Dynamic and Granular Loss Modeling With Copulae

[SAV 2018]

Matúš Maciak[1], Ostap Okhrin[2], and Michal Pešta[1]

Prague, May 11, 2018

[1]Charles University, Prague, Czech Republic
[2]Dresden University of Technology, Germany

FACULTY
OF MATHEMATICS
AND PHYSICS
Charles University

Agenda

1. Aims
2. Methodology
3. Results
4. Conclusions

Aims

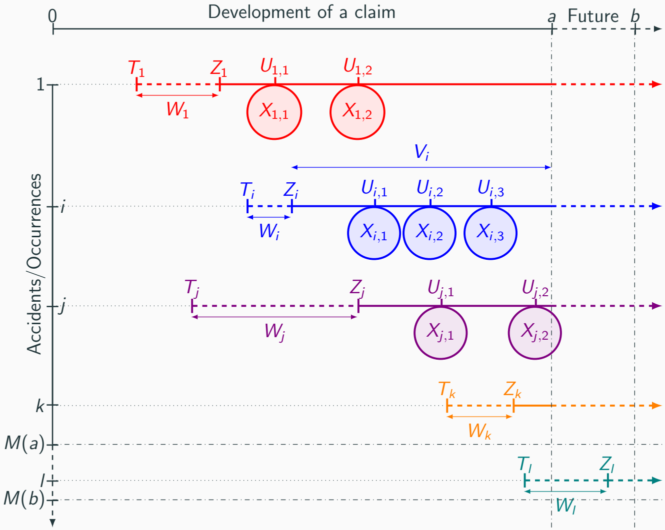
Two Main Goals

- Primary: predict future claim cash flows in non-life insurance and their uncertainty
- Secondary: back-predict incurred but not reported claims due to truncated data

Aggregated vs Granular

- Pitfalls of the conventional reserving techniques:
  - loss of information from the policy and the claim's development due to the aggregation, cf. Norberg (1993)
  - usually small number of observations in the triangle
  - only few observations for recent accident years
  - sensitivity to the most recent paid claims
- How to possibly overcome the issues:
  - individual/claim-by-claim/micro-level/granular data, which do not represent a mainstream in the reserving field, e.g., Antonio & Plat (2014)

Illustration



Methodology

Reporting Dates

- The time ordered reporting dates $\{Z_i\}_{i \in \mathbb{N}}$ are arrival times of a non-homogeneous Poisson process $\{M(t)\}_{t \geq 0}$ with a parametric intensity $\psi(t; \boldsymbol{\rho})$ such that

$$M(t) = \sum_{i=1}^{\infty} \mathbb{1}\{Z_i \leq t\}$$

- The cumulative intensity $\Psi(t; \boldsymbol{\rho}) := \int_0^t \psi(v; \boldsymbol{\rho}) \mathrm{d}v$ diverges if $t \to \infty$

Reporting Delays

- The reporting delays W_i's are independent random variables
- Sequence $\{W_i\}_{i \in \mathbb{N}}$ is stochastically independent of $\{Z_i\}_{i \in \mathbb{N}}$
- Given $Z_n = z$, W_n has a parametric density $f_W(\cdot, z; \boldsymbol{\theta})$

Payment Dates

- The time ordered payment delays $\{U_{i,1} - Z_i, U_{i,2} - Z_i, \ldots\}$ of the ith claim are arrival times of a non-homogeneous Poisson process $\{N_i(t)\}_{t \geq 0}$
- $N_i(t) | W_i, Z_i$
- Processes $\{N_i(t)\}_{t \geq 0}$, $i = 1, 2, \ldots$ are independent with a parametric intensity $\lambda_i(t; \boldsymbol{\nu}, \boldsymbol{\beta})$ such that

$$N_i(t) = \sum_{k=1}^{\infty} \mathbb{1}\{U_{i,k} - Z_i \leq t\}$$

- The cumulative intensity $\Lambda_i(t; \boldsymbol{\nu}, \boldsymbol{\beta}) := \int_0^t \lambda_i(v; \boldsymbol{\nu}, \boldsymbol{\beta}) \mathrm{d}v$ converges if $t \to \infty$

