

Metody pro výpočet kapitálu

Interní modely v životním pojištění

Petr Bednařík



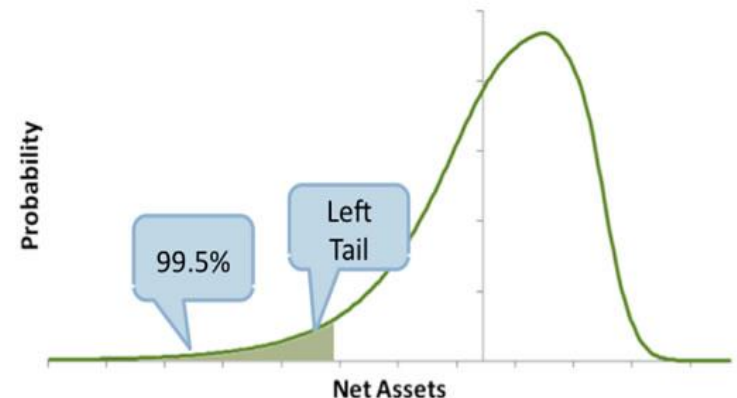
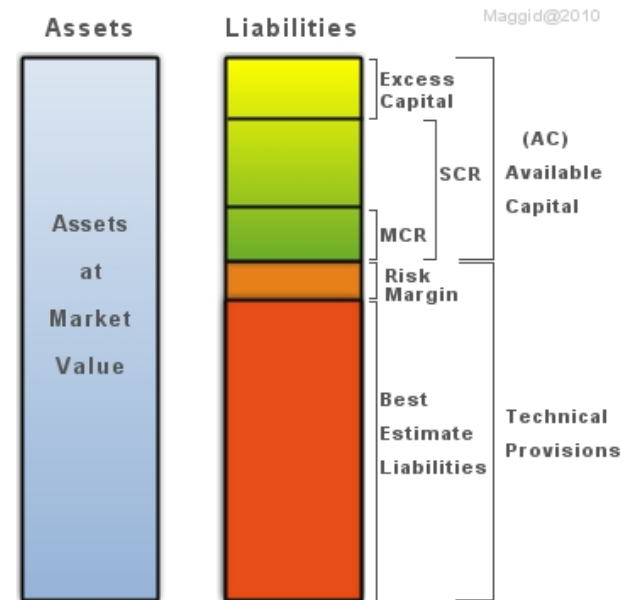
What I will talk about

- General comments
 - A lot of the topics are general (i.e. it doesn't matter if it's a life or non-life company)
 - However, many of the problems I will talk about are mostly relevant for market risk and handling stochastic economic scenarios
 - It is therefore more interesting for life than non-life companies
- Agenda
 - The ideal (non-simplified) method to calculate the SCR – nested stochastic
 - Very complex calculations
 - There are some tricks that can be done to make it feasible
 - Show what other options there are – proxy calculations
 - The standard formula – what are its assumptions and problems
 - More sophisticated proxy calculations
 - Curve fitting
 - Least squares Monte Carlo
 - Replicating portfolios

Solvency II Requirements

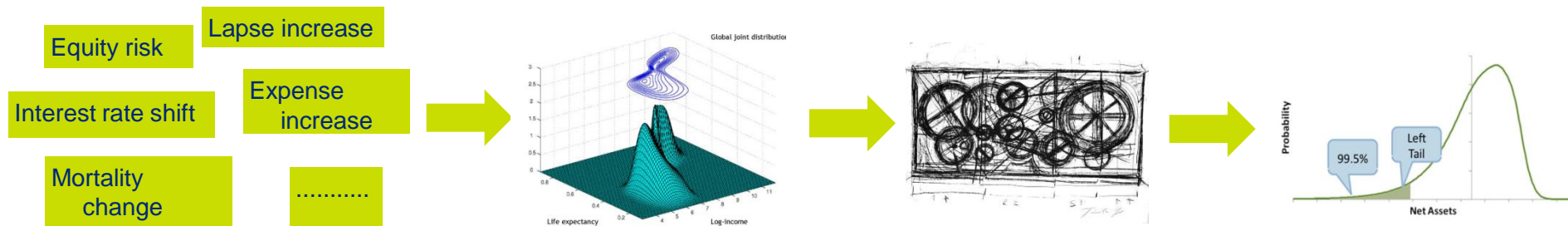
- Solvency Capital Requirement (SCR)
 - Own funds are the difference between value of assets and liabilities (the green part)
 - Valuation of assets and liabilities according to SII requirements – idea is that it is the fair value
 - We are interested in the change of own funds over a one year period – this can be thought of as a random variable in relation to future uncertain development
 - SCR = 99.5% VaR of the change in own funds over one year = 99.5% quantile of the distribution of the decrease in own funds over one year
- There are discussions if this is a good measure of risk but we will not deal with those now, let's consider this definition as a good one
 - For example it does not depend at all on the tail after 99.5%

Solvency II Balance Sheet



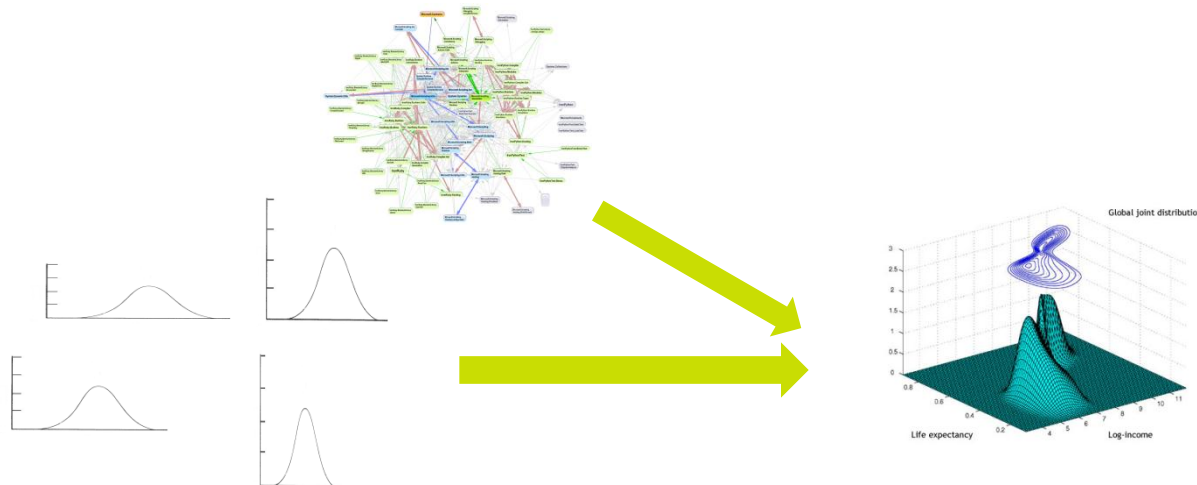
Capital calculation steps

- Theoretical steps
 1. Find the sources of uncertainty – risk factors
 2. Find the joint probability distribution of the development of the risk factors over the first year
 3. Be able to value the own funds (difference between assets and liabilities) for every risk factor scenario – asset and liability model – this is a transformation of the distribution of risk factors (from n dimensions to 1)
 4. Find the quantile of the transformed distribution – this is the capital
- Practical aspects
 - Doing this analytically is only possible in special cases
 - The joint distribution is multivariate normal and value of assets and liabilities is a linear function of the risk factors
 - The result of this is the standard formula – more on this later
 - In other cases it is necessary to use Monte Carlo simulations to get the transformed distribution



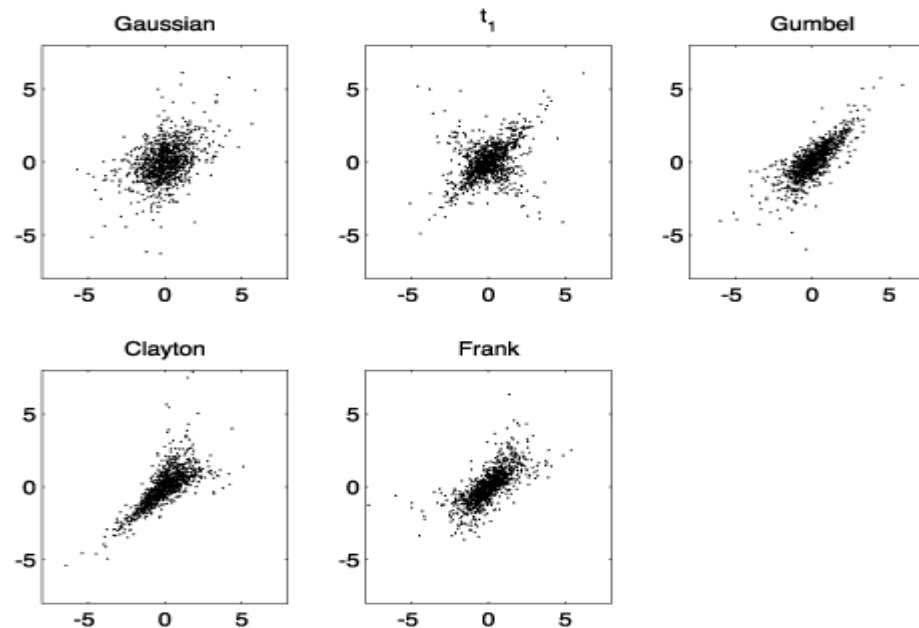
Joint distribution of risk factors

- In the Monte Carlo approach scenarios of the development of risk factors over the first year are needed – these are the outer scenarios
- In practise the joint distribution is not constructed (observed) directly but based on
 - Marginal distributions for individual risk factors
 - Dependency structure between risk factors
- It is usually possible to get reasonable marginal distributions based on existing data, however the data is often scarce in the regions that interest us most – the tails
 - It is quite difficult to collect data about extreme economic events (big stock market crashes, etc.)
 - However, for non-economic data it's probably even worse (pandemics, catastrophes, etc.)



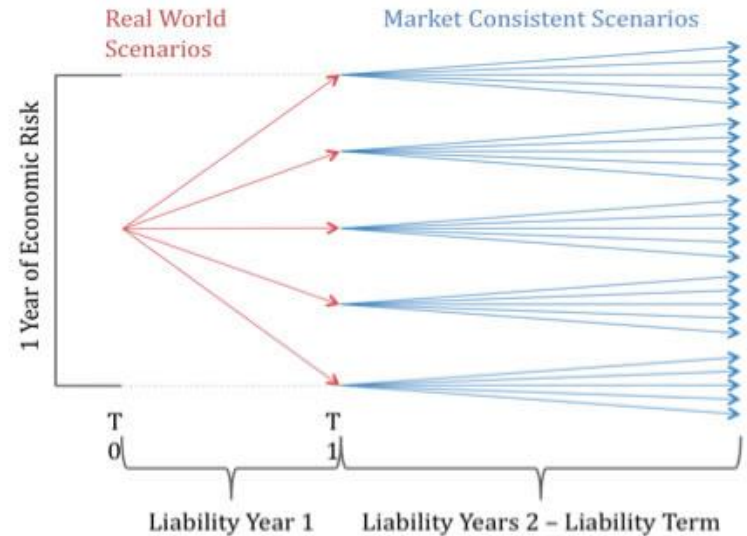
Joint distribution of risk factors

- All of this is however very easy compared to estimating a reasonable dependency structure
 - Very little data available about the correlation of different risk factors – as before, for economic risks there is some data, though little when under stress, for other risks it's worse
 - Even if we had data, it's not easy mathematically - dependency can be defined analytically only for some multivariate distributions (for example normal)
 - If we want a mix of marginal distributions the only solution is to use copulas (for example the Gauss copulas carries over the dependency structure of the multivariate normal distribution to any set of marginal distributions that we desire)
 - What copula to use is highly judgemental



Own funds impact - Asset and Liability model

- To calculate the change in own funds in a given scenario, we need an asset and liability model
 - In each outer scenario run to the end of the year
 - The assets and liabilities will be impacted by the outer scenario
 - Value the assets and liabilities at the end of the year
 - When options and guarantees are present, you need to perform a risk-neutral valuation of the liabilities – many economic scenarios (inner) needed for each outer scenario
- This leads to the nesting problem
 - Calculating the MCEV is like having one outer scenario (the best estimate)
 - Imagine now that you have a few tens of risk factors and you want scenarios, which would reasonably cover the joint distribution – better not to imagine (usual count is e.g. 100 000)
 - Performing 100 000 MCEV calculations is probably not possible
- Some tricks you can do
 - Ignore some risk drivers (calculate them separately using the standard formula) – when nested stochastic is done it is usually only for market risks (economic scenarios)
 - Reduce the number of scenarios that you run – in the end you want only the 99.5% VaR so you only need to look at the really bad scenarios – filtering

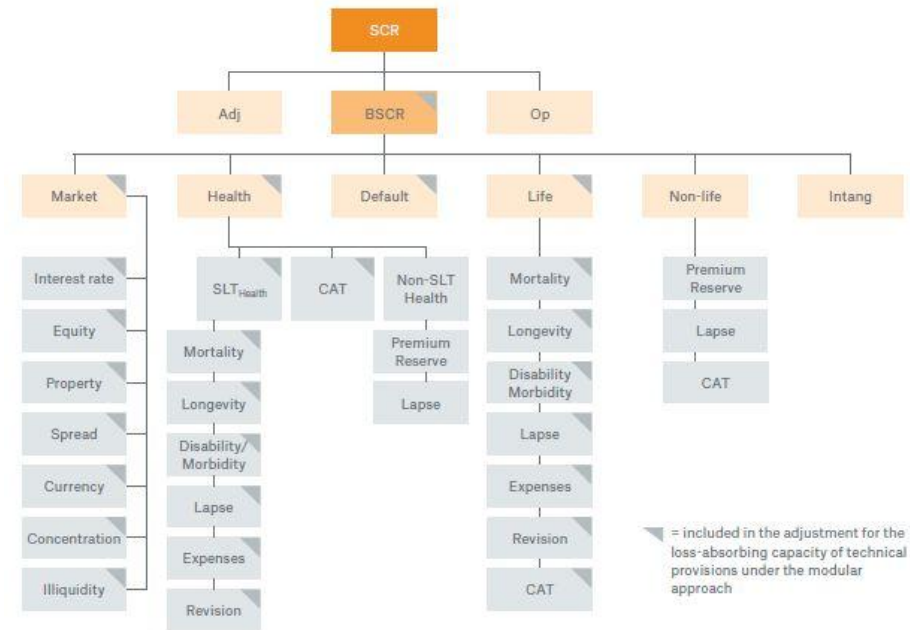


Basic proxy method – the standard formula

- The calculation is done first per each risk factor separately based on one 99.5% quantile value of the risk factor
- The total quantile is gained by aggregