Sustainable Retirement Spending

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Sustainable retirement spending: the Czech case

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Agenda

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Motivation

Population ageing phenomenon

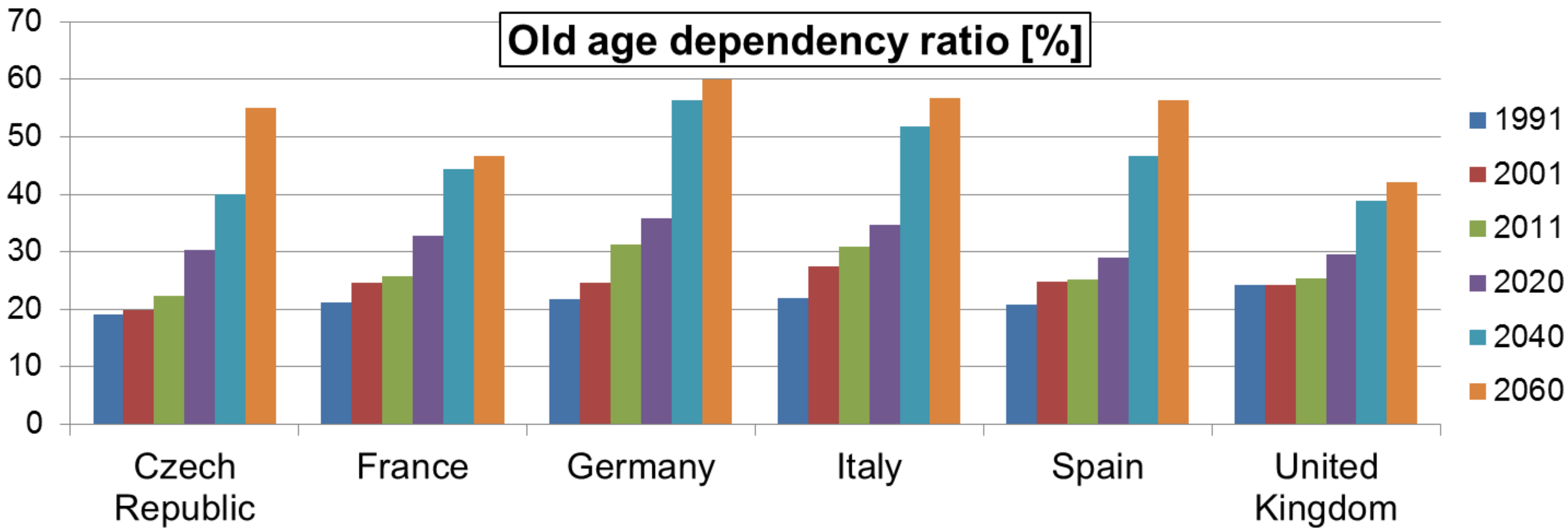
- Decreasing fertility
- Increasing longevity

Social insurance systems predicted to face sustainability problems

- Financed via pay-as-you-go scheme

Future retirees relying on their own lifetime savings to finance retirement consumption

- Need for informed decisions



Source: Eurostat, 2011

2 distinct ways to transform savings into periodic income stream…

Buying a life annuity

- The easiest way to secure a life-contingent cash flow
- Income stream can not be outlived ✓
- Loss of liquidity/flexibility ✗
- Not possible to leave a bequest ✗
- Theoretically should maximize consumer's utility – Yaari (1965)
- Actual worldwide demand for annuities is rather low - "annuity puzzle"

"Self-annuitization"

- i.e. discretionary management of pension funds with periodic withdrawals for purposes of consumption
- Flexibility ✓
- Possibility of leaving a bequest ✓
- Risk that the pension capital runs out prior to death ✗

2 distinct ways to transform savings into periodic income stream…

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- Possibility of leaving a bequest ✓
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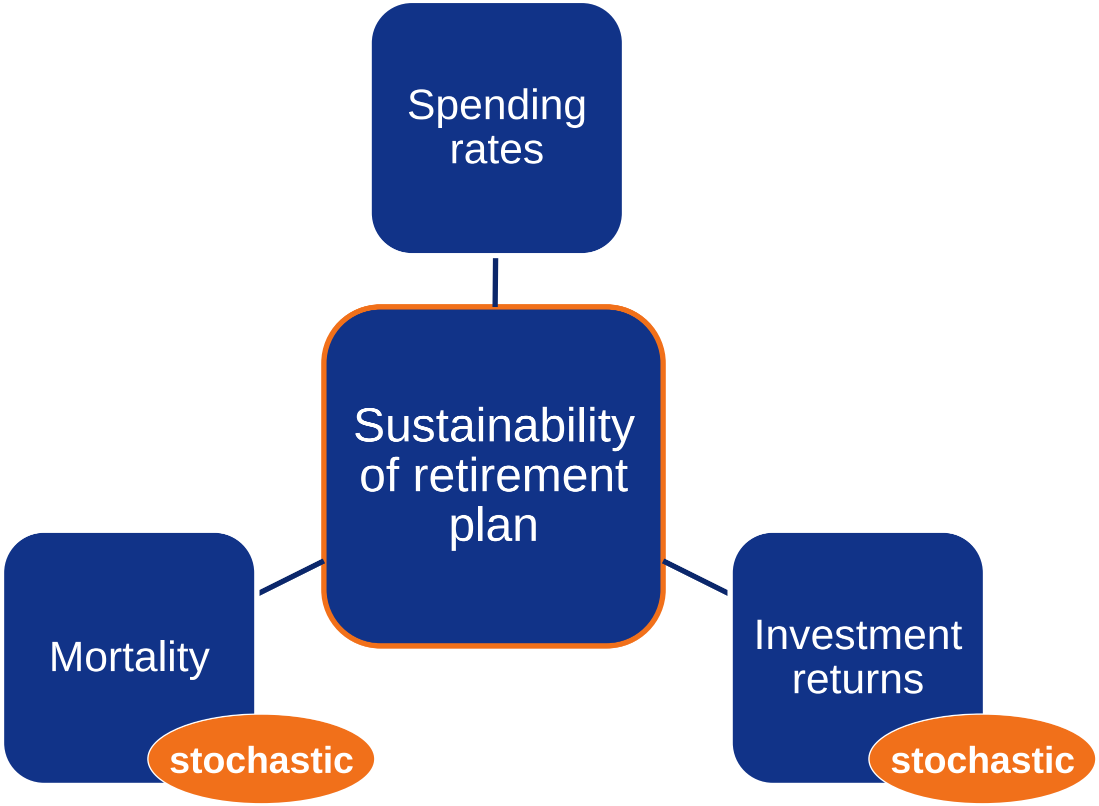
Probability of ruin (PoR)

Key variables of retirement planning



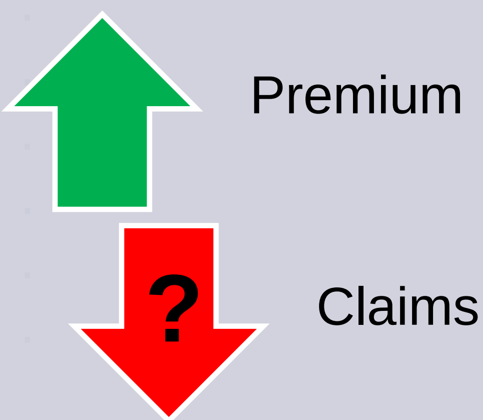
We aim to:

- Link these three key factors in a simple analytic model where the probability that a given retirement plan is sustainable can be determined
- Complement to this probability is the **PoR** (probability of ruin)

Connection to classical ruin theory in insurance

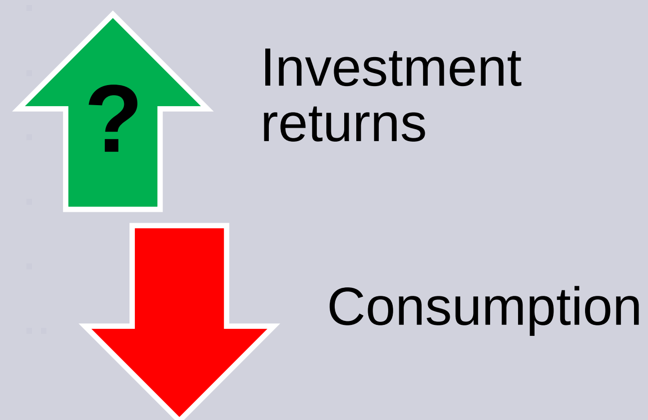
Classical ruin theory

- From perspective of an insurer
- Deterministic income: premium
- Stochastic outcome: claims



Sustainable spending model

- From a consumer perspective
- Stochastic income: investment returns
- Deterministic outcome: consumption



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Simplified deterministic situation

Assume:

- r … fixed inflation-free interest rate
- T … fixed remaining length of life
- c … desired lifelong consumption stream (*c* real CZK per annum)
- w … initial capital (retirement savings)

Then:

Present value of desired future consumption (PV):

$$c\,\bar{a}_{\overline{T}|} = c\int_0^T e^{-rt}\,dt$$

Probability of ruin:

- *PoR(w)* = 1, if *w* < PV
- *PoR(w)* = 0, if *w* >= PV

Stochastic model for mortality

Gompertz-Makeham law of mortality:

$$\lambda(x) = \lambda + \frac{1}{b}\exp\left\{\frac{x-m}{b}\right\}, \qquad x \geq 0$$

- $\lambda(x)$ … instantaneous force of mortality at age x
- $m > 0$ … location parameter
- $b > 0$ … scale parameter
- $\lambda \geq 0$ … component attributable to accidental deaths

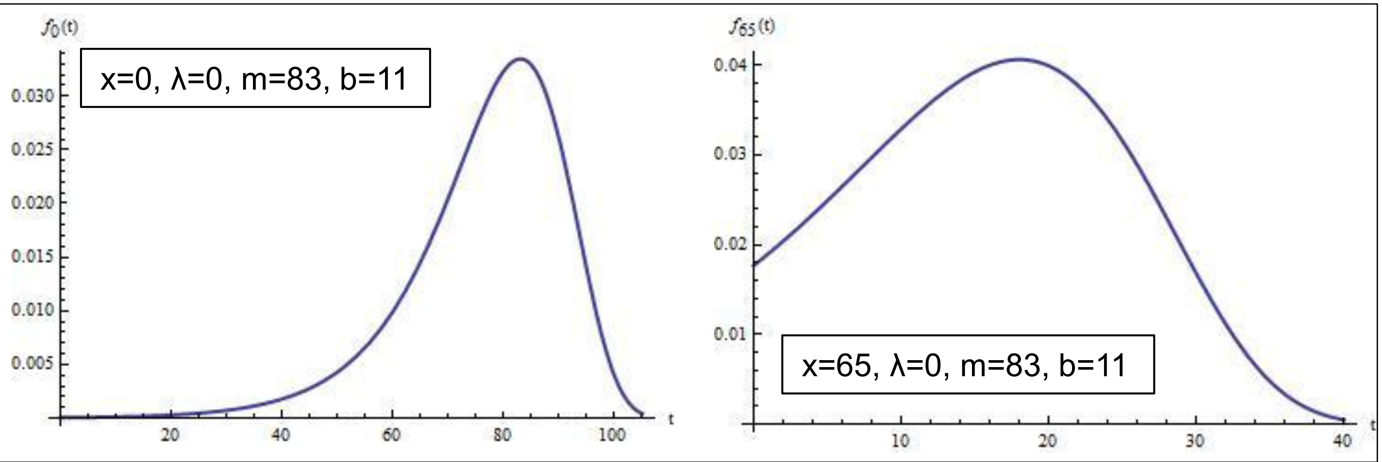
Probability of survival to age *x+t*, conditional on a life at age *x*:

$$_tp_x = \mathrm{P}[T_x > t] = \exp\left\{-\lambda t + e^{(x-m)/b}\left(1 - e^{t/b}\right)\right\}$$

- T_x … remaining lifetime at age *x*

Figure:

Density of T_x



Stochastic model for investment returns

We assume the standard geometric Brownian motion (GBM) model of market prices, i.e. the real value of investment portfolio obeys the SDE:

$$dS_t = \mu S_t \, dt + \sigma S_t \, dB_t, \qquad S_0 = 1$$

- S_t … real (after-inflation) portfolio value at time t
- B_t … standard Brownian motion (Wiener process)
- μ … rate of return (drift coefficient)
- σ … volatility (diffusion coefficient)

