<td>1 599</td>
</tr>
<tr>
<td>750 000</td>
<td>1 000 000</td>
<td>810 399 944</td>
<td>936</td>
</tr>
<tr>
<td>1 000 000</td>
<td>1 500 000</td>
<td>697 830 194</td>
<td>563</td>
</tr>
<tr>
<td>1 500 000</td>
<td>2 500 000</td>
<td>403 707 061</td>
<td>199</td>
</tr>
<tr>
<td>2 500 000</td>
<td>5 000 000</td>
<td>106 697 299</td>
<td>32</td>
</tr>
<tr>
<td>5 000 000</td>
<td>10 000 000</td>
<td>40 104 436</td>
<td>8</td>
</tr>
</tbody>
</table>

**Total Premium** 20 512 584

**Step 2**

<table>
<thead>
<tr>
<th>Average SI/PML</th>
<th>R in % SI</th>
<th>R+L in % SI</th>
</tr>
</thead>
<tbody>
<tr>
<td>83 906</td>
<td>100.0%</td>
<td>100.0%</td>
</tr>
<tr>
<td>159 878</td>
<td>100.0%</td>
<td>100.0%</td>
</tr>
<tr>
<td>255 291</td>
<td>100.0%</td>
<td>100.0%</td>
</tr>
<tr>
<td>327 920</td>
<td>100.0%</td>
<td>100.0%</td>
</tr>
<tr>
<td>717 283</td>
<td>69.7%</td>
<td>100.0%</td>
</tr>
<tr>
<td>865 812</td>
<td>57.7%</td>
<td>100.0%</td>
</tr>
<tr>
<td>1 239 485</td>
<td>40.3%</td>
<td>100.0%</td>
</tr>
<tr>
<td>2 028 679</td>
<td>24.6%</td>
<td>100.0%</td>
</tr>
<tr>
<td>3 334 291</td>
<td>15.0%</td>
<td>90.0%</td>
</tr>
<tr>
<td>5 013 055</td>
<td>10.0%</td>
<td>59.8%</td>
</tr>
</tbody>
</table>

- Total Gross Loss Ratio is 60%
- for the sake of simplicity we assume that Loss Ratio is equal to 60% for all bands and one exposure curve is appropriate for all bands
# Pricing of Reinsurance Contracts with Exposure Curves

## Example – Quotation of XL 2M € xs 0.5M €

<table>
<thead>
<tr>
<th>Band</th>
<th>Premium</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>7 502 888</td>
</tr>
<tr>
<td>#2</td>
<td>4 158 031</td>
</tr>
<tr>
<td>#3</td>
<td>3 053 667</td>
</tr>
<tr>
<td>#4</td>
<td>1 150 935</td>
</tr>
<tr>
<td>#5</td>
<td>1 668 885</td>
</tr>
<tr>
<td>#6</td>
<td>1 280 817</td>
</tr>
<tr>
<td>#7</td>
<td>941 983</td>
</tr>
<tr>
<td>#8</td>
<td>523 651</td>
</tr>
<tr>
<td>#9</td>
<td>190 575</td>
</tr>
<tr>
<td>#10</td>
<td>41 152</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Step 4</th>
<th>Step 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Band</td>
<td>RI Premium</td>
</tr>
<tr>
<td>#1</td>
<td>100.0%</td>
</tr>
<tr>
<td>#2</td>
<td>100.0%</td>
</tr>
<tr>
<td>#3</td>
<td>100.0%</td>
</tr>
<tr>
<td>#4</td>
<td>100.0%</td>
</tr>
<tr>
<td>#5</td>
<td>91.2%</td>
</tr>
<tr>
<td>#6</td>
<td>85.2%</td>
</tr>
<tr>
<td>#7</td>
<td>73.1%</td>
</tr>
<tr>
<td>#8</td>
<td>56.4%</td>
</tr>
<tr>
<td>#9</td>
<td>39.5%</td>
</tr>
<tr>
<td>#10</td>
<td>25.6%</td>
</tr>
</tbody>
</table>

Total Premium: 20 512 584

Loss Ratio: 60%

RI Rate: 2.798%

RI Premium: 956 614

VIG Re
Exposure Models in Reinsurance

Increased Limit Factors
Increased Limit Factors

Introduction

• another type of exposure rating that can be used in liability non-proportional reinsurance
• the object of insurance is not known in advance
• the maximum possible loss is hardly to be estimated and can be much higher than sum insured
• helps
  ▪ when not enough historical claims are available
  ▪ if any of the experience rating based approaches is not possible to be applied reasonably (e.g. limited data to develop charges for high limits of liability coverages – these may represent very significant potential loss)

• parameters based on enough market data need to be applied

Increased Limit Factors
Increased Limit Factors

Definition

- usually available in tabular form
- An **increased limit factor** (ILF) at limit $L$ related to basic limit $B$ is defined as:

$$ILF(L) = \frac{\mathbb{E}[Y^L]}{\mathbb{E}[Y^B]},$$

- where
  - $\mathbb{E}[Y^L]$ denotes mean expected aggregate loss at the policy limit $L$
  - $\mathbb{E}[Y^B]$ denotes the mean aggregate loss at the basic limit $B$

- Both denote aggregate losses assuming all original policies had limits $L$ or $B$ respectively, i.e.

  - $Y^L = \sum_{i=1}^{N} \min(X_i,L)$
  - $Y^B = \sum_{i=1}^{N} \min(X_i,B)$
Increased Limit Factors

Definition

- It is assumed, that claims frequency is independent of claim severity and the frequencies are equal independently on the purchased limit.

  Therefore

  \[
  ILF(L) = \frac{\mathbb{E}[Y^L]}{\mathbb{E}[Y^B]} = \frac{\mathbb{E}[N^L]\mathbb{E}[X^L]}{\mathbb{E}[N^B]\mathbb{E}[X^B]} = \frac{\mathbb{E}[X^L]}{\mathbb{E}[X^B]}
  \]

- Assumptions need to be verified.
  - ILF should be constructed for different classes of liability separately.

- In notation as we had for exposure curves:

  \[
  ILF(L) = \frac{LAS(L)}{LAS(B)}
  \]
### Increased Limit Factors

#### Example

- **ILF Table**

<table>
<thead>
<tr>
<th>Claim Severity</th>
<th>Loss at 100 000 Basic Limit</th>
<th>Loss at 250 000 Increased Limit</th>
<th>Loss at 500 000 Increased Limit</th>
<th>Loss at 750 000 Increased Limit</th>
<th>Loss at 1 000 000 Increased Limit</th>
<th>Loss at 1 250 000 Increased Limit</th>
<th>Loss at 1 500 000 Increased Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 000</td>
<td>50 000</td>
<td>50 000</td>
<td>50 000</td>
<td>50 000</td>
<td>50 000</td>
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<td>60 000</td>
<td>60 000</td>
<td>60 000</td>
<td>60 000</td>
<td>60 000</td>
</tr>
<tr>
<td>120 000</td>
<td>100 000</td>
<td>120 000</td>
<td>120 000</td>
<td>120 000</td>
<td>120 000</td>
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<tr>
<td>475 000</td>
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<td>250 000</td>
<td>475 000</td>
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<td>475 000</td>
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<tr>
<td>580 000</td>
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<td>250 000</td>
<td>500 000</td>
<td>750 000</td>
<td>1 000 000</td>
<td>1 100 000</td>
<td>1 100 000</td>
</tr>
<tr>
<td>2 000 000</td>
<td>100 000</td>
<td>250 000</td>
<td>500 000</td>
<td>750 000</td>
<td>1 000 000</td>
<td>1 250 000</td>
<td>1 500 000</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>910 000</strong></td>
<td><strong>1 895 000</strong></td>
<td><strong>3 140 000</strong></td>
<td><strong>3 970 000</strong></td>
<td><strong>4 500 000</strong></td>
<td><strong>4 850 000</strong></td>
<td><strong>5 100 000</strong></td>
</tr>
<tr>
<td><strong>LAS</strong></td>
<td><strong>91 000</strong></td>
<td><strong>189 500</strong></td>
<td><strong>314 000</strong></td>
<td><strong>397 000</strong></td>
<td><strong>450 000</strong></td>
<td><strong>485 000</strong></td>
<td><strong>510 000</strong></td>
</tr>
<tr>
<td><strong>ILF</strong></td>
<td><strong>1</strong></td>
<td><strong>2.08</strong></td>
<td><strong>3.45</strong></td>
<td><strong>4.36</strong></td>
<td><strong>4.95</strong></td>
<td><strong>5.33</strong></td>
<td><strong>5.60</strong></td>
</tr>
</tbody>
</table>

**Interpretation:**
If the limit increases 10 times, the loss will increase only 4.95 times.
### Increased Limit Factors

**Example**

- **ILF Curve**

**Table:**

<table>
<thead>
<tr>
<th>Limits</th>
<th>ILF</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 000</td>
<td>0.55</td>
</tr>
<tr>
<td>100 000</td>
<td>1</td>
</tr>
<tr>
<td>250 000</td>
<td>2.08</td>
</tr>
<tr>
<td>500 000</td>
<td>3.45</td>
</tr>
<tr>
<td>750 000</td>
<td>4.36</td>
</tr>
<tr>
<td>1 000 000</td>
<td>4.95</td>
</tr>
<tr>
<td>1 250 000</td>
<td>5.33</td>
</tr>
<tr>
<td>1 500 000</td>
<td>5.60</td>
</tr>
</tbody>
</table>

**Diagram:**

Interpolation between limits is necessary.
Increased Limit Factors

**Limited Average Severity**

- For any limit $L \geq 0$, the limited average severity (LAS), at a limit of $L$, in the case of a continuous distribution with distribution function $F$, can be expressed as

$$\text{LAS}(L) = \mathbb{E}[X^L] = \int_0^L x \, dF(x) + L \cdot [1 - F(L)] = \int_0^L (1 - F(x)) \, dx$$
Increased Limit Factors

Properties

• for derivative of ILF it holds:

\[
\frac{d}{dL} ILF(L) = \frac{1}{\mathbb{E}[Y^B]} \cdot \frac{d}{dL} \left( \int_0^L x \, dF(x) + L \cdot [1 - F(L)] \right) \\
= \frac{1}{\mathbb{E}[Y^B]} \left( L \frac{dF(L)}{dL} + [1 - F(L)] - L \frac{dF(L)}{dL} \right)
\]

• because

\[ ILF'(L) = \frac{1 - F(L)}{LAS(B)} \geq 0 \]

\[ ILF(L) \text{ is an increasing function of } L \]

• as

\[ ILF''(L) = \frac{-f(L)}{LAS(B)} \leq 0 \]

\[ ILF \text{ is concave} \]
Increased Limit Factors

**Limited Average Severity**

- *LAS* can be used to express the **loss into XL layer**
- if expected number of losses from ground up is $\lambda$ then expected loss into XL layer with retention $R$ and limit $L$ is

$$
\mathbb{E}[Y_{R}^{R+L}] := \lambda \cdot \mathbb{E}[X_{R}^{R+L}] := \lambda \left[ \int_{R}^{R+L} (x - R) \, dF(x) + L \cdot [1 - F(R + L)] \right]
$$

![Diagram showing losses with indemnity above retention, limited by retention + limit.](image-url)
Limited Average Severity

previous can be expressed as

\[ \mathbb{E}[Y_R^{R+L}] = \lambda \cdot \left[ \int_R^{R+L} (x - R)dF(x) + L \cdot [1 - F(R + L)] \right] \]

\[ = \lambda \cdot \left[ \int_R^{R+L} x \, dF(x) + (R + L) \cdot [1 - F(R + L)] - R[1 - F(R)] \right] \]

\[ = \lambda \cdot [LAS(R + L) - LAS(R)] \]
Increased Limit Factors

Ceded Share

- ILF determine the ratio in which original loss from policy with sum insured \( SI \) and deductible \( D \) is divided between reinsurer and cedent

- ceded ratio \( C \) can be expressed as

\[
C = \frac{\mathbb{E}[Y_{R+L}^R]}{\mathbb{E}[Y_{SI}^D]} = \frac{\lambda \cdot \mathbb{E}[X_{R+L}^R]}{\lambda \cdot \mathbb{E}[X_{SI}^D]} = \frac{\text{LAS}(R + L) - \text{LAS}(R)}{\text{LAS}(SI) - \text{LAS}(D)} = \frac{\text{ILF}(R + L) - \text{ILF}(R)}{\text{ILF}(SI) - \text{ILF}(D)}
\]

- in case \( D = 0 \)

\[
C = \frac{\text{ILF}(R + L) - \text{ILF}(R)}{\text{ILF}(SI)}
\]
Increased Limit Factors

Inflation

• disadvantage of the liability exposure rating method is the sensitivity of $LAS$ and therefore also $ILF$ on inflation

• Assuming a constant inflation applied on all sizes of losses, the basic limit losses ($LAS(B)$) will be inflated by lower rate than losses limited at higher limits of liability ($LAS(L)$)

  ➢ will lead to even higher inflation in excess layers

• This phenomenon is called as “Leveraged Effect of Inflation”
### Increased Limit Factors

#### Inflation Example

<table>
<thead>
<tr>
<th>Claim Severity</th>
<th>Loss at 100,000</th>
<th>Loss at 250,000</th>
<th>Loss at 500,000</th>
<th>Loss at 750,000</th>
<th>Loss at 1,000,000</th>
<th>Loss at 1,250,000</th>
<th>Loss at 1,500,000</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Basic Limit</td>
<td>Increased Limit</td>
<td>Increased Limit</td>
<td>Increased Limit</td>
<td>Increased Limit</td>
<td>Increased Limit</td>
<td>Increased Limit</td>
</tr>
<tr>
<td>50,000</td>
<td>50,000</td>
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<td>50,000</td>
<td>50,000</td>
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<td>60,000</td>
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<tr>
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<td>165,000</td>
<td>165,000</td>
<td>165,000</td>
</tr>
<tr>
<td>270,000</td>
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<td>270,000</td>
<td>270,000</td>
<td>270,000</td>
<td>270,000</td>
<td>270,000</td>
<td>270,000</td>
</tr>
<tr>
<td>475,000</td>
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<tr>
<td>580,000</td>
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<td>580,000</td>
<td>580,000</td>
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<tr>
<td>780,000</td>
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<tr>
<td>1,100,000</td>
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<td>1,100,000</td>
<td>1,100,000</td>
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<td>1,100,000</td>
</tr>
<tr>
<td>2,000,000</td>
<td>2,000,000</td>
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<td>2,000,000</td>
<td>2,000,000</td>
<td>2,000,000</td>
<td>2,000,000</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>910,000</td>
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<td>3,140,000</td>
<td>3,970,000</td>
<td>4,500,000</td>
<td>4,850,000</td>
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**LAS**
- 91,000
- 189,500
- 314,000
- 397,000
- 450,000
- 485,000
- 510,000

**ILF**
- 1
- 2.08
- 3.45
- 4.36
- 4.95
- 5.33
- 5.60

#### Inflated Claim Severity

<table>
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<tr>
<th>Claim Severity</th>
<th>Loss at 100,000</th>
<th>Loss at 250,000</th>
<th>Loss at 500,000</th>
<th>Loss at 750,000</th>
<th>Loss at 1,000,000</th>
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<tr>
<td><strong>Total</strong></td>
<td>921,000</td>
<td>1,934,500</td>
<td>3,231,500</td>
<td>4,142,000</td>
<td>4,750,000</td>
<td>5,210,000</td>
<td>5,460,000</td>
</tr>
</tbody>
</table>

**LAS**
- 92,100
- 193,450
- 323,150
- 414,200
- 475,000
- 521,000
- 546,000

**ILF**
- 1
- 2.13
- 3.55
- 4.55
- 5.22
- 5.73
- 6.00

**LAS Inflation Effect**
- 1.21%
- 2.08%
- 2.91%
- 4.33%
- 5.56%
- 7.42%
- 7.06%

**ILF Inflation Effect**
- 1.21%
- 2.08%
- 2.91%
- 4.33%
- 5.56%
- 7.42%
- 7.06%

---

**Inflated by 10%**

**Leveraged Effect of Inflation**
Increased Limit Factors

Leveraged Effect of Inflation

![Graph showing the increased limit factors as a leveraged effect of inflation. The x-axis represents limits in thousands of dollars, ranging from 0 to 1,500,000. The y-axis represents increased limit factors, ranging from 0 to 6. There are two lines on the graph: one black line labeled ILF and one red line labeled ILF Inflated. The red line shows a greater increase in limit factors compared to the black line, indicating the leveraged effect of inflation.]
Increased Limit Factors

