Optimization approaches to multiplicative tariff of rates estimation in non-life insurance

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Introduction

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The contribution combines four methodologies:

- **Data-mining** – data preparation.
- **Mathematical statistics** – random distribution estimation using generalized linear models.
- **Insurance mathematics** – pricing of non-life insurance contracts.
- **Operations research** – (stochastic) optimization approach to tariff of rates estimation based on the previous methodologies.
Practical experiences

- More than 3 years at Actuarial Department, Head Office of Vienna Insurance Group Czech Republic.
- VIG CR – the largest group on the market: 2 universal insurance companies (Kooperativa pojišťovna, Česká podnikatelská pojišťovna) and 1 life-oriented (Česká spořitelna).
- Kooperativa & ČPP MTPL: 2.5 mil. cars from 7 mil.
- Kooperativa & ČPP: completely different portfolios and strategies, e.g. flat MTPL rates vs. strict segmentation.
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Tariff classes/segmentation criteria

Tariff of rates based on $S + 1$ segmentation criteria:

- $i_0 \in \mathcal{I}_0$, e.g. tariff classes $\mathcal{I}_0 = \{\text{engine up to 1000, up to 1350, up to 1850, up to 2500, over 2500 ccm}\}$,
- $i_1 \in \mathcal{I}_1, \ldots, i_S \in \mathcal{I}_S$, e.g. age $\mathcal{I}_1 = \{18–30, 31–65, 66$ and more years\}

We denote $l = (i_0, i_1, \ldots, i_S)$, $l \in \mathcal{I}$ a tariff class, where $\mathcal{I} = \mathcal{I}_0 \otimes \mathcal{I}_1 \otimes \cdots \otimes \mathcal{I}_S$ denotes all combinations of criteria values. Let $W_l$ be the number of contracts in $l$. 
Compound distribution of aggregated losses

Aggregated losses over one year for risk cell $I$

$$L^T_I = \sum_{w=1}^{W_I} L_{I,w}, \quad L_{I,w} = \sum_{n=1}^{N_{I,w}} X_{I,n,w},$$

where all r.v. are assumed to be independent ($N_I, X_I$ denote independent copies)

- $N_{I,w}$ is the random number of claims for a contract during one year with the same distribution for all $w$
- $X_{I,n,w}$ is the random claims severity with the same distribution for all $n$ and $w$

Well-known formulas for the mean and the variance:

$$\mu^T_I = \mathbb{E}[L^T_I] = W_I \mu_I = W_I \mathbb{E}[N_I] \mathbb{E}[X_I],$$
$$\sigma^2_I = \text{var}(L^T_I) = W_I \sigma^2_I = W_I (\mathbb{E}[N_I] \text{var}(X_I) + (\mathbb{E}[X_I])^2 \text{var}(N_I)).$$
Multiplicative tariff of rates

We assume that the risk (office) premium is composed in a multiplicative way from

- **basic premium levels** $Pr_{i0}$ and
- nonnegative **surcharge coefficients** $e_{i1}, \ldots, e_{iS}$,

i.e. we obtain the decomposition

$$Pr_I = Pr_{i0} \cdot (1 + e_{i1}) \cdots (1 + e_{iS}).$$

We denote the **total premium** $TP_I = W_I Pr_I$ for the risk cell $I$.

**Example**: engine between 1001 and 1350 ccm, age 18–30, region over 500 000:

$$130 \cdot (1 + 0.5) \cdot (1 + 0.4)$$
Prescribed loss ratio – random constraints

Our goal is to find optimal basic premium levels and surcharge coefficients with respect to a prescribed loss ratio $\hat{LR}$, i.e. to fulfill the random constraints

$$\frac{LT}{TP_I} \leq \hat{LR} \text{ for all } I \in I,$$

(1)

and/or the random constraint

$$\frac{\sum_{I \in I} LT}{\sum_{I \in I} TP_I} \leq \hat{LR}.$$

(2)

The prescribed loss ratio $\hat{LR}$ is usually based on a management decision. If $\hat{LR} = 1$, we obtain the netto-premium. It is possible to prescribe a different loss ratio for each tariff cell.
Sources of risk

Two sources of risk:

1. **Expectation risk**: different *expected losses* for tariff cells.
2. **Distributional risk**: different *shape of the probability distribution* of losses, e.g. standard deviation.
Prescribed loss ratio – expected value constraints

Usually, the **expected value** of the loss ratio is bounded

\[
\frac{\mathbb{E}[L_i^T]}{TP_i} = \frac{\mathbb{E}[L_i]}{Pr_i} \leq \hat{LR} \text{ for all } i \in \mathcal{I}.
\]  

(3)

The distributional risk is not taken into account.
Prescribed loss ratio – chance constraints

A natural requirement: the inequalities are fulfilled with a \textbf{prescribed probability} leading to individual chance (probabilistic) constraints

\[
P \left( \frac{L_T^I}{TP_I} \leq \hat{LR} \right) \geq 1 - \varepsilon, \text{ for all } I \in \mathcal{I},
\]

where $\varepsilon \in (0, 1)$, usually $\varepsilon \in \{0.1, 0.05, 0.01\}$, or a constraint for the whole line of business:

\[
P \left( \frac{\sum_{I \in \mathcal{I}} L_T^I}{\sum_{I \in \mathcal{I}} TP_I} \leq \hat{LR} \right) \geq 1 - \varepsilon.
\]

Distributional risk allocation to tariff cells will be discussed later.
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Generalized linear models

A standard approach based on GLM with the logarithmic link function \( g(\mu) = \ln \mu \) without the intercept:

- **Poisson (overdispersed) or Negative-binomial regression**
  - the expected number of claims:
    \[
    \mathbb{E}[N_i] = \exp\{\lambda_{i_0} + \lambda_{i_1} + \cdots + \lambda_{i_S}\},
    \]

- **Gamma or Inverse Gaussian regression**
  - the expected claim severity:
    \[
    \mathbb{E}[X_i] = \exp\{\gamma_{i_0} + \gamma_{i_1} + \cdots + \gamma_{i_S}\},
    \]

where \( \lambda_i, \gamma_i \) are the regression coefficients for each \( l = (i_0, i_1, \ldots, i_S) \). For the **expected loss** we obtain

\[
\mathbb{E}[L_i] = \exp\{\lambda_{i_0} + \gamma_{i_0} + \lambda_{i_1} + \gamma_{i_1} + \cdots + \lambda_{i_S} + \gamma_{i_S}\}.
\]
The **basic premium levels** and the **surcharge coefficients** can be estimated as a product of normalized coefficients

\[
Pr_{i_0} = \frac{\exp\{\lambda_{i_0} + \gamma_{i_0}\}}{LR} \cdot \prod_{s=1}^{S} \min_{i \in I_s} \exp(\lambda_i) \cdot \prod_{s=1}^{S} \min_{i \in I_s} \exp(\gamma_i),
\]

\[
e_{i_s} = \frac{\exp(\lambda_{i_s})}{\min_{i_s \in I_s} \exp(\lambda_{i_s})} \cdot \frac{\exp(\gamma_{i_s})}{\min_{i_s \in I_s} \exp(\gamma_{i_s})} - 1,
\]

Under this choice, the constraints on loss ratios are fulfilled with respect to the expectations.
Generalized linear models

The GLM approach is highly dependent on using GLM with the logarithmic link function. It can be hardly used if other link functions are used, interaction or other regressors than the segmentation criteria are considered.

For the total losses modelling, we can employ generalized linear models with the logarithmic link and a **Tweedie distribution** for $1 < p < 2$, which corresponds to the compound Poisson–gamma distributions.
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Advantages of the optimization approach

- GLM with other than logarithmic link functions can be used,
- business requirements on surcharge coefficients can be ensured,
- total losses can be decomposed and modeled using different models, e.g. for bodily injury and property damage,
- other modelling techniques than GLM can be used to estimate the distribution of total losses over one year, e.g. generalized additive models, classification and regression trees,
- not only the expectation of total losses can be taken into account but also the shape of the distribution,
- costs and loadings (commissions, tax, office expenses, unanticipated losses, cost of reinsurance) can be incorporated when our goal is to optimize the combined ratio instead of the loss ratio, we obtain final office premium as the output,
Total loss – decomposition

We can assume that $L_I$ contains not only losses but also various costs and loadings, thus we can construct the tariff rates with respect to a prescribed combined ratio. For example, the total loss over one year can be composed as follows

$$L_I = (1 + vc_I)[(1 + inf_s)L^s_I + (1 + inf_l)L^l_I] + fc_I,$$

where small $L^s_I$ and large claims $L^l_I$ are modeled separately, inflation of small claims $inf_s$ and large claims $inf_l$, proportional costs $vc_I$ and fixed costs $fc_I$ are incorporated.

We only need estimates of $\mathbb{E}[L^T_I]$ and $\text{var}(L^T_I)$ for all $I$.
The premium is minimized\(^1\) under the conditions on the prescribed loss ratio and a maximal possible surcharge\( (r^{\text{max}})\):

\[
\min_{Pr,e} \sum_{l \in I} w_l Pr_{i_0}(1 + e_{i_1}) \cdots (1 + e_{i_S})
\]

s.t. \( \hat{LR} \cdot Pr_{i_0} \cdot (1 + e_{i_1}) \cdots (1 + e_{i_S}) \geq \mathbb{E}[L_{i_0,i_1,\ldots,i_S}] \),

\( (1 + e_{i_1}) \cdots (1 + e_{i_S}) \leq 1 + r^{\text{max}} \),

\( e_{i_1}, \ldots, e_{i_S} \geq 0, (i_0, i_1, \ldots, i_S) \in I \).

This problem is nonlinear nonconvex, thus very difficult to solve. Other constraints can be included.

\(^1\)A profitability is ensured by the constraints on the loss ratio. The optimization leads to minimal levels and surcharges.
Optimization model – expected value approach

Using the **logarithmic transformation** of the decision variables $u_{i_0} = \ln(P_{r_{i_0}})$ and $u_{i_s} = \ln(1 + e_{i_s})$ and by setting

$$b_{i_0, i_1, \ldots, i_S} = \ln(\mathbb{E}[L_{i_0, i_1, \ldots, i_S}]/\hat{L}_R),$$

the problem can be rewritten as a **nonlinear convex programming problem**, which can be efficiently solved by standard software tools:

$$\min_u \sum_{l \in \mathcal{I}} w_l e^{u_{i_0} + u_{i_1} + \cdots + u_{i_S}}$$

subject to $u_{i_0} + u_{i_1} + \cdots + u_{i_S} \geq b_{i_0, i_1, \ldots, i_S},$ (6)

$u_{i_1} + \cdots + u_{i_S} \leq \ln(1 + r_{\text{max}})$,

$u_{i_1}, \ldots, u_{i_S} \geq 0, \quad (i_0, i_1, \ldots, i_S) \in \mathcal{I}.$

The problems (5) and (6) are equivalent.
Optimization over a net of coefficients

Let the surcharge coefficients be selected from a discrete net and \( r_s > 0 \) be a step, usually 0.1 or 0.05. We set \( J_s = \left\lceil \frac{r_{\text{max}}}{r_s} \right\rceil \) and

\[
u_{i_s} = \sum_{j=0}^{J_s} y_{i_s,j} \ln(1 + j \cdot r_s),\]

together with the conditions \( \sum_{j=0}^{J_s} y_{i_s,j} = 1, y_{i_s,j} \in \{0, 1\} \).
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The goal is to minimize $f : \mathbb{R}^n \rightarrow \mathbb{R}$ under the conditions

$$g_j(x) \geq \xi_j, \ j = 1, \ldots, m,$$

where $g_j : \mathbb{R}^n \rightarrow \mathbb{R}$ and $\xi_j$ are real random variables. **Chance (probabilistic, VaR) constraints**

$$P(g_j(x) \geq \xi_j) \geq 1 - \varepsilon, \ j = 1, \ldots, m,$$

can be reformulated using the quantile function leading to

$$g_j(x) \geq F_{\xi_j}^{-1}(1 - \varepsilon), \ j = 1, \ldots, m.$$
Chance constraints are **nonconvex** in general.

It can be even **difficult to verify feasibility** of a points.

**Solution approaches:**
- Discrete distribution and mixed-integer programming
- Sample approximation technique (numerical integration)
- Penalty methods
- Distributional assumptions
- Convex approximations
- ...

See Prékopa (1995), Shapiro and Ruszczyński (2003), Shapiro et al. (2009), Branda and Dupačová (2012), Branda (2012, 2013)
If we prescribe a small probability level $\varepsilon \in (0, 1)$ for violating the loss ratio in each tariff cell, we obtain the following chance constraints

$$P \left( L^T_{i_0, i_1, \ldots, i_S} \leq \hat{\text{LR}} \cdot W_{i_0, i_1, \ldots, i_S} \cdot Pr_{i_0} \cdot (1 + e_{i_1}) \cdots (1 + e_{i_S}) \right) \geq 1 - \varepsilon,$$

which can be rewritten using the quantile function $F_{L^T_i}^{-1}$ of $L^T_i$ as

$$\hat{\text{LR}} \cdot W_{i_0, i_1, \ldots, i_S} \cdot Pr_{i_0} \cdot (1 + e_{i_1}) \cdots (1 + e_{i_S}) \geq F_{L^T_{i_0, i_1, \ldots, i_S}}^{-1}(1 - \varepsilon).$$

By setting

$$b_i = \ln \left[ \frac{F_{L^T_i}^{-1}(1 - \varepsilon)}{W_i \cdot \hat{\text{LR}}} \right],$$

the formulation (6) can be used.
Optimization approaches to rates estimation

Optimization models – individual chance constraints

Optimization model – individual chance constraints

\[
\min_u \sum_{l \in \mathcal{I}} w_l e^{u_{i_0} + u_{i_1} + \cdots + u_{i_S}}
\]

s.t.

\[
\begin{align*}
    u_{i_0} + u_{i_1} + \cdots + u_{i_S} & \geq b_{i_0, i_1, \ldots, i_S}, \\
    u_{i_1} + \cdots + u_{i_S} & \leq \ln(1 + r^{\text{max}}), \\
    u_{i_1}, \ldots, u_{i_S} & \geq 0, \quad (i_0, i_1, \ldots, i_S) \in \mathcal{I},
\end{align*}
\]

with

\[
b_l = \ln \left[ \frac{F_{L_l}^{-1}(1 - \varepsilon)}{W_l \cdot \hat{LR}} \right].
\]
It can be very difficult to compute the quantiles $F^{-1}_{L_i}$, see, e.g., Withers and Nadarajah (2011). We can employ the one-sided Chebyshev’s inequality based on the mean and variance of the compound distribution:

$$P \left( \frac{L_i^T}{TP_i} \geq \hat{LR} \right) \leq \frac{1}{1 + (\hat{LR} \cdot TP_i - \mu_i^T)^2/(\sigma_i^T)^2} \leq \varepsilon,$$

(7) for $\hat{LR} \cdot TP_i \geq \mu_i^T$. Chen et al. (2011) showed that the bound is tight for all distributions $\mathcal{D}$ with the expected value $\mu_i^T$ and the variance $(\sigma_i^T)^2$:

$$\sup_{\mathcal{D}} P(L_i^T \geq \hat{LR} \cdot TP_i) = \frac{1}{1 + (\hat{LR} \cdot TP_i - \mu_i^T)^2/(\sigma_i^T)^2},$$

for $\hat{LR} \cdot TP_i \geq \mu_i^T$. 

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Optimization model – individual reliability constraints

The inequality (7) leads to the following constraints, which serve as conservative approximations:

$$\mu_I^T + \sqrt{\frac{1-\varepsilon}{\varepsilon}} \sigma_I^T \leq \hat{LR} \cdot TP_I.$$

Finally, the constraints can be rewritten as reliability constraints

$$\mu_I + \sqrt{\frac{1-\varepsilon}{\varepsilon}} \frac{\sigma_I}{\sqrt{W_I}} \leq \hat{LR} \cdot Pr_I. \quad (8)$$

If we set

$$b_I = \ln \left[ \left( \mu_I + \sqrt{\frac{1-\varepsilon}{\varepsilon W_I}} \sigma_I \right) / \hat{LR} \right],$$

we can employ the linear programming formulation (6) for rate estimation.
Optimization approaches to rates estimation

Optimization models – individual chance constraints

Optimization model – individual reliability constraints

\[
\begin{align*}
m\in\mathbb{R} \\
\min_{u} \sum_{I \in \mathcal{I}} w_I e^{u_{i_0} + u_{i_1} + \cdots + u_{i_S}} \\
\text{s.t.} \quad u_{i_0} + u_{i_1} + \cdots + u_{i_S} & \geq b_{i_0, i_1, \ldots, i_S}, \\
u_{i_1} + \cdots + u_{i_S} & \leq \ln(1 + r_{\text{max}}), \\
u_{i_1}, \ldots, u_{i_S} & \geq 0, \ (i_0, i_1, \ldots, i_S) \in \mathcal{I},
\end{align*}
\]

with

\[
b_I = \ln \left[ \left( \mu_I + \sqrt{\frac{1 - \varepsilon}{\varepsilon W_I} \sigma_I} \right) / \hat{L}_R \right].
\]
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In the collective risk model, a probability is prescribed for ensuring that the total losses over the **whole line of business** (LoB) are covered by the premium with a high probability, i.e.

\[
P \left( \sum_{l \in I} L_l^T \leq \sum_{l \in I} W_l Pr_l \right) \geq 1 - \varepsilon.
\]
Zaks et al. (2006) proposed the following program for rate estimation, where the **mean square error** is minimized under the reformulated **collective risk constraint** using the Central Limit Theorem:

\[
\min_{r_{I}} \sum_{I \in \mathcal{I}} \frac{1}{r_{I}} \mathbb{E} \left[ (L_{I}^T - W_{I}P_{r_{I}})^2 \right]
\]

s.t.

\[
\sum_{I \in \mathcal{I}} W_{I}P_{r_{I}} = \sum_{I \in \mathcal{I}} W_{I}\mu_{I} + z_{1-\varepsilon} \sqrt{\sum_{I \in \mathcal{I}} W_{I}\sigma_{I}^2},
\]

where \( r_{I} > 0 \) and \( z_{1-\varepsilon} \) denotes the quantile of the Normal distribution. Various premium principles can be obtained by the choice of \( r_{I} \) (\( r_{I} = 1 \) or \( r_{I} = W_{I} \) leading to semi-uniform or uniform risk allocations).
According to Zaks et al. (2006), Theorem 1, the program has a unique solution

$$\hat{Pr}_I = \mu_I + z_{1-\varepsilon} \frac{r_I \sigma}{rW_I},$$

with $r = \sum_{I \in I} r_I$ and $\sigma^2 = \sum_{I \in I} W_I \sigma^2_I$. If we want to incorporate the prescribed loss ratio $\hat{LR}$ for the whole LoB into the rates, we can set

$$b_I = \ln \left[ \left( \mu_I + z_{1-\varepsilon} \frac{r_I \sigma}{rW_I} \right) / \hat{LR} \right],$$

within the problem (6).
Optimization model – a collective risk constraint

\[
\begin{align*}
\min_u & \quad \sum_{l \in I} w_l e^{u_{i_0} + u_{i_1} + \cdots + u_{i_S}} \\
\text{s.t.} & \quad u_{i_0} + u_{i_1} + \cdots + u_{i_S} \geq b_{i_0, i_1, \ldots, i_S}, \\
& \quad u_{i_1} + \cdots + u_{i_S} \leq \ln(1 + r^{max}), \\
& \quad u_{i_1}, \ldots, u_{i_S} \geq 0, \quad (i_0, i_1, \ldots, i_S) \in I,
\end{align*}
\]

with

\[
b_l = \ln \left[ \left( \mu_l + z_{1-\varepsilon} \frac{r_l \sigma}{\hat{r} W_l} \right) / \hat{LR} \right].
\]
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We consider 60 000 policies with settled claims simulated using characteristics of real MTPL portfolio. The following segmentation variables are used:

- **tariff group**: 5 categories (engine up to 1000, up to 1350, up to 1850, up to 2500, over 2500 ccm),
- **age**: 3 cat. (18-30, 31-65, 66 and more years),
- **region** (reg): 4 cat. (over 500 000, over 50 000, over 5 000, up to 5 000 inhabitants),
- **gender** (gen): 2 cat. (men, women).

Many other available indicators related to a driver (marital status, type of licence), vehicle (engine power, mileage, value), policy (duration, no claim discount). **Real data** for MTPL models: 120 columns and over 8 millions rows.
Software

SAS Enterprise Guide:

- SAS GENMOD procedure (SAS/STAT 9.3) – generalized linear models
- SAS OPTMODEL procedure (SAS/OR 9.3) – nonlinear convex optimization
## Parameter estimates

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Employed models

- **GLM** – The approach based on generalized linear models
- **EV model** – Deterministic optimization model with expected value constraints
- **SP model (ind.)** – Stochastic programming problem with individual reliability constraints $\varepsilon = 0.1$
- **SP model (col.)** – Stochastic programming problem with collective risk constraint $\varepsilon = 0.1$
## Multiplicative tariff of rates

<table>
<thead>
<tr>
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<th>GLM G</th>
<th>GLM IG</th>
<th>EV model G</th>
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### Optimization approaches to rates estimation

- **Numerical comparison**
- **Multiplicative tariff of rates**

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**Martin Branda**

Optimization approaches to rates estimation
Conclusions (open for discussion)

- **EV model** – good start
- **SP model (ind.)** – appropriate for less segmented portfolios with high exposures of tariff cells
- **SP model (col.)** – appropriate for heavily segmented portfolios with low exposures of tariff cells
Optimization approaches to rates estimation

Numerical comparison

References


References


Thank you for your attention. Questions?

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