Parameterisation, calibration and realisation of the full Internal Model

Author: Jan Martinek
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Agenda

- Reasons for Internal model
  - Advantages/Disadvantages of the Internal model
  - How to build Internal Model
  - Calculation Engine structure
- Calculation engine modules
  - Dependencies
  - ESG & payment patterns
  - CAT risk
  - Gross UW
  - Outwards RI & RI Default
  - Reserve Risk
  - Gross Emergence
  - Other Risk modules
  - Financial Statements
  - Capital allocation
- Discussion & Questions
Reasons for Internal Model

• Capital requirement
  • Standard formula vs. Internal Model

• Use of the model
  • Regulatory requirements & Rating Agencies requirements
    • ICA, SCR/MCR, Solvency II to ultimate
    • Decision making process
    • Business planning
    • Underwriting, Reinsurance & Investment strategy testing
    • Capital allocation -> pricing

• Understanding of the business - Internal
  • Parameterisation and analysis consistency between departments (e.g. outwards reinsurance team, underwriters/pricing actuaries, capital modelling team)
## Advantages/Disadvantages of Internal Model

### + Capital requirement
- Internal Model SCR 2-3 times lower than Standard Formula SCR

### + Use of the model
- Possibility to get higher rating with lower capital
- Reinsurance optimisation - possibility of reducing reinsurance costs
- Business planning - better view on business mix, manage the underwriting process, change in risk appetite
- Allow for capital allocation in pricing

### + Understanding of the business - Internal
- Parameterisation and analysis consistency between departments

### - Costs, Resources & Time
- Software & Hardware costs, Consultancy
- Dedicated team of 2-4 people (Actuaries) + 2 people (Risk Management)
- Model build + documentation min of 12 months
How to build the Internal Model

• **What do we want?**
  • SCR, ICA - Financial statements (incl. GAAP)
  • Risk category split (standalone / attribution)
  • Additional reports/analysis - based on user requirements

• **Model design**
  • Determine and design individual model components
    • Standard vs. non-traditional
    • Top->Down or Bottom->Up
  • Number of Simulations and Memory constraints
  • Importance of the run-time
    • Dependencies
    • Parallel x Serial
  • Scenario/Sensitivity testing intelligent design
    • Recalculate minimal number of model components

• Model Map
Model Structure - example
Dependencies - general

- **Dependency types**
  - Gaussian copula, Student-T copula
  - Archimedean copula (Clayton, Gumbel, Frank) - [example](#)

- **Dependency groups**
  - **Within Risk Categories**
    - UW risk (Classes of business, attritional x large losses)
    - Reserve Risk (Classes of business, adjacent years)
    - CAT Risk (Similar type of Catastrophes)
    - Credit Risk (Reinsurers)
    - Operational Risk (Scenarios within same category)
  - **Between Risk Categories**
    - Market Risk & Cat Risk (short tail CATs - Man made & Natural)
    - UW risk x Reserve risk
    - Cat risk x Credit risk
    - Operational Risk x Insurance risks
Dependencies - parameterisation

- **Estimate using data**
  - Problematic parameterisation due to lack of data

- **Expert Judgement**
  - Low x Medium x High - symmetric and positive semidefinite matrix
    - Archimedean Copulas parameter (e.g. Gumbel > 1)
  - Adjusting matrix to be positive semidefinite - [example](#)

\[
C = \begin{pmatrix}
1 & 0.9 & 0.7 \\
0.9 & 1 & 0.3 \\
0.7 & 0.3 & 1 \\
\end{pmatrix}
\]

\[
C = \begin{pmatrix}
1 & 0.89458 & 0.69662 \\
0.89458 & 1 & 0.30254 \\
0.69662 & 0.30254 & 1 \\
\end{pmatrix}
\]

\[
\{ 2.2967 \ 0.710625 \ -0.00735 \}
\]

- **Sensitivity testing (decrease/increase by 10%)**
ESG & Payment Patterns and Inflation

• ESG - External model
  • Link to Internal model
  • Exchange rate calculation
  • Inflation weights - Class specific
    \[ \text{Inflation}_{i,\text{ClassA}} = \alpha_{\text{ClassA}} \text{Inf}_{i,\text{Prev}} + (1 - \alpha_{\text{ClassA}}) \text{Inf}_{i,\text{Rate}} + \text{SuperimposedInf} \]

• Payment patterns
  • Stochastic decay patterns
    • Pct of reserve, sum always 1 - example .xls
    • Common assumption - Mid year payments
    • Inflation adjusted payment patterns