Or equivalently:

$$S_t = S_0 e^{\left(\mu - \sigma^2/2\right)t + \sigma B_t}$$

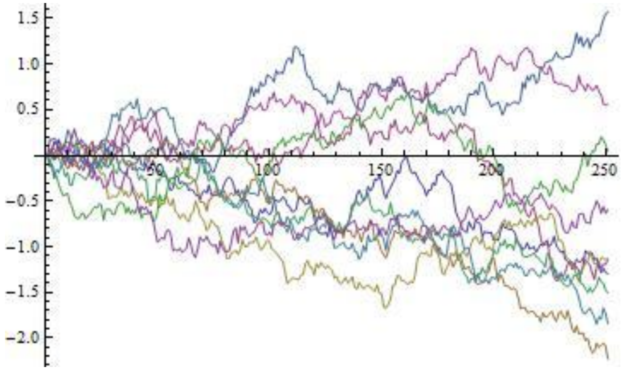
Growth condition:

$$\mu - \sigma^2/2 > 0$$

Brownian motion - illustration

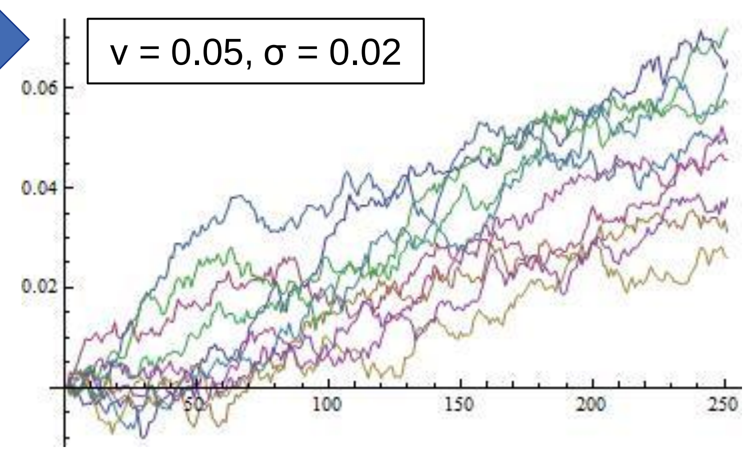
Standard Brownian motion

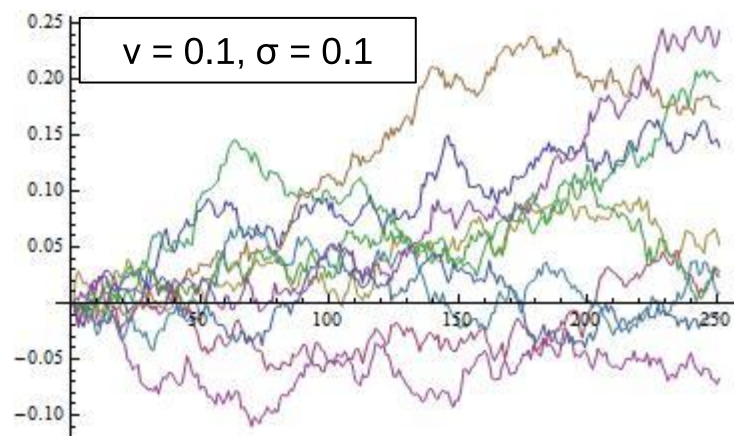
- $B_t \sim N(0, t)$



Non-standard Brownian motion

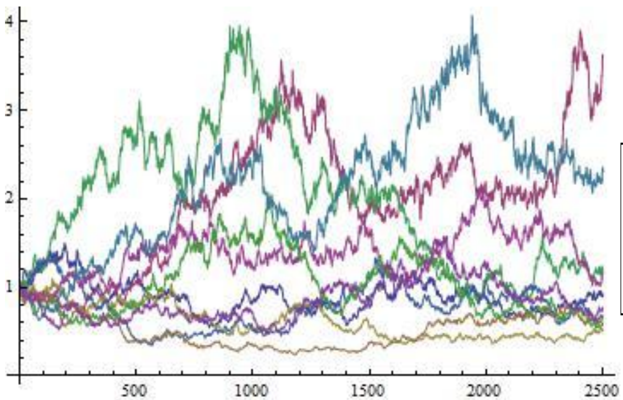
- $B_t^{(\nu,\sigma)} = \nu t + \sigma B_t$
- $B_t^{(\nu,\sigma)} \sim N(\nu t, \sigma^2 t)$





Geometric Brownian motion

- $S_t = S_0 exp\{\nu t + \sigma B_t\}$
- $S_t | S_0 \sim LN(\log S_0 + \nu t, \sigma^2 t)$



v = 0.02
σ = 0.3
S_0 = 1

Net wealth process SDE

If the retiree invests his initial capital *w* in a portfolio satisfying the SDE

$$dS_t = \mu S_t \, dt + \sigma S_t \, dB_t, \qquad S_0 = 1$$

and than consumes a fixed real amount *k* per year, than his net wealth process W_t obeys the following SDE:

$$dW_t = (\mu W_t - k) \, dt + \sigma W_t \, dB_t, \qquad W_0 = w \, .$$

This equation can be solved using a stochastic analogy to the variation of coefficients method (see e.g. Karatzas & Shreve, 1991) to yield an explicit formula for the net wealth process:

$$W_t = S_t \left[w - k \int_0^t \frac{1}{S_u} \, du \right], \qquad t \geq 0 \, .$$

Observe that for all T ∈ $(0, \infty]$ it holds that

$$\mathrm{P}\left[\inf_{0 \leq t \leq T} W_t \leq 0 \middle| W_0 = w \right] = \mathrm{P}\left[W_T \leq 0 | W_0 = w \right] .$$

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Reciprocal gamma distribution

Random variable Y obeys the *reciprocal gamma (RG) distribution* with shape parameter $\alpha > 0$ and scale parameter $\beta > 0$, denoted by $Y \sim RG(\alpha, \beta)$, if $Y = \frac{1}{X}$, where $X \sim Gamma(\alpha, \beta)$.

Consequently, we have for any x>0:

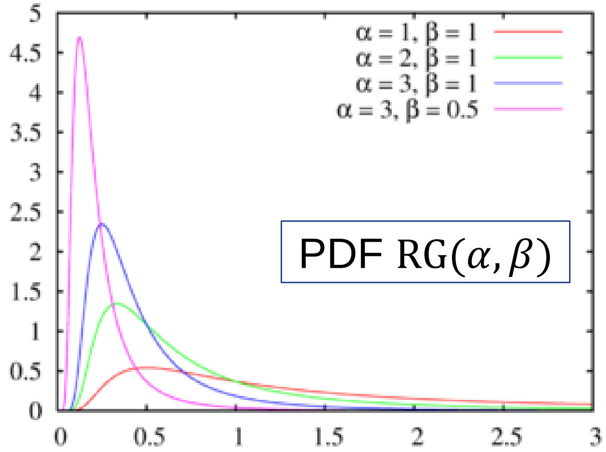
$$G(x;\ \alpha, \beta) = 1 - G_R(1/x;\ \alpha, \beta)$$

- $G(x;\ \alpha, \beta)$ … CDF of gamma distribution
- $G_R(x;\ \alpha, \beta)$ … CDF of reciprocal gamma distribution.

The first two moments of the RG distribution are:

$$E[Y] = \frac{1}{\beta(\alpha - 1)},$$

$$E[Y^2] = \frac{1}{\beta^2(\alpha - 1)(\alpha - 2)}$$



Eventual probability of ruin (EPoR) (1/2)

EPoR = probability that the net wealth process will ever hit zero:

$$\begin{aligned}\mathrm{EPoR}(w) &= \mathsf{P}\left[\inf_{0\le t\le\infty} W_t \le 0 \middle| W_0 = w\right] \\ &= \mathsf{P}\left[W_\infty \le 0 | W_0 = w\right] \\ &= \mathsf{P}\left[\frac{w}{k} \le \underbrace{\int_0^\infty \frac{1}{S_u}\, du}_{=:Z}\right]\end{aligned}$$

- Z … present value of a stochastic perpetuity (PVSP)

Theorem:

The PVSP random variable obeys RG distribution: $Z \sim \mathrm{RG}\left(\frac{2\mu}{\sigma^2} - 1, \frac{\sigma^2}{2}\right)$

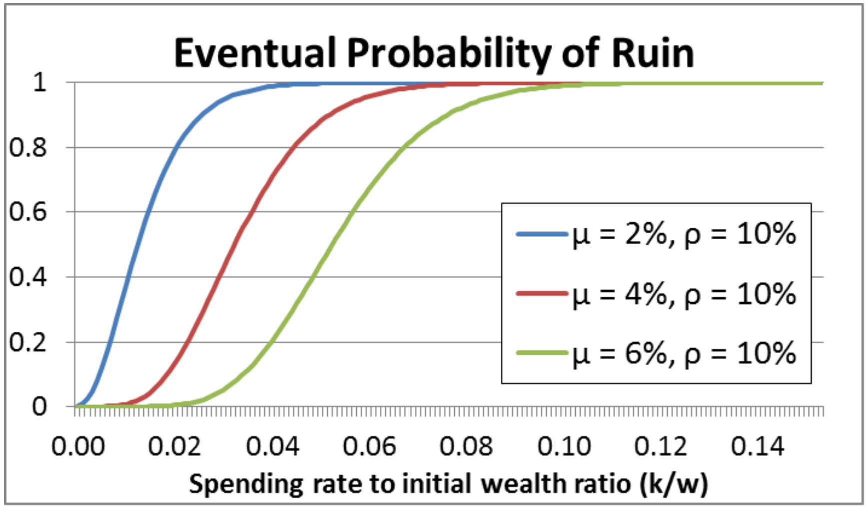
Proof: Milevsky (1997).

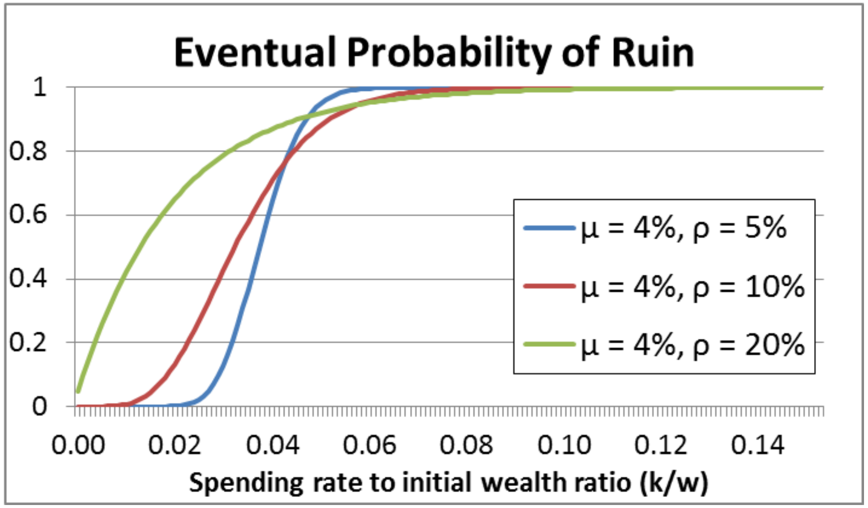
Eventual probability of ruin (EPoR) (2/2)

Since the PVSP random variable Z follows the reciprocal gamma distribution, we can calculate the eventual probability of ruin easily by evaluating the CDF of gamma distribution at $\frac{k}{w}$:

$$\begin{aligned}\mathrm{EPoR}\,(w) &= \mathsf{P}\left[\frac{w}{k} \le Z\right] \\ &= 1 - G_R\left(\frac{w}{k};\ \frac{2\mu}{\sigma^2} - 1, \frac{\sigma^2}{2}\right) = G\left(\frac{k}{w};\ \frac{2\mu}{\sigma^2} - 1, \frac{\sigma^2}{2}\right)\end{aligned}$$

Note: Gamma distribution is available in MS Excel.





Lifetime probability of ruin (LPoR)

LPoR = probability that the net wealth process will hit zero while the retiree is still living.