Payment Amounts

- Sets of the payment amounts $\{X_{ij}\}_j$'s are independent random sequences
- Sequence $\{X_{ij}\}_j$ forms an AR process
- Sequence $\{\{X_{ij}\}_j\}_{i\in\mathbb{N}}$ is stochastically independent of $\{Z_i\}_{i\in\mathbb{N}}$
- Given $Z_n = z$, the first payment X_{n1} has a parametric density $f_X(\cdot, z; \zeta)$

Accident Dates

- Accident dates as displacement of the reporting dates
- Reporting dates are fully observed, accident dates are truncated
- The displacement theorem (Kingman, 1993) provides that accident dates T_i's are arrival times of another non-homogeneous Poisson process with a parametric intensity

$$\mu(t; \boldsymbol{\rho}, \boldsymbol{\theta}) = \int_{\mathbb{R}} \psi(z; \boldsymbol{\rho}) f_W(t, z; \boldsymbol{\theta}) \mathrm{d}z$$

- Back-fit the incurred but not reported claims

Copula

- Suppose that $Y_t^{(\ell)}$ is a loss amount for a time time period t (e.g., month) and for a LoB ℓ
- Assume that the dependence between lines of business (LoBs) is modeled via a parametric copula (possibly time-varying in order to capture dynamic behavior)
- E.g., $\ell \in \{1, 2\}$ ... two LoBs (material damage and bodily injury)

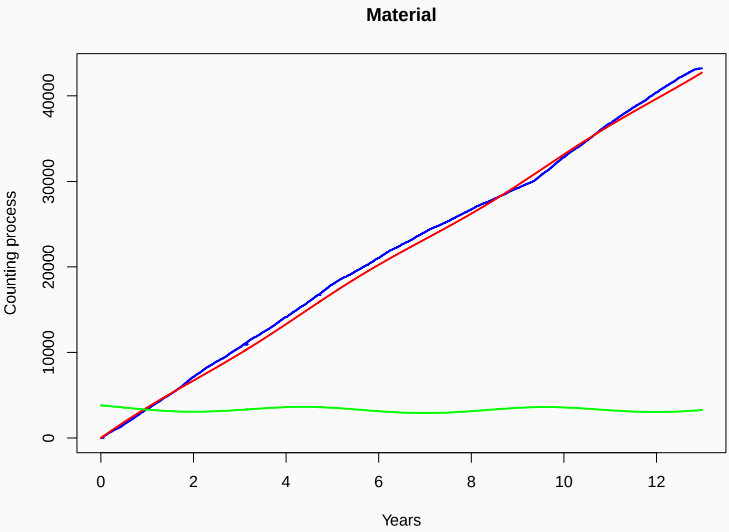
$$\mathbb{P}\left[Y_t^{(1)} \leq y^{(1)}, Y_t^{(2)} \leq y^{(2)}\right] = C\left(\mathbb{P}\left[Y_t^{(1)} \leq y^{(1)}\right], \mathbb{P}\left[Y_t^{(2)} \leq y^{(2)}\right]; \alpha(t)\right)$$

By-product for Stochastic Theory

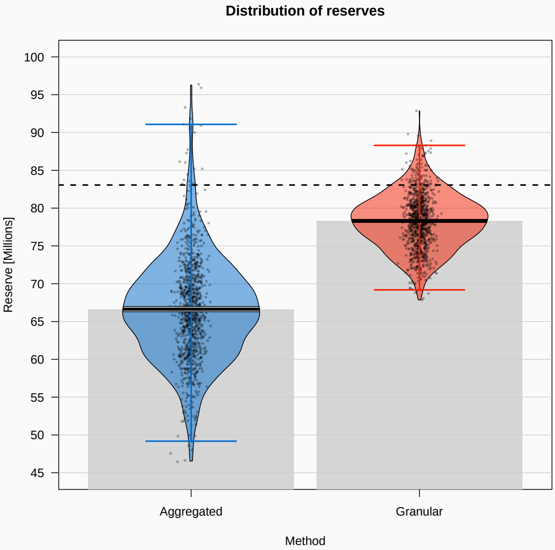
- Probabilistic framework for the n.i.n.i.d. observations
- Time-varying models in an unbalanced panel data setup
- Maximum likelihood estimators derived
- Proved consistency and asymptotic normality of the estimators
- Justification for usage of the method