• Calculation of Ultimate factors/rates
  • “Ultimate” Inflation factors
  • “Ultimate” Exchange rates
  • “Ultimate” Discount factor
Catastrophic Risk

- RMS/AIR (externally modelled) Perils x Other Perils
  - Definition (Loss in excess of x mil USD, high RP)
  - Natural Catastrophes vs Man-Made, Short tail vs Long tail

- Modelling of RMS/AIR Perils
  - Natural catastrophes (earthquake, windstorm)
  - Loss to company and market losses (predominant currency)
  - Loss dates
  - 1st approach: ELTs - example
  - 2nd approach: Pre-simulated values from external models

- Modelling of “other” Perils
  - Man-made (Terrorism, nuclear, medical procedures) and non-RMS modelled perils (Australia/Thai flood etc.)
  - Frequency per Peril x aggregated loss by each class

- Modelling extras
  - Modelled on AY basis and then allocated to UW years - applying exposure weighted scaling factor
Catastrophic Risk - parameterisation

• RMS/AIR (externally modelled) Perils
  • Produced by Aggregate modelling team - on AY basis
  • 1st approach: Import the portfolio and produce the ELTs
  • 2nd approach: Import the portfolio and Pre-simulate the losses

• Non-RMS/AIR Perils
  • Parameterised on AY basis
  • Lack and low data quality
  • Setting the mean return period for each peril (>RP 25/10 years)
  • Selection of different RP and estimation of corresponding max loss to class
    • Fitting the distribution - example
Gross UW Risk

- **Non-Cat Losses**
  - Attritional losses - aggregate distribution
  - Large losses - frequency x severity
  - Loss Dates & UW Dates
  - Modelled on AY's or UW Years and allocated down to UW Years or AY's
    - Allocation - stochastic assignation or deterministic split

- **Expenses & Premium**
  - Acquisition, operating, fixed expenses
  - Gross Premium
  - Dominant currency

- **Parameterisation & Calibration**
  - Analysis of historical losses (MM or MLE)
  - Large losses threshold and cap
  - Calibration to business plan (AY or UW Year basis)
    - Roll forward by change in exposure
Outwards Reinsurance & RI Default

- **Traditional Programmes**
  - QS, XoL, SL, Cat Aggs, Fac
  - Loss ordering (Net of previous programmes) & recovery cashflows
  - Modelled functionalities (LOD x RAD, contractual exchange rates, program currency, programmes orderings, payments lags, event caps, term/event limits, ceding commission, profit commission)

- **non-Traditional Programmes**
  - ILW, CILW, Cat-bonds, Franchise, Reverse Franchise

- **RI Default**
  - Rating by Reinsurer, default rate & loss given default
  - Credit migration
  - Next year programmes placements

- **Parameterisation**
  - Simplifications
## Reserve Risk

- **Gross Reserve**
  - Aggregate distribution by Historical UW Year or AY
  - Deterministic split to different currencies or dominant currency

- **Net Reserve**
  - Gross to Net ratio - vary by percentile
  - Additionally there are imported Gross and Net paid to date losses
    - ADC (LPT) modelling purposes
    - Intra-group reinsurance modelling purposes (Stop-Loss/QS covers)

- **Historical premium & expenses**
  - Modelling future payments/receivables/balances of opening outstandings

- **Reserve Perils**
  - Frequency x Severity
  - Factor of reserves
Reserve Risk - parameterisation

- **Gross Reserve Risk**
  - **Bootstrap**
    - Frequently used approach to get some initial estimate of volatility
    - Simple and flexible approach
  - **Mack & Munich Chain Ladder**
  - **Actuarial Hi-Low**
    - Changing of assumptions to estimate level of volatility
  - Gross Reserve modelled by LogNormal distribution

- **Net Reserve Risk**
  - Modelled by Gross to Net Ratio
  - These could vary by different percentiles to allow for non-proportional reinsurance contracts
Gross Emergence

- **Gross Loss Emergence**
  - Complete Re-Reserving in model - not recommended
  - Perfect view on ultimate - for some risks
  - Chain ladder development factors and/or BF method
  - Factor approach (weights between mean and true ultimate) - preferred approach
    - Simple and easy to validate
    \[ EX + \alpha(X - EX) = (1 - \alpha)EX + \alpha X \]

- **RI Recoveries Emergence**
  - Calculate recoveries for 1 year gross views
    - Same proportion between emergence and ultimate for all losses
  - Apply the gross loss emergence factors to ultimate RI recoveries
    - Limitations (ART, non-proportional contracts)
Other Models

- **Operational Risk**
  - Market, Credit & Insurance scenarios
  - cca 10% of capital
  - Frequency x Severity approach by scenario

- **IntraGroup Reinsurance**
  - QS, SL
  - Similar as outwards reinsurance modelling
Assets and Financial Statements

• Solvency II (1 year)
  • Balance sheet (opening & closing)
  • P&L statement
  • SCR & MCR
  • Functional currency

• GAAP (1 year)
  • Balance sheet (opening & closing)
  • P&L statement
  • Functional currency

• ICA (to ultimate)

• Additional modelled risks
  • Broker balances
  • Liquidity risk (selling asset portfolio, borrow at penal rate)
  • Asset portfolio rebalancing (duration and/or currency match, minimal holdings per Asset class)
    • No transaction costs

• Examples
Capital allocation

- **Allocation**
  - Risk categories - example
    - Insurance Risk
      - UW Risk - Cat risk x non-Cat risk
      - Reserve Risk
    - Market Risk
      - FX risk
      - Spread risk
      - Interest risk
      - Default risk
    - Operational Risk
  - Classes of business

- **Allocation techniques**
  - co-TVaR
  - Shapley (last-in, first-in)
  - Allocation by standalone risk profile
  - example
Thank you

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Dependencies - example 1

- Gaussian
- Multivariate Student's-t
- Gumbel
- Clayton
Dependencies - example 2

• **Method 1**
  - **Formula**
    
    \[
    M' = \frac{1}{1 - \lambda_{\text{max}}} (M - \lambda_{\text{max}})I
    \]

    \[
    M = \begin{bmatrix}
    1 & 0.9 & 0.7 \\
    0.9 & 1 & 0.3 \\
    0.7 & 0.3 & 1
    \end{bmatrix}
    \]

    \[
    M' = \begin{bmatrix}
    1 & 0.8934 & 0.6949 \\
    0.8934 & 1 & 0.2978 \\
    0.6949 & 0.2978 & 1
    \end{bmatrix}
    \]

  

• **Method 2**
  - Calculate eigenvalues and right hand side eigenvectors of M
  - Set all negative eigenvalues to 0
  - Set the length of the eigenvector to its associated eigenvalue
  - Arrange the eigenvectors as the columns of the matrix C
  - C' results from C by normalising the row vectors of C to unit length
  - Calculate \( M' = C'C^T \)

    \[
    C = \begin{bmatrix}
    1 & 0.9 & 0.7 \\
    0.9 & 1 & 0.3 \\
    0.7 & 0.3 & 1
    \end{bmatrix}
    \]

    \[
    C = \begin{bmatrix}
    1 & 0.89458 & 0.69662 \\
    0.89458 & 1 & 0.30254 \\
    0.69662 & 0.30254 & 1
    \end{bmatrix}
    \]

    \[
    \begin{bmatrix}
    2.2967 & 0.710625 & -0.00735 \\
    2.287 & 0.713 & 0
    \end{bmatrix}
    \]
Catastrophic Risk - example 1

- Simulations using ELT

<table>
<thead>
<tr>
<th>EVENT ID</th>
<th>RATE</th>
<th>PERSVALUE</th>
<th>STDEVI</th>
<th>STDEVC</th>
<th>EXPVALUE</th>
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<tbody>
<tr>
<td>151076</td>
<td>0.00927</td>
<td>15629.0</td>
<td>48775.8</td>
<td>97551.6</td>
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<td>52045.9</td>
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</tr>
</tbody>
</table>

- Frequency - Poisson
- Secondary uncertainty - Damage Ratio (Beta distribution)
  \[ \alpha = \frac{EX(1-EX)}{VarX} \]
  \[ \beta = \frac{EX(1-EX)}{VarX} \]

- Loss = EXPVALUE*Damage Ratio

* back
Catastrophic Risk - example 2
• non-RMS/AIR parameterisation

<table>
<thead>
<tr>
<th>Name of Peril</th>
<th>Max Loss</th>
<th>PML 1</th>
<th>RP 1</th>
<th>PML 2</th>
<th>RP 2</th>
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</thead>
<tbody>
<tr>
<td>Flood Australia</td>
<td>22 000 000</td>
<td>5 000 000</td>
<td>25</td>
<td>15 000 000</td>
<td>250</td>
</tr>
</tbody>
</table>