We have:

$$\begin{aligned}\mathrm{LPoR}(w) &= \mathsf{P}\left[\inf_{0 \le t \le T_x} W_t \le 0 \middle| W_0 = w\right] \\ &= \mathsf{P}\left[W_{T_x} \le 0 | W_0 = w\right] \\ &= \mathsf{P}\left[\frac{w}{k} \le \underbrace{\int_0^{T_x} \frac{1}{S_u}\, du}_{=:Z_{T_x}}\right]\end{aligned}$$

- Z_{Tx} … stochastic present value (SPV) of a life annuity

There is no closed-form density function for SPV. However, its distribution can be estimated…

Reciprocal gamma approximation (1/3)

SPV random variable: $$Z_{T_x} := \int_0^{T_x} e^{-(\mu-\sigma^2/2)s-\sigma B_s}\, ds$$

True distribution of Z_{T_x}: unknown

Approximation technique: moment matching

Candidate distribution: reciprocal gamma distribution (2 degrees of freedom)

Assumptions used: T_x independent of the Brownian motion $\{B_t, t \geq 0\}$ driving S_u

Recall the first two moments of $Y \sim RG(\alpha, \beta)$:

$$M_1 = \mathrm{E}[Y] = \frac{1}{\beta(\alpha-1)},$$

$$M_2 = \mathrm{E}[Y^2] = \frac{1}{\beta^2(\alpha-1)(\alpha-2)}$$

Solving for α and β yields:

$$\alpha = \frac{2M_2 - M_1^2}{M_2 - M_1^2}, \qquad \beta = \frac{M_2 - M_1^2}{M_2 M_1}$$

Reciprocal gamma approximation (2/3)

Calculate the first two moments of Z_{T_x}:

$$\tilde{M}_1 = \mathrm{E}\left[Z_{T_x}\right] = \cdots = A(\mu - \sigma^2|\cdot)$$

$$\tilde{M}_2 = \mathrm{E}\left[Z_{T_x}^2\right] = \cdots = \frac{2}{\mu - 2\sigma^2}\left[A(\mu - \sigma^2|\cdot) - A(2\mu - 3\sigma^2|\cdot)\right]$$

where we defined

$$A\left(\xi|\cdot\right) := \int_0^\infty \exp\left\{-\xi s\right\}{}_s p_x \, ds$$

as the price of a continuous life annuity under a continuously compounded interest rate ξ.

In case T_x follows the Gompertz-Makeham law of mortality, we have

$$A\left(\xi|\lambda, m, b, x\right) = b \exp\left\{e^{(x-m)/b} + (\xi + \lambda)(x - m)\right\}\Gamma(-(\xi + \lambda)b, e^{(x-m)/b})$$

where

$$\Gamma\left(a, c\right) = \int_c^\infty e^{-t} t^{a-1} \, dt$$

stands for the (upper) incomplete Gamma function.

Reciprocal gamma approximation (3/3)

Now the fitted parameter values of RG distribution can be calculated as

$$\hat{\alpha} = \frac{2\tilde{M}_2 - \tilde{M}_1^2}{\tilde{M}_2 - \tilde{M}_1^2}, \qquad \hat{\beta} = \frac{\tilde{M}_2 - \tilde{M}_1^2}{\tilde{M}_2 \tilde{M}_1}$$

and we can conclude that the approximate distribution of the SPV random variable Z_{T_x} is

$$Z_{T_x} \dot{\sim} \mathrm{RG}(\hat{\alpha}, \hat{\beta}).$$

Finally, we can express the approximated lifetime probability of ruin:

$$\begin{aligned} \mathrm{LPoR}(w) &= \mathsf{P}\left[\frac{w}{k} \le Z_{T_x}\right] \\ &\cong 1 - G_R\left(\frac{w}{k};\ \hat{\alpha}, \hat{\beta}\right) = G\left(\frac{k}{w};\ \hat{\alpha}, \hat{\beta}\right) \end{aligned}$$

Figure: RG approximation of the LPoR

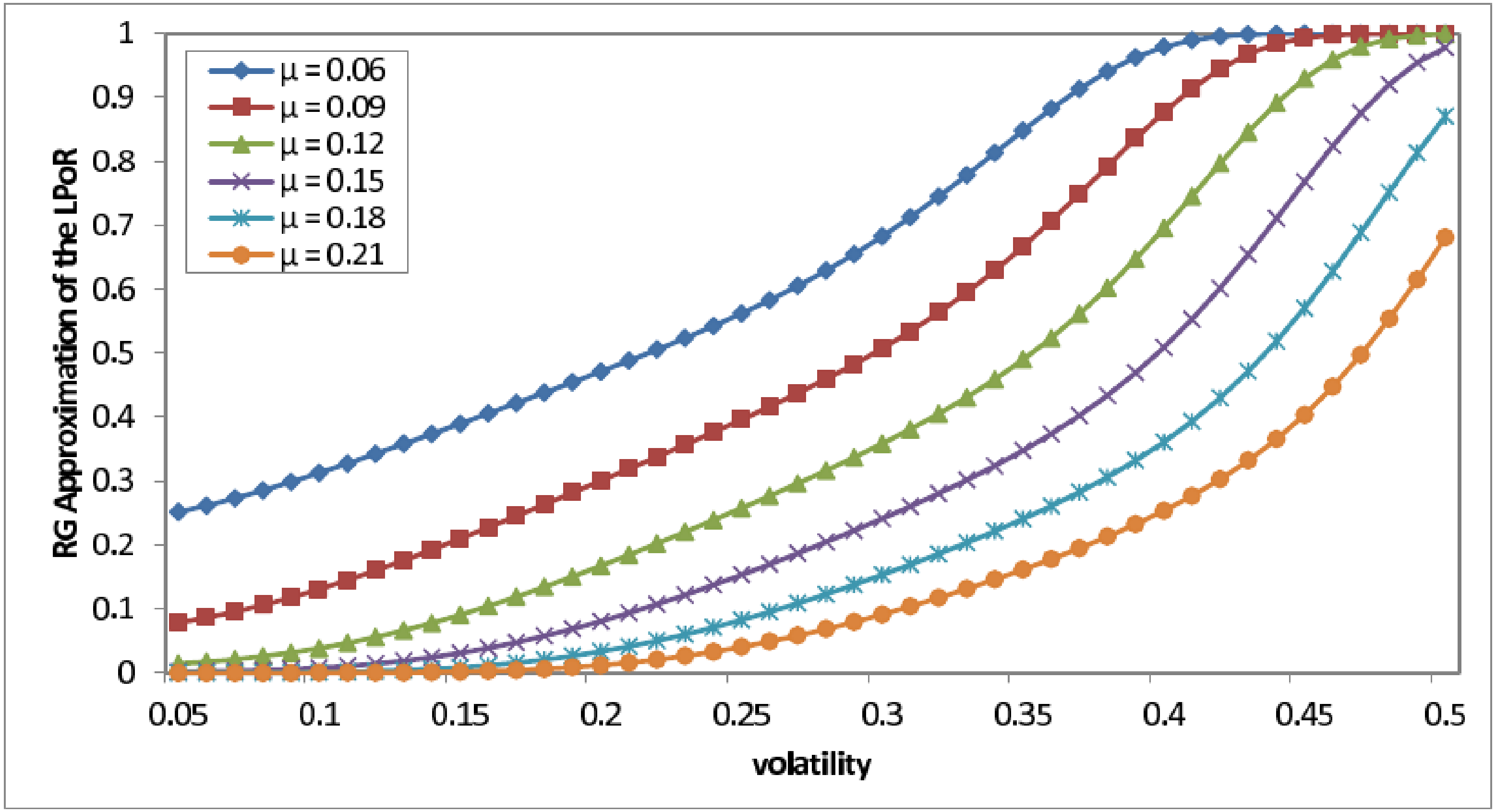


Figure 4.1: The RG approximation of the LPoR as a function of the investment volatility σ, for various values of the expected real rate of return μ. A 65-year-old individual with an initial wealth $w = 12$ CZK, who plans to consume $k = 1$ real CZK per year was assumed. The parameters of the Gompertz distribution were set to $m = 85$ and $b = 9$.

Sustainable spending rate

The RG formula for life time probability of ruin

$$\mathrm{LPoR}(w) \cong G\left(\frac{k}{w};\ \hat{\alpha}, \hat{\beta}\right)$$

can be inverted to yield the so-called maximal sustainable spending rate (SSR):

$$k \cong w \cdot G^{-1}\left(\mathrm{LPoR}(w);\ \hat{\alpha}, \hat{\beta}\right).$$

Interpretation:

For a fixed initial capital *w, SSR* is the maximal annual spending rate, so that the corresponding consumption plan results in a chosen tolerated probability of ruin *LPoR(w)*.

Note:

$G^{-1}(\cdot\,;\ \alpha, \beta)$ … quantile function of gamma distribution (available in MS Excel)

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Monte Carlo simulations

1.
- Generate time of death T_x
- Sampling from Gompertz distribution

2.
- Generate path of the net wealth process W_t
- Repeated sampling from normal distribution

3.
- Decide if ruin of the individual occurred
- Did W_t cross zero for some $t_0 < T_x$?

- For a given set of parameters independently repeat steps 1. to 3. for *n* times
- Monte Carlo LPoR approximation:

$$MC\ LPoR = \frac{\#\ observed\ events\ of\ ruin}{n}$$

- In the following analysis: n = 10000

Convergence of the MC method

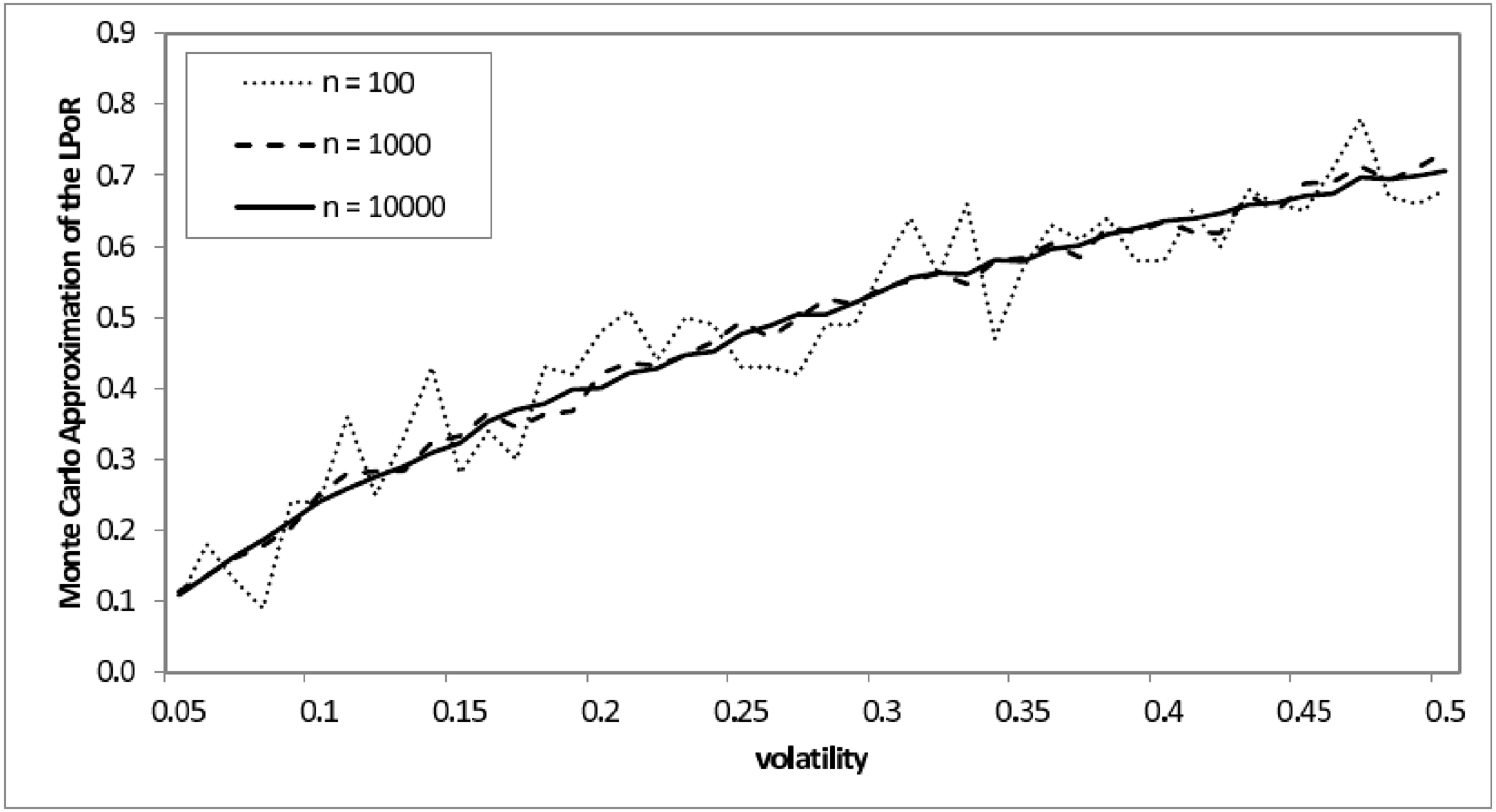


Figure 5.1: The speed of convergence of the Monte Carlo LPoR approximation. The figure displays the MC approximation of the LPoR as a function of the investment volatility σ, for various number of simulations n. A 65-year-old individual with an initial wealth $w = 10$ CZK, who plans to consume $k = 1$ real CZK per year was assumed. The parameters of the Gompertz distribution were set to $m = 85$ and $b = 9$. The assumed expected real rate of investment return is $\mu = 0.1$.

Monte Carlo approximation of the LPoR

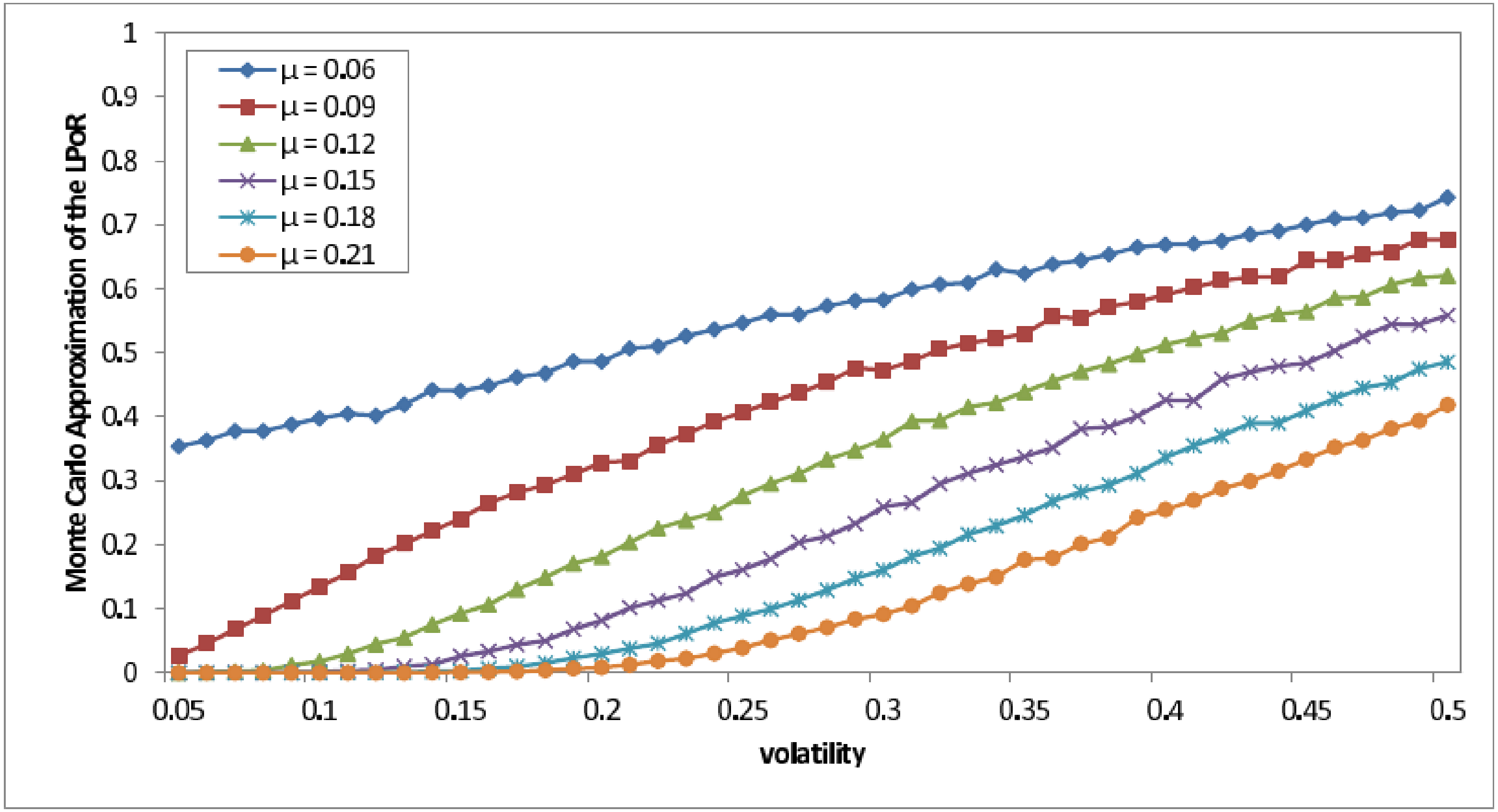


Figure 5.2: The MC approximation of the LPoR as a function of the investment volatility σ, for various values of the expected real rate of return μ. A 65-year-old individual with an initial wealth $w = 12$ CZK, who plans to consume $k = 1$ real CZK per year was assumed. The parameters of the Gompertz distribution were set to $m = 85$ and $b = 9$.

Difference between the RG and MC approximation (1/3)

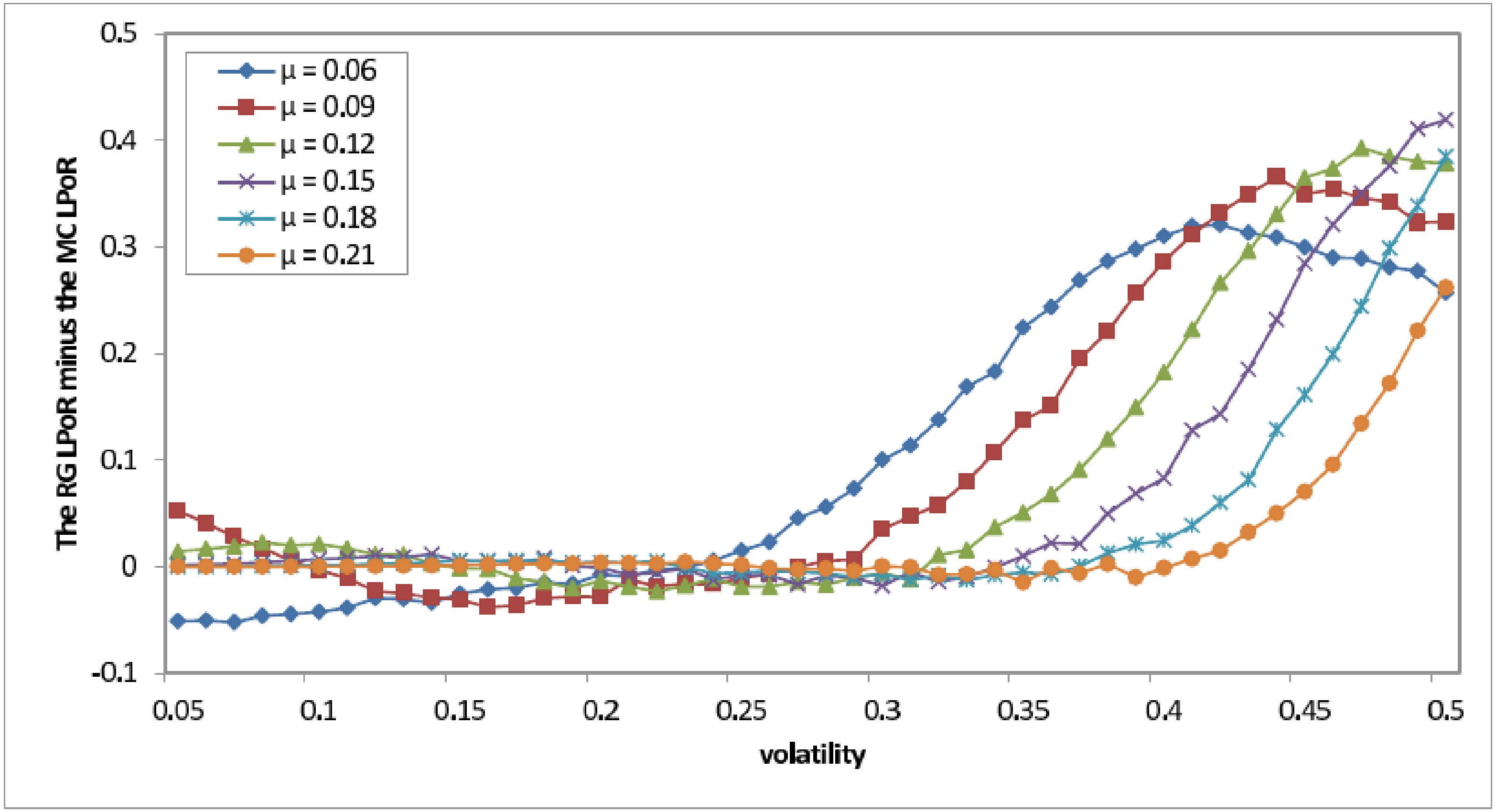


Figure 5.3: The discrepancy between the RG and the MC approximations of the LPoR as a function of the investment volatility σ, for various values of the expected real rate of return μ. A 65-year-old individual with an initial wealth $w = 12$ CZK, who plans to consume $k = 1$ real CZK per year was assumed. The parameters of the Gompertz distribution were set to $m = 85$ and $b = 9$.

Difference between the RG and MC approximation (2/3)

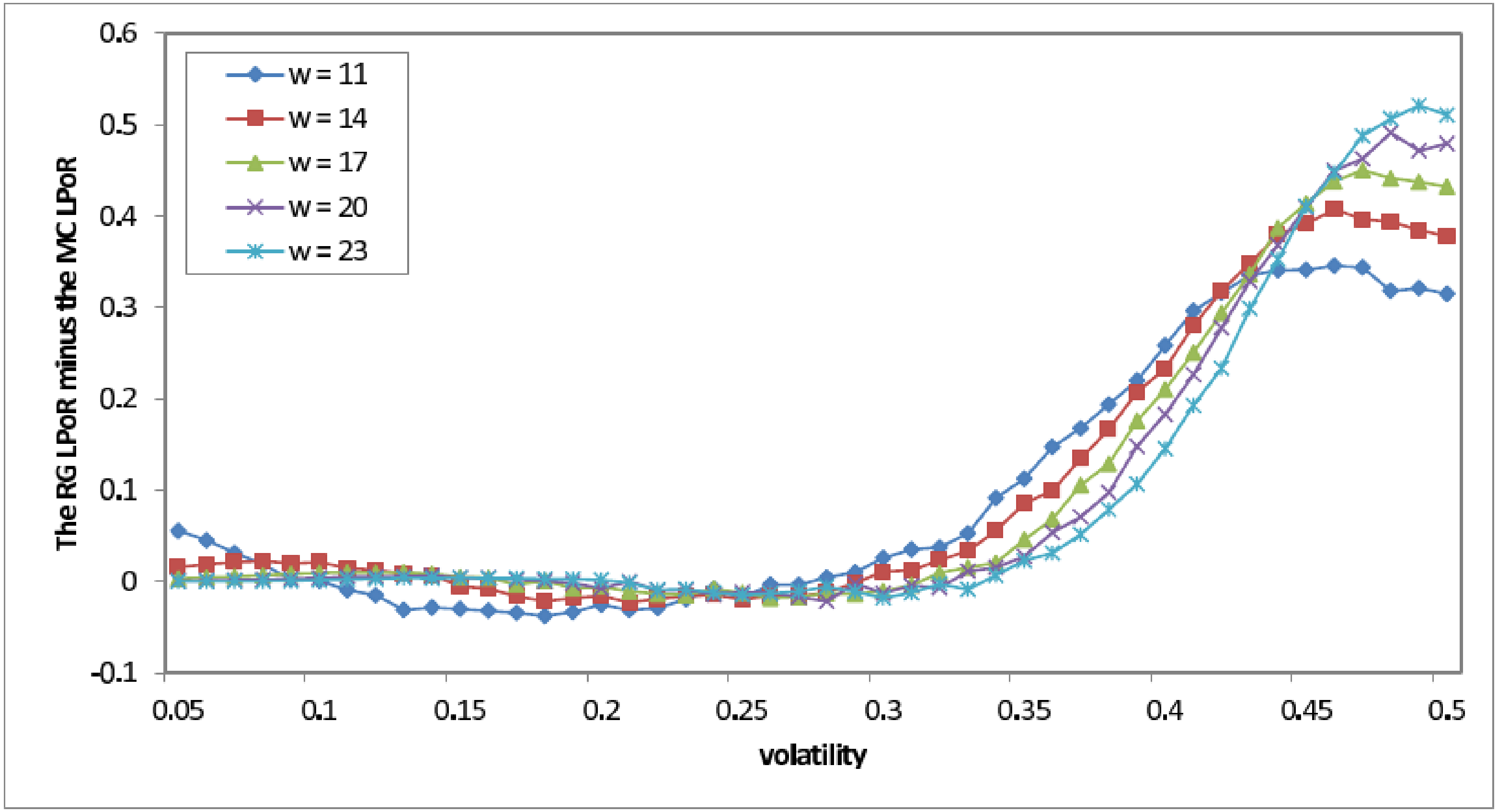


Figure 5.4: The discrepancy between the RG and the MC approximations of the LPoR as a function of the investment volatility σ, for various values of the initial wealth w. A 65-year-old individual, who plans to consume $k = 1$ real CZK per year was assumed. The parameters of the Gompertz distribution were set to $m = 85$ and $b = 9$. The expected real rate of investment return was set to $\mu = 0.1$.

Difference between the RG and MC approximation (3/3)

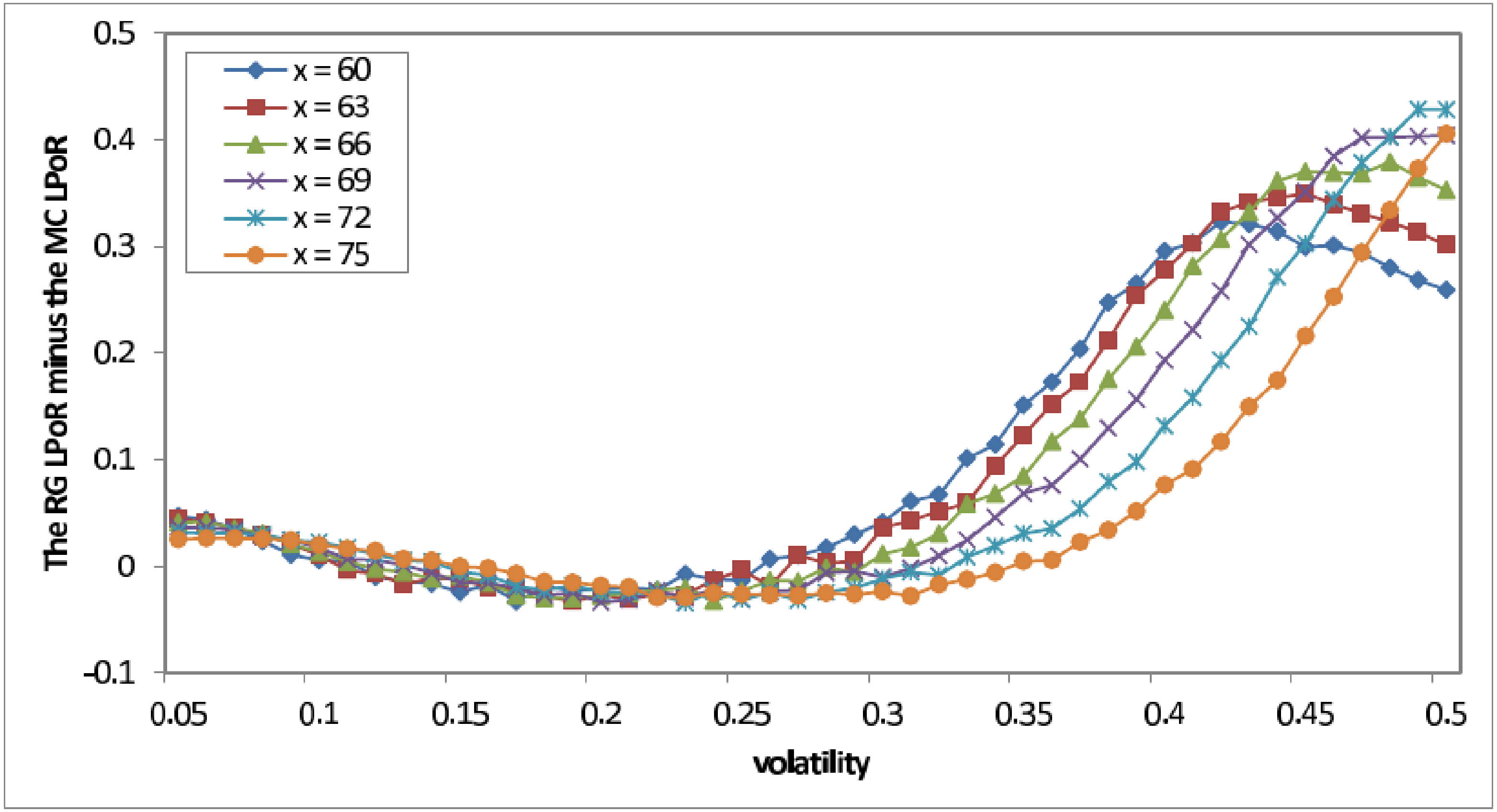


Figure 5.5: The discrepancy between the RG and the MC approximations of the LPoR as a function of the investment volatility σ, for various values of the age x of the retiree. An individual with the initial wealth of $w = 12$ CZK, who plans to consume $k = 1$ real CZK per year was assumed. The parameters of the Gompertz distribution were set to $m = 85$ and $b = 9$. The expected real rate of investment return was set to $\mu = 0.1$.

Accuracy of the RG approximation

In general, the RG approximation is reasonably accurate as long as *σ* does not increase beyond a certain level, at which the approximation starts to overrate the *LPoR*.

Recall the second moment of SPV:

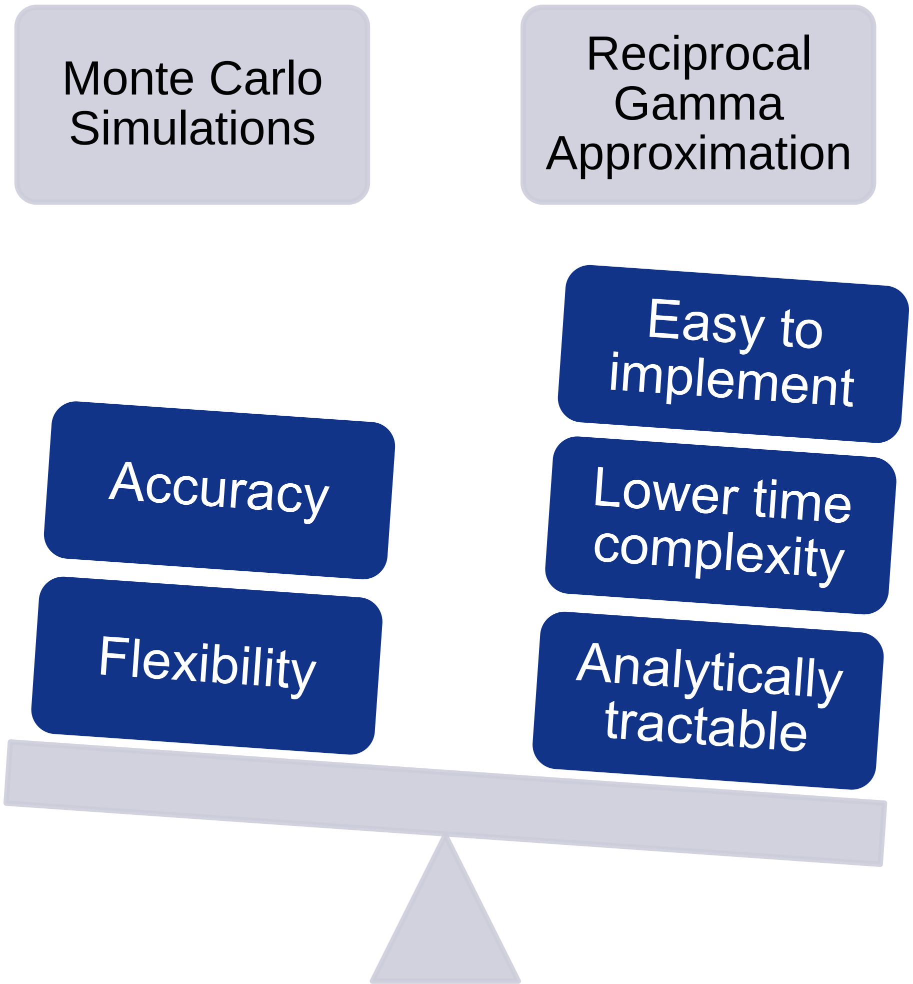
$$\mathsf{E}\left[Z_{T_x}^2\right] = \frac{2}{\mu - 2\sigma^2}\int_0^\infty \left(e^{-(\mu-\sigma^2)s} - e^{-(2\mu-3\sigma^2)s}\right) {}_sp_x \, ds.$$

In case of high *σ* relative to *μ*, the second moment of SPV becomes large because of the second exponential term in the integrand and hence the moment matching approximation deteriorates. Due to the ${}_sp_x$ factor, the exact threshold depends also on *x* and on mortality law parameters.

Rule of thumb:

The RG approximation of *LPoR* deteriorates for $\sigma > \sqrt{2\mu/3}$.

Benefits of RG approximation over Monte Carlo



4

Estimation of Gompertz parameters

Define loss function:

$$L(m,b) := \sum_{x=x_0}^{\omega} \sqrt{D_x} \left| 1 - \frac{q_x(m,b)}{\hat{q}_x} \right|$$

- $\hat{q}_x,\ x = x_0, \dots, \omega$ … death rate at age *x* (life table, ω … ultimate age)
- $D_x,\ x = x_0, \dots, \omega$ … total amount of death claims associated with rate $\hat{q}_x$
- $\hat{q}_x(m,b)$ … analytic Gompertz death rate at age *x*

and set:

$$(\hat{m}, \hat{b}) := \arg\min_{m \geq 0,\, b \geq 0} L(m,b)$$

Results:

Life table Czech Republic (2011), $x_0 = 60,\ \omega = 105$

- female: $\hat{m}_F = 87.9,\ \hat{b}_F = 7.6$
- male: $\hat{m}_M = 82.5,\ \hat{b}_M = 10.5$

Fit to female life table (Czech Republic, 2011)