Riebessel’s Parameterization of ILFs

- approach which has been often used by German insurance and reinsurance companies
- it is inflation resistant
- Riebessel’s Curves are based on the assumption that each time $i \in \mathbb{N}$ the sum insured doubles, the risk cost increases by constant factor of $(1 + z)$ with $z \in (0,1)$, i.e.

$$P(2^iL) = P_L \cdot (1 + z)^i,$$

where $P_L = P(L)$ denotes standard risk premium for a limit of $L$
- here the sum insured acts only as limit of indemnity and not as a measure of the size of the risk like in Property insurance, the premium increases less than the sum insured
Increased Limit Factors

Riebessel’s Parameterization of ILFs

- $z$ is set according to the type of underlying portfolio
- by using a substitution $a = 2^i$ (i.e. $i = \log_2 a$) we have

\[
P(aL) = P_L \cdot (1 + z)^{\log_2 a}
\]

- can be rewritten more helpful to give the premium for any desired limit in terms of the relativity to the base ($y = aL$)

\[
P(y) = P_L \cdot (1 + z)^{\log_2 \left(\frac{y}{L}\right)} = P_L \cdot \left(\frac{y}{L}\right)^{\log_2 (1+z)}
\]

- this is called **Riebessel’s formula** with $z \in (0,1)$ for net premium $P(y)$ at any sum insured $y > 0$
Increased Limit Factors

Riebessel’s Parameterization of ILFs

- according to collective model

\[ P(y) = \mathbb{E}[N] \cdot \mathbb{E}[\min(X, y)] \]

- Then for ILF we have

\[ ILF(L) = \frac{LAS(L)}{LAS(B)} = \frac{P(L)}{P(B)} = \left(\frac{L}{B}\right)^{\log_2(1+z)} \]

- not affected by currency changes or inflation

- Mack & Fackler [2003] demonstrated that there exist loss distributions that lead to Riebesell's formula and that the formula is consistent with the assumption that the tail of severity distribution has a Pareto tail above a certain threshold

- generalizations which offer more flexibility regarding the severity distribution can be found in Riegel [2008]
Increased Limit Factors

Riebessel Parameterization of ILFs

- with Base Limit = 1 we have

\[ ILF(SI) = (1 + z)^{\log_2 SI} \]
Exposure Models in Reinsurance

Summary & Bibliography
Summary

- Exposure Rating gives us consistent pricing of reinsurance contracts
  - without intensive computational simulation runs
  - it is indispensible in case of insufficient loss history
  - in case of sufficient number of historical losses it can serve for creating second opinion on the final rate after performing experience ratings - Historical experience alone is NOT necessarily the best predictor of future experience

- MBBEFD distributions only depend on two parameters and are suitable for many property insurance branches
  - Not sensitive to inflation
  - One of the weaknesses is the uncertainty about choice of the appropriate exposure curve. The choice of the curve is always subjective and requires an in-depth knowledge of the analyzed portfolio.
  - aggregated risk profile is provided - might be also helpful to use some blended curves for some bands of risk profile which includes mix of various types of risks
Summary

• ILF curves suitable for casualty branches are always company specific
  ▪ no standard curves
  ▪ sensitive to inflation
  ▪ Riebessel Parameterization of ILFs
    ▪ it is inflation resistant

• Further development can be found in Desmedt et al. [2012]
  ▪ show methods to overcome the different limitations using a combination of experience and exposure rating techniques if historical profile information is available
  ▪ propose an experience rating method in which the measure for frequency and the as-if claims are determined using the evolutions observed in the risk profiles
  ▪ For pricing unused capacity exposure rating calibrated on the experience rate for a working layer is used
Bibliography


Dutang Christophe, Gesmann Markus & Spedicato Giorgio. [2016] ‘Exposure rating, destruction rate models and the mbbefd package’,
https://cran.r-project.org/web/packages/mbbefd/vignettes/Introduction_to_mbbefd.pdf


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