- **Frequency - Poisson**
  \[ \lambda = \frac{1}{RP1} \]

- **Severity - Pareto**
  \[ F(x) = 1 - \left( \frac{\beta}{x} \right)^{\alpha}; \alpha > 0, \beta > 0, x > \beta \]
  \[ \beta = \text{PML1} \]
  \[ \alpha = \ln \left( \frac{-\ln \left( \frac{1 - \frac{1}{RP2}}{\lambda} \right)}{\ln \left( \frac{PML2}{PML1} \right)} \right) = 2.094 \]
### Financial Statements - example 1

<table>
<thead>
<tr>
<th>Category</th>
<th>Year Opening</th>
<th>Year Closing</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Assets</strong></td>
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<td></td>
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<tr>
<td>Current Assets</td>
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<td></td>
</tr>
<tr>
<td>Non-current Assets</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Liabilities</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Current Liabilities</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-current Liabilities</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Shareholders’ Equity</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For detailed financial data, please refer to the report.
<table>
<thead>
<tr>
<th>UK CAMP BIL STATEMENT</th>
<th>Year</th>
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<tbody>
<tr>
<td>WORTHY PREMIUM</td>
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<tr>
<td>Gross Written Premium</td>
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</tr>
<tr>
<td>Less: Claim Incurred</td>
<td>0</td>
</tr>
<tr>
<td>Less: Policy Premium</td>
<td>0</td>
</tr>
<tr>
<td>Outward Premium</td>
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</tr>
<tr>
<td>Net Worthy Premium</td>
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<tr>
<td>SARBIC PREMIUM</td>
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<tr>
<td>Gross Earned Premium</td>
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<tr>
<td>Inwards reinforcer</td>
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<tr>
<td>Outward reinforcer</td>
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<tr>
<td>Total Earned Premium</td>
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<tr>
<td>EXPENSES</td>
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<tr>
<td>Gross Claims Incurred</td>
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<tr>
<td>Less: Claim Incurred</td>
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<tr>
<td>Less: Admin Cost</td>
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<td>Inwards Admin Cost</td>
<td>0</td>
</tr>
<tr>
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</tr>
<tr>
<td>Total Net Expenses</td>
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<tr>
<td>TOTAL NET LOSSES</td>
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<tr>
<td>TOTAL NET CLAIMS</td>
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</tr>
<tr>
<td>UNDERWRITING PROFIT</td>
<td>0</td>
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<tr>
<td>OTHER REVENUE</td>
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<td>PRE-TAX PROFIT</td>
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<thead>
<tr>
<th>ECONOMIC BIL STATEMENT</th>
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<tbody>
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<td>Change in NI Work of Premium Pre</td>
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<td>EXPENSES</td>
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<td>0</td>
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<tr>
<td>PRE-TAX PROFIT</td>
<td>0</td>
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</table>
Capital Allocations - example 1

<table>
<thead>
<tr>
<th>Risk Type</th>
<th>Interest Rate Risk</th>
<th>Market and Trading Risk</th>
<th>Calculative Risk</th>
<th>Credit Default Risk</th>
<th>Market Risk</th>
<th>Liquidity Risk</th>
<th>Operational Risk</th>
<th>Funding Risk</th>
<th>Market Risk</th>
<th>Model Risk</th>
<th>Valuation Risk</th>
<th>Transaction Risk</th>
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<tbody>
<tr>
<td>Value Cap</td>
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<td>789</td>
<td>012</td>
<td>345</td>
<td>678</td>
<td>900</td>
<td>123</td>
<td>456</td>
<td>789</td>
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<td>GHI</td>
<td>JKL</td>
<td>MNO</td>
<td>PQR</td>
<td>STU</td>
<td>VWF</td>
<td>XYS</td>
<td>Z01</td>
<td>234</td>
<td>567</td>
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</table>

* back
## Capital Allocations - example 2

<table>
<thead>
<tr>
<th>Class of Business</th>
<th>STD</th>
<th>Capital Calcs</th>
<th>Capital Allocation</th>
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</thead>
<tbody>
<tr>
<td></td>
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<td>Multiplier</td>
<td>Standalone Capital</td>
</tr>
<tr>
<td>A</td>
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<td>1,000</td>
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<tr>
<td>B</td>
<td>866</td>
<td>2,33</td>
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<td>C</td>
<td>1,288</td>
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<tr>
<td>D</td>
<td>5,717</td>
<td>2,33</td>
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<tr>
<td>Aggregated</td>
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