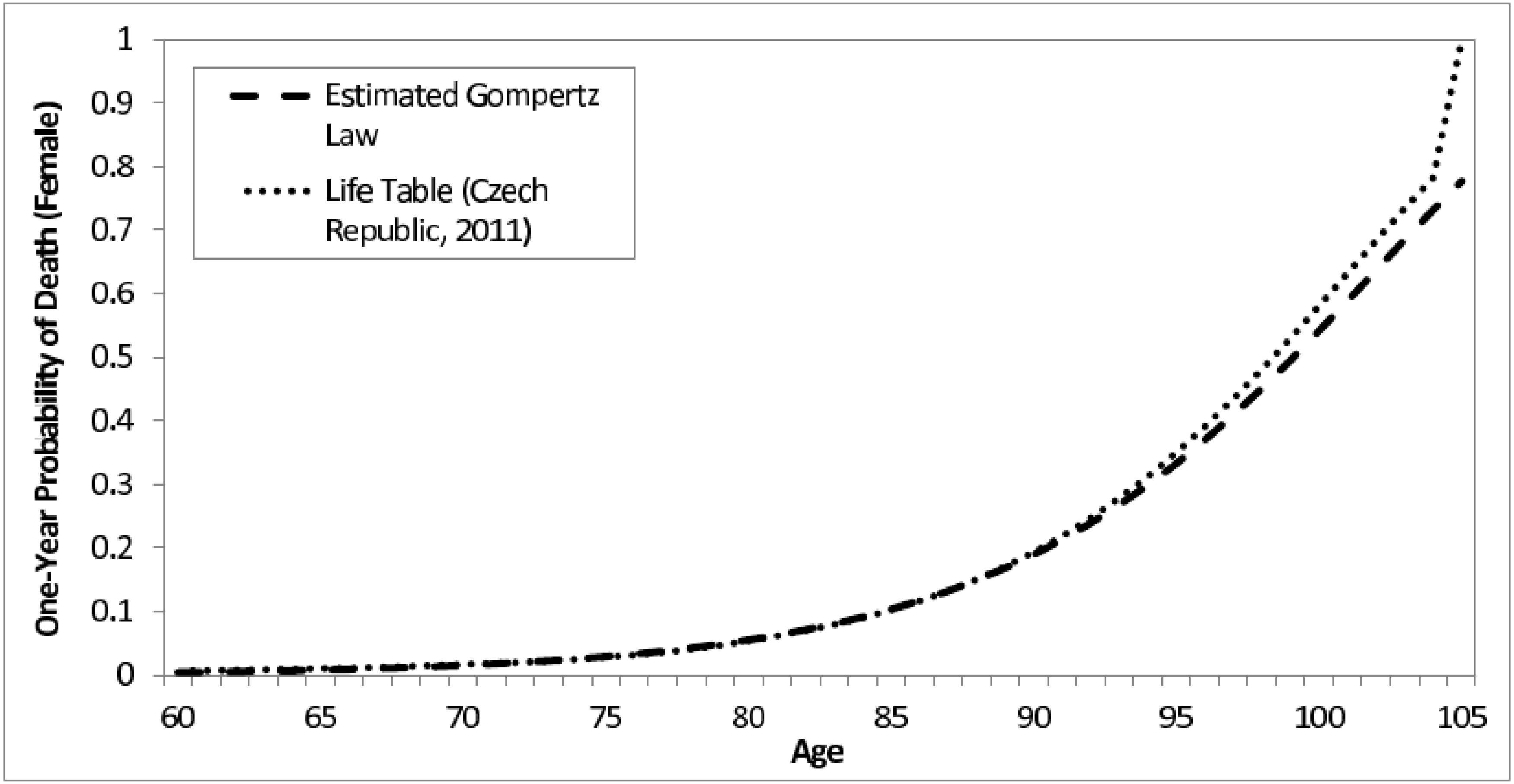


Figure 6.2: An illustration of the Gompertz law fit to the female life table (Czech Republic, 2011). The estimated parameter values are $\hat{m}_F = 87.87$ and $\hat{b}_F = 7.64$.

Fit to male life table (Czech Republic, 2011)

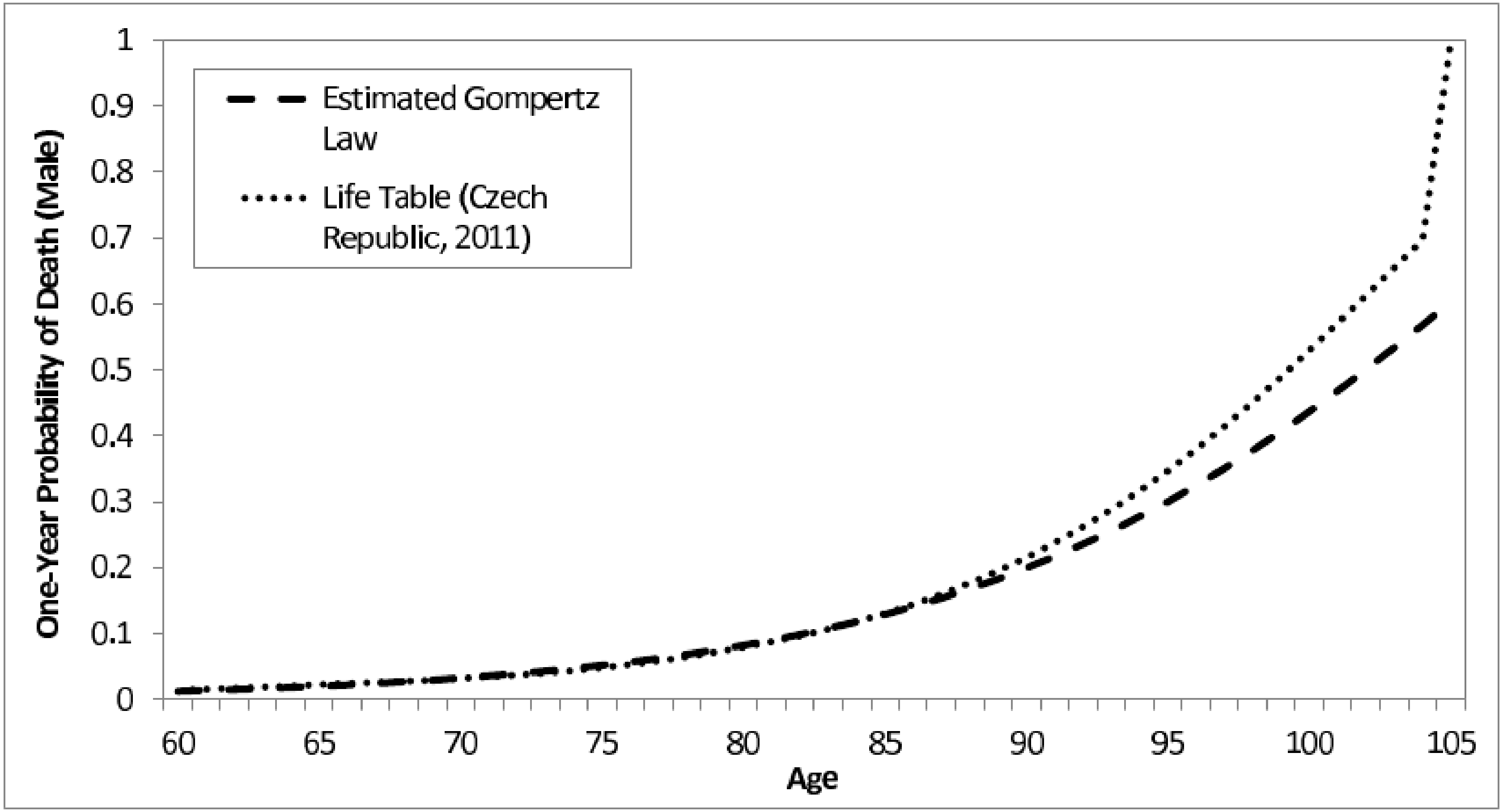


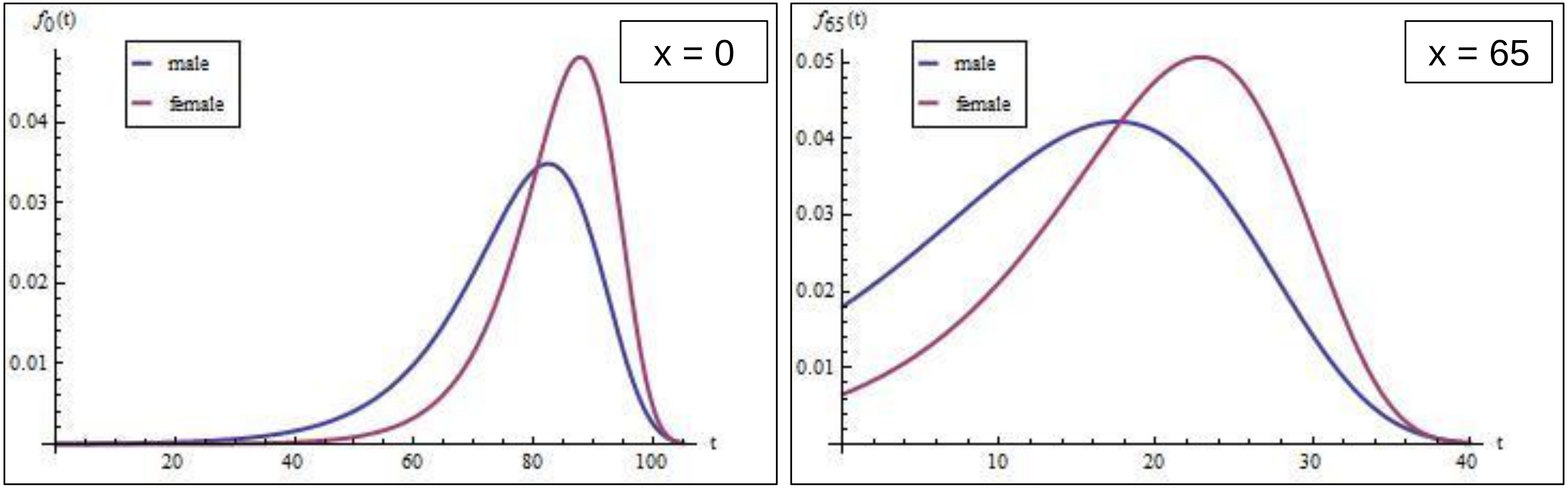
Figure 6.1: An illustration of the Gompertz law fit to the male life table (Czech Republic, 2011). The estimated parameter values are $\hat{m}_M = 82.51$ and $\hat{b}_M = 10.54$.

Estimated density of remaining lifetime

Gompertz law of mortality

- Life table Czech Republic (2011)
- Female: m = 87.9, b = 7.7
- Male: m = 82.5, b = 10.5

Figure: density of T_x



Financial portfolio constructed by combination of 2 assets

Reinvestment Savings Government Bond of the Czech Republic

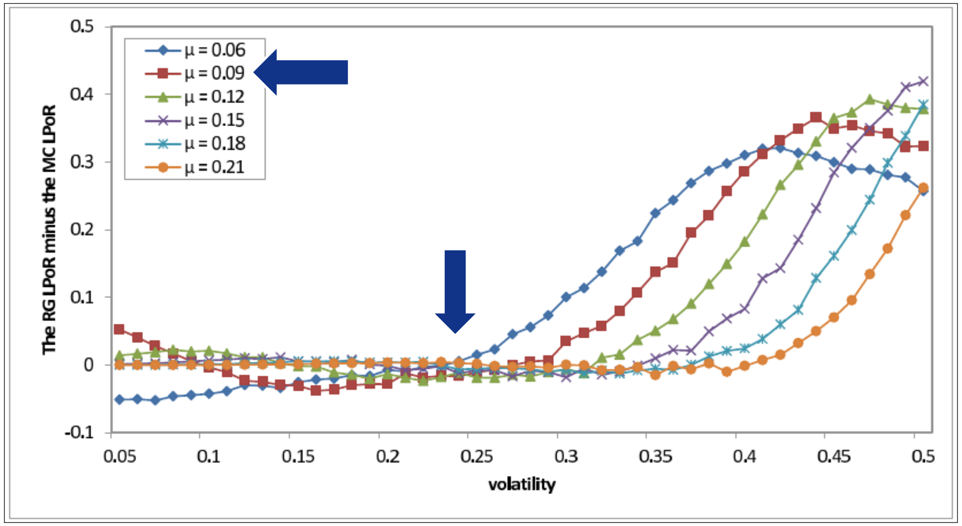
- Issued by the Ministry of Finance
- 5-year maturity; tranche from May, 2013
- Fixed avg. real rate of return: $\mu_B = 1.074\%$

PX index (official index of the Prague Stock Exchange)

- Historical (after-inflation) prices observed between 2/1/2003 and 28/12/2012
- Expected return and volatility estimated by moment matching technique (see e.g. Cipra, 2008)
- $\hat{\mu}_{PX} = 8.79\%,\ \hat{\sigma}_{PX} = 24.5\%$

Allocation to bonds	μ (drift)	σ (volatility)
0%	0.08787	0.24475
20%	0.07244	0.19580
40%	0.05702	0.14685
60%	0.04159	0.09790
80%	0.02616	0.04895
100%	0.01074	0.00000

Note: The residual is allocated to the PX index.



Historical prices: Prague Exchange index

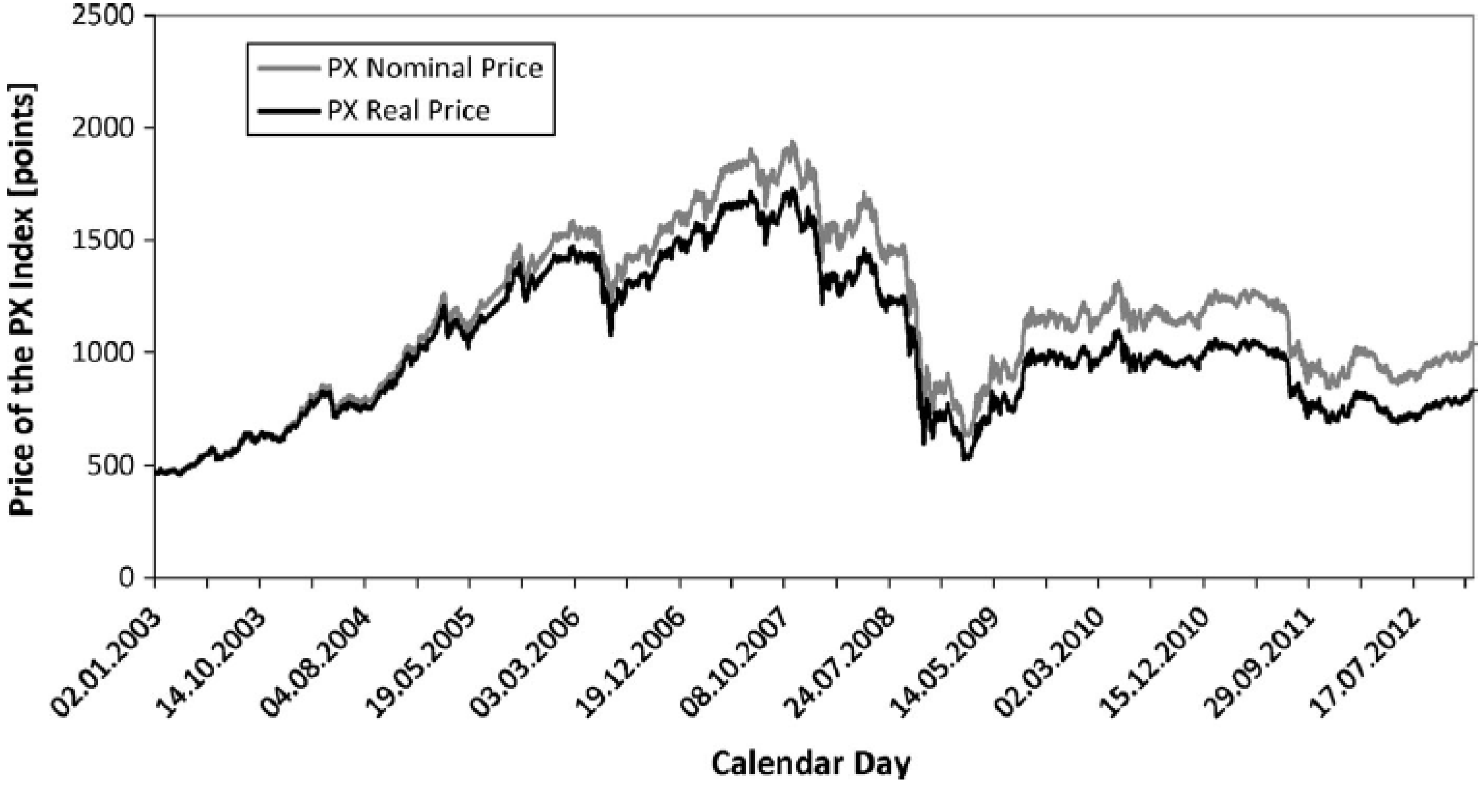


Fig. 3 Development of the daily Prague Exchange (PX) index price over the last ten years (2/1/2003–28/12/2012). The real price was calculated by subtracting the (monthly) inflation from the nominal price. *Source*: PSE (http://www.bcpp.cz), CZSO (http://www.czso.cz)

Figure: Sustainable spending rate in the Czech Republic

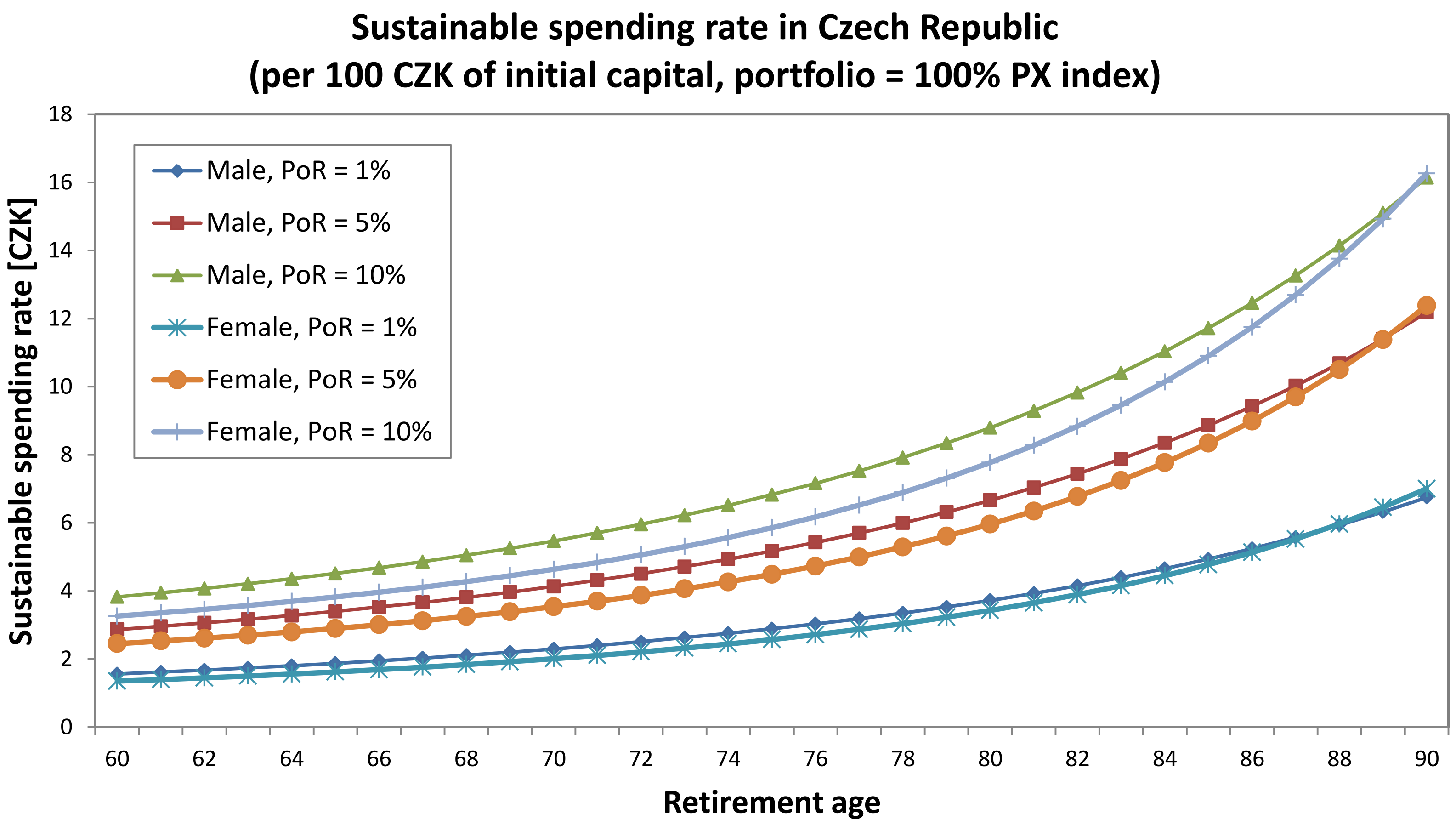


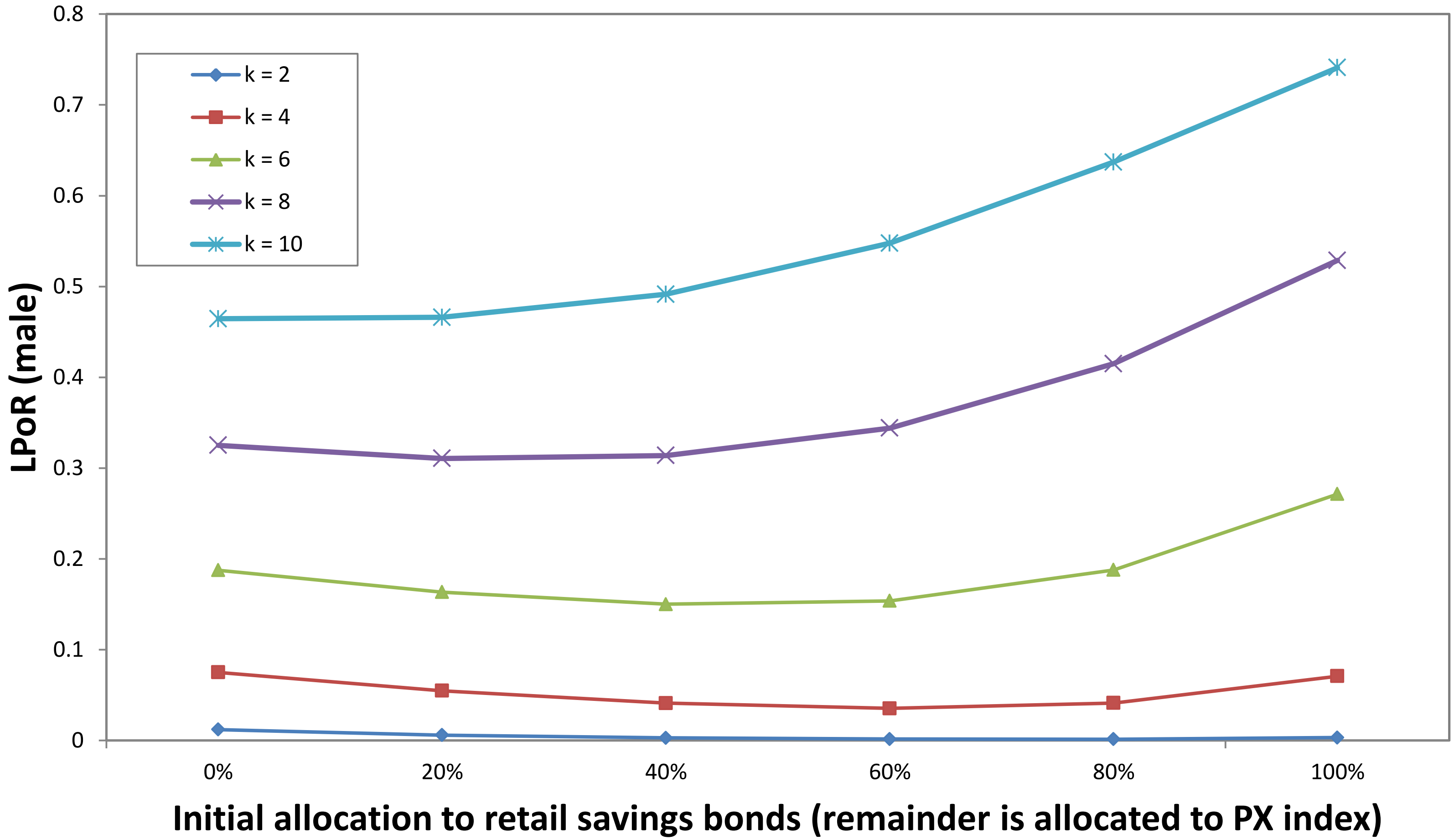
Table: Sustainable spending rate in the Czech Republic

Age at retirement	Tolerated probability of ruin							
	1 %		5 %		10 %		20 %	
	M	F	M	F	M	F	M	F
60	2.69	2.64	3.85	3.56	4.60	4.13	5.63	4.91
61	2.74	2.69	3.95	3.64	4.72	4.23	5.79	5.03
62	2.80	2.74	4.04	3.72	4.84	4.33	5.96	5.16
63	2.85	2.79	4.14	3.81	4.98	4.44	6.14	5.31
64	2.91	2.85	4.25	3.90	5.12	4.56	6.33	5.46
65	2.98	2.90	4.37	4.00	5.27	4.69	6.54	5.63
66	3.04	2.97	4.49	4.10	5.44	4.82	6.76	5.80
67	3.12	3.03	4.62	4.22	5.61	4.97	6.99	6.00
68	3.19	3.10	4.76	4.34	5.79	5.13	7.24	6.21
69	3.27	3.18	4.91	4.47	5.99	5.30	7.51	6.43
70	3.36	3.26	5.07	4.61	6.21	5.48	7.80	6.68
71	3.46	3.34	5.25	4.77	6.44	5.68	8.12	6.94
72	3.56	3.43	5.43	4.93	6.68	5.89	8.45	7.23
73	3.66	3.53	5.63	5.11	6.95	6.13	8.81	7.55
74	3.78	3.64	5.85	5.31	7.23	6.39	9.20	7.89
75	3.90	3.76	6.08	5.52	7.54	6.67	9.62	8.27
76	4.04	3.89	6.32	5.75	7.87	6.97	10.07	8.68
77	4.18	4.03	6.59	6.01	8.22	7.31	10.56	9.14
78	4.34	4.18	6.88	6.29	8.61	7.68	11.09	9.64
79	4.51	4.35	7.19	6.59	9.03	8.08	11.66	10.19

The table displays the maximal real annual spending (k) [in CZK] per 100 CZK of initial wealth (w) for a given tolerated lifetime probability of ruin. An investment portfolio with initial 60 % allocation to retail savings bonds and 40 % to PX index was assumed

M male, *F* female. The values are in CZK

Impact of investment strategy on the LPoR for various spending rates (per 100 CZK of initial capital)



Optimal investment allocation for retiring Czech females

Age at retirement	PoR = 1 %		PoR = 5 %		PoR = 10 %		PoR = 20 %	
	RSB (%)	SSR	RSB (%)	SSR	RSB (%)	SSR	RSB (%)	SSR
60	75	2.75	67	3.58	60	4.13	45	4.97
61	75	2.79	67	3.65	59	4.23	45	5.10
62	74	2.83	66	3.73	59	4.33	45	5.23
63	74	2.88	66	3.82	59	4.44	45	5.38
64	73	2.93	65	3.91	58	4.56	45	5.54
65	73	2.98	65	4.01	58	4.69	44	5.70
66	72	3.03	64	4.11	57	4.82	44	5.89
67	72	3.09	64	4.22	57	4.97	44	6.08
68	71	3.16	63	4.34	57	5.13	44	6.29
69	71	3.23	63	4.47	56	5.30	43	6.52
70	70	3.30	62	4.61	56	5.49	43	6.77
71	69	3.38	62	4.77	55	5.69	43	7.04
72	69	3.47	61	4.93	55	5.91	43	7.33
73	68	3.56	61	5.11	54	6.14	42	7.65
74	68	3.67	60	5.31	54	6.40	42	8.00
75	67	3.78	60	5.52	53	6.68	42	8.38
76	67	3.90	59	5.75	53	6.99	41	8.80
77	66	4.04	58	6.01	52	7.33	41	9.26
78	66	4.19	58	6.29	52	7.70	41	9.77
79	65	4.36	57	6.60	51	8.11	40	10.33

For a given age and tolerated PoR the table displays an optimal initial allocation to retail savings bonds and the corresponding sustainable spending rate (SSR). The rest of the portfolio is allocated to the PX index. The SSR values are in Czech crowns (CZK) per 100 CZK of initial capital

RSB Retail Savings Bonds, *SSR* Sustainable Spending Rate

Optimal investment allocation for retiring Czech males

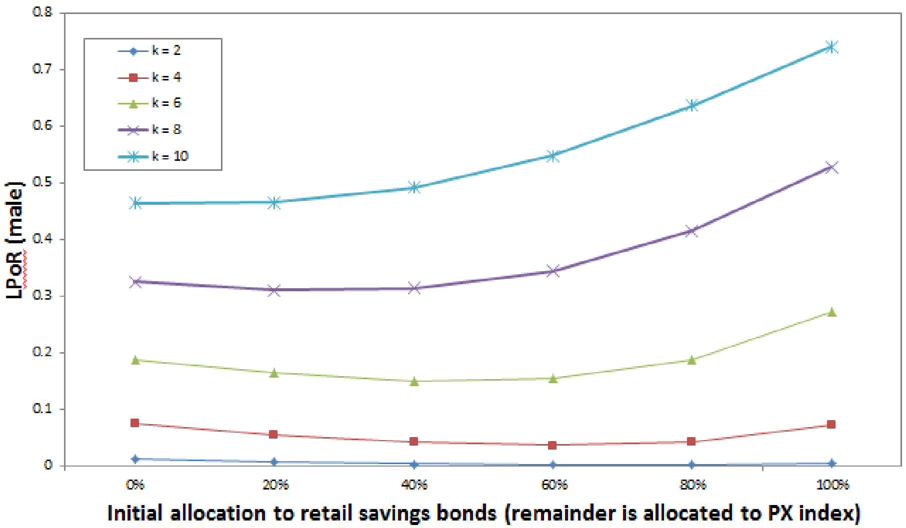
Age at retirement	PoR = 1 %		PoR = 5 %		PoR = 10 %		PoR = 20 %	
	RSB (%)	SSR	RSB (%)	SSR	RSB (%)	SSR	RSB (%)	SSR
60	69	2.73	61	3.86	54	4.61	42	5.74
61	69	2.78	61	3.95	54	4.73	42	5.90
62	68	2.83	61	4.04	54	4.86	41	6.08
63	68	2.88	60	4.14	54	4.99	41	6.26
64	68	2.94	60	4.25	53	5.14	41	6.45
65	67	3.00	59	4.37	53	5.29	41	6.66
66	67	3.06	59	4.49	53	5.46	41	6.88
67	66	3.13	59	4.62	52	5.63	40	7.12
68	66	3.21	58	4.76	52	5.82	40	7.38
69	66	3.29	58	4.91	52	6.02	40	7.65
70	65	3.37	58	5.08	51	6.24	40	7.94
71	65	3.46	57	5.25	51	6.47	40	8.26
72	64	3.56	57	5.44	51	6.71	39	8.60
73	64	3.67	56	5.64	50	6.98	39	8.96
74	64	3.78	56	5.85	50	7.27	39	9.36
75	63	3.91	56	6.08	50	7.58	39	9.78
76	63	4.04	55	6.33	49	7.91	39	10.23
77	62	4.19	55	6.60	49	8.27	38	10.73
78	62	4.34	55	6.89	49	8.66	38	11.26
79	62	4.51	54	7.21	48	9.08	38	11.83

For a given age and tolerated PoR the table displays an optimal initial allocation to retail savings bonds and the corresponding sustainable spending rate. The rest of the portfolio is allocated to the PX index. The SSR values are in CZK per 100 CZK of initial capital

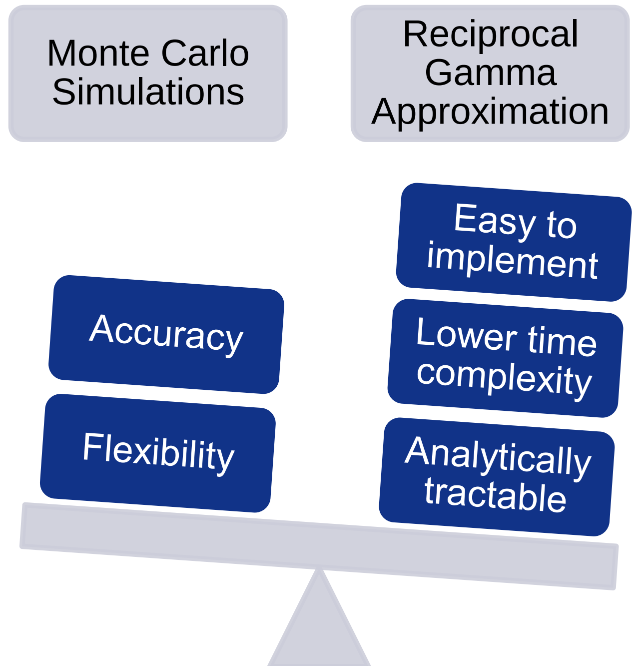
RSB Retail Savings Bonds, *SSR* Sustainable Spending Rate

5

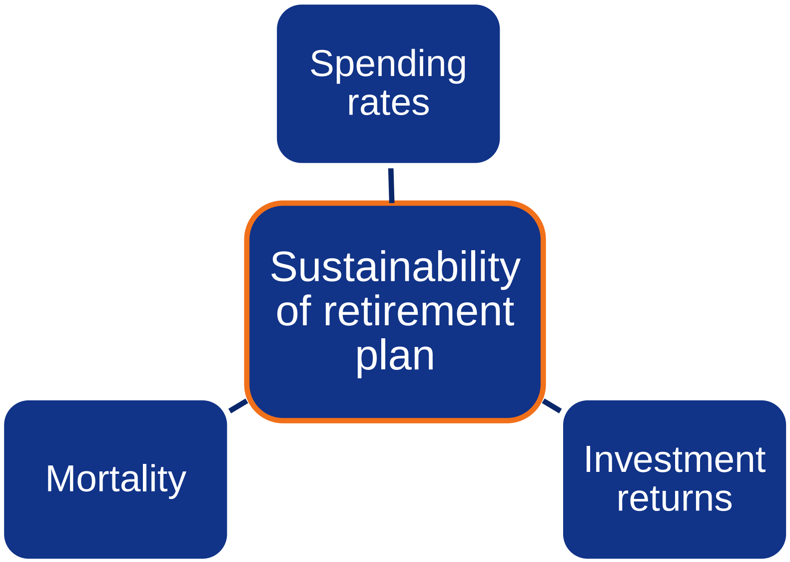
Summary



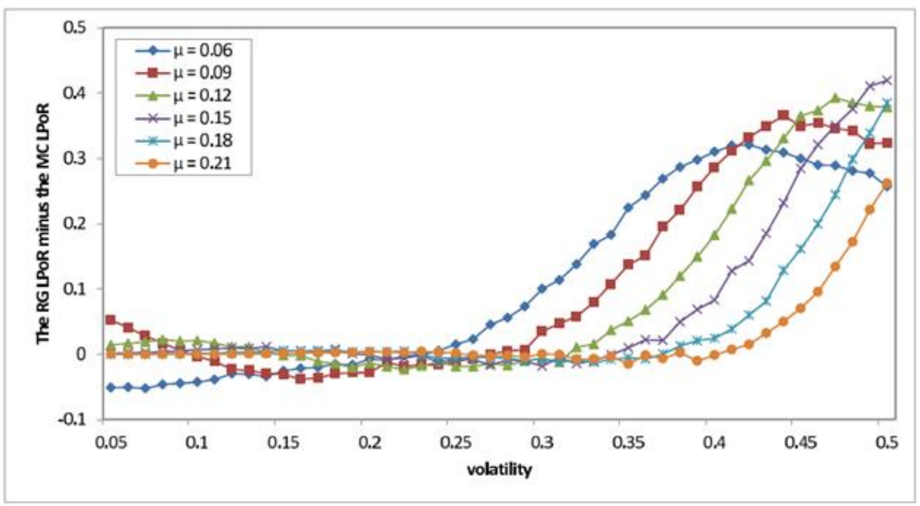
k = 2
k = 4
k = 6
k = 8
k = 10
LPoR (male)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0%
20%
40%
60%
80%
100%
Initial allocation to retail savings bonds (remainder is allocated to PX index)



Monte Carlo Simulations
Reciprocal Gamma Approximation
Accuracy
Flexibility
Easy to implement
Lower time complexity
Analytically tractable



Spending rates
Sustainability of retirement plan
Mortality
Investment returns



μ = 0.06
μ = 0.09
μ = 0.12
μ = 0.15
μ = 0.18
μ = 0.21
The RG LPoR minus the MC LPoR
-0.1
0
0.1
0.2
0.3
0.4
0.5
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
volatility

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