Results

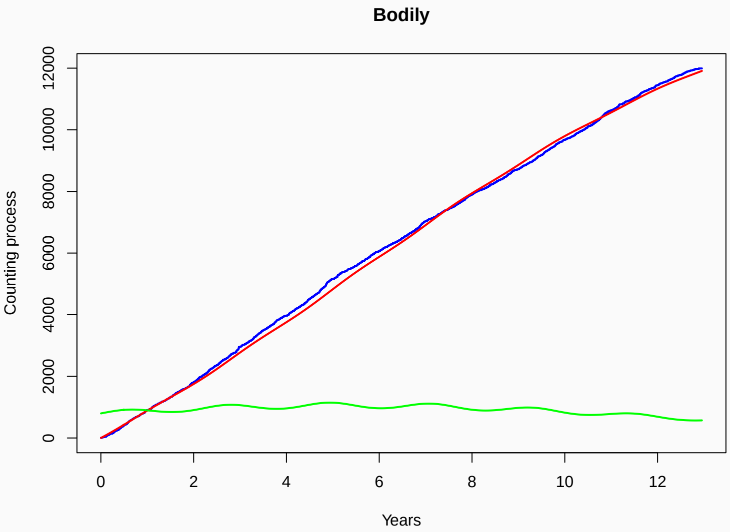
$M(t)$ as NHPP (Material)



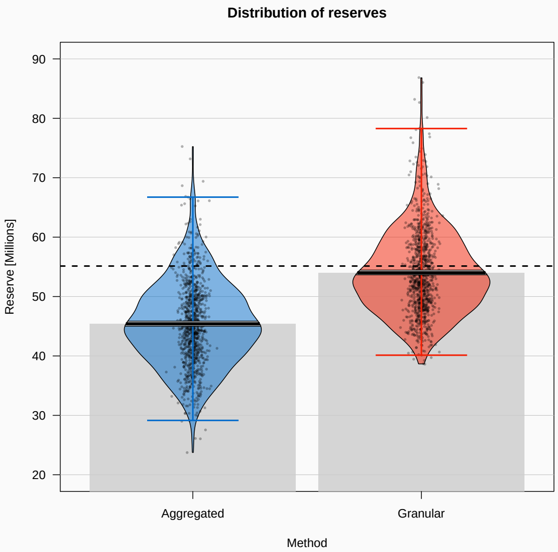
Material


Distribution of reserves
Reserve [Millions]
100
95
90
85
80
75
70
65
60
55
50
45
Aggregated
Granular
Method

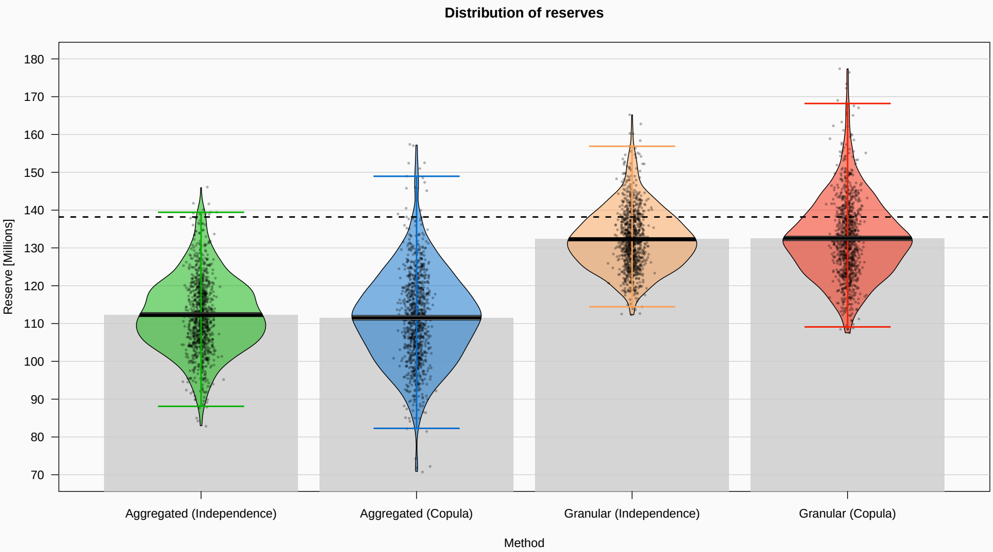
$M(t)$ as NHPP (Bodily)



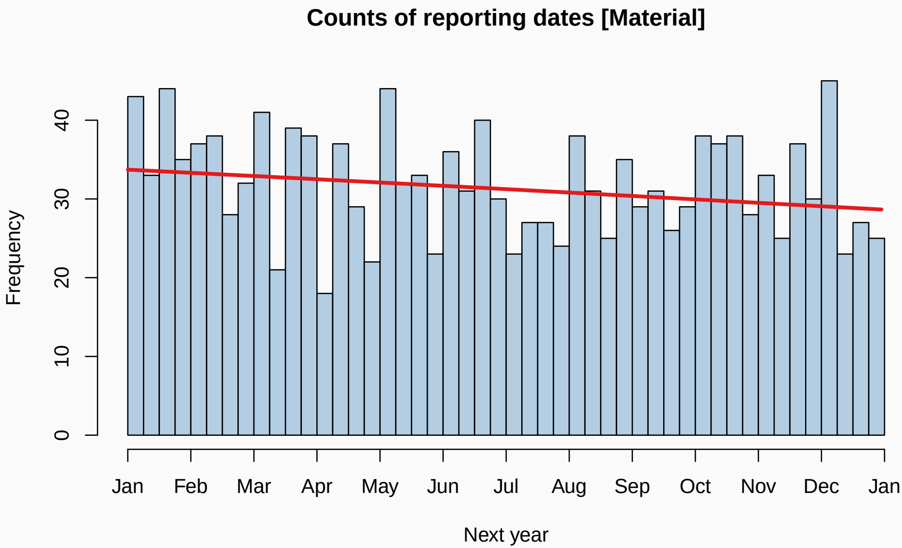
Bodily


Distribution of reserves
Reserve [Millions]
90
80
70
60
50
40
30
20
Aggregated
Granular
Method

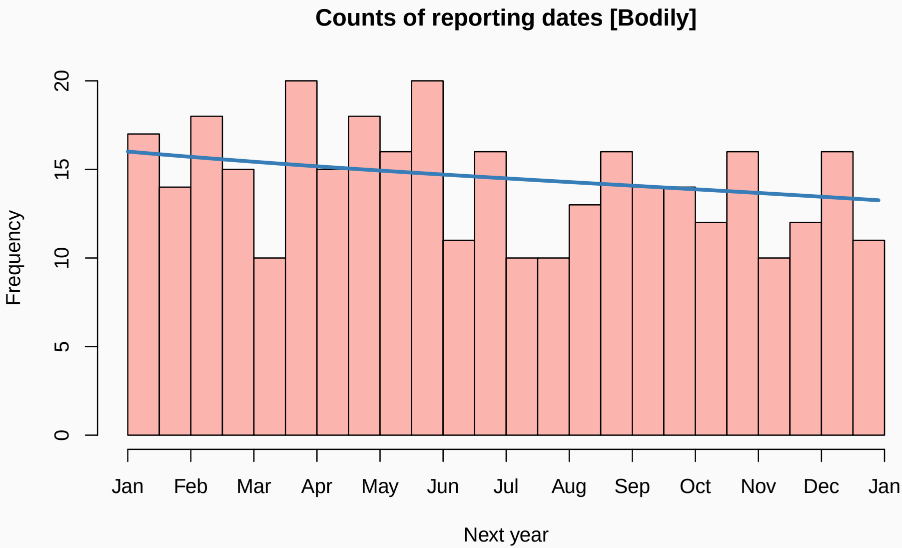
Total


Distribution of reserves
180
170
160
150
140
130
120
110
100
90
80
70
Reserve [Millions]
Aggregated (Independence)
Aggregated (Copula)
Granular (Independence)
Granular (Copula)
Method

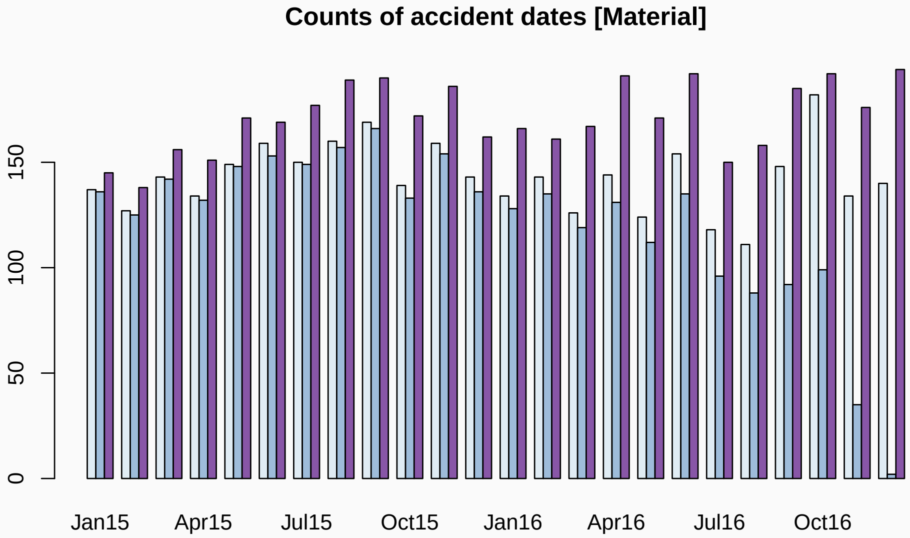
Predicted Future Reportings



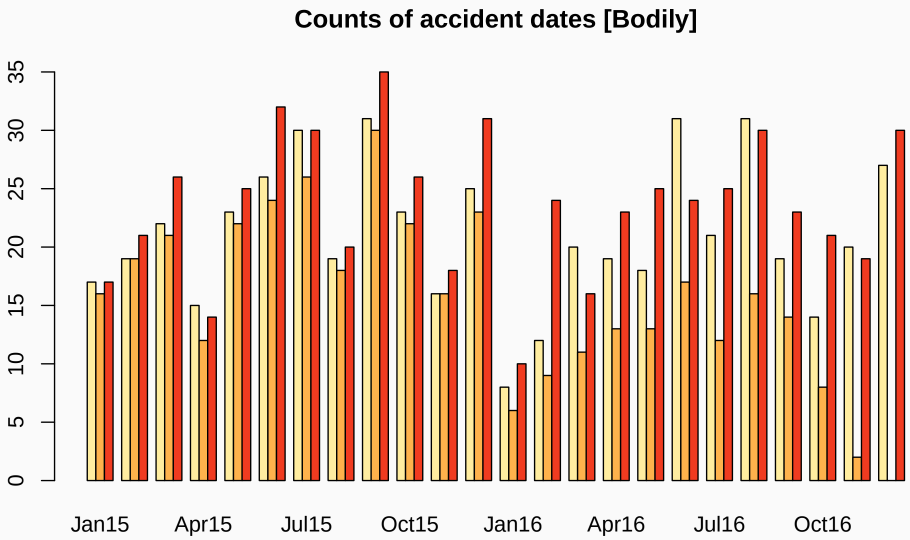
Predicted Future Reportings



Back-fitted Recent Accidents



Back-fitted Recent Accidents



Conclusions

Summary

- Focus on three synergic research areas:

1. Inventing stochastic methods for loss reserving based on claim-by-claim data
2. Using dynamic copulae for modeling dependencies among types of claims
3. Deriving appropriate statistical inference for these approaches

Questions?

michal.pesta@mff.cuni.cz

References

- Antonio, K. and Plat, R. (2014): Micro-level stochastic loss reserving for general insurance, Scand. Actuar. J. 2014(7), 649–669
- Kingman, J.F.C. (1993): Poisson Processes. Oxford University Press, New York, NY
- Maciak, M., Okhrin, O., and Pešta, M. (2018): Micro and Dynamic Claims Reserving Embracing Dependencies, Submitted.
- Norberg, R. (1993): Prediction of outstanding liabilities in non-life insurance, ASTIN Bull. 23(1), 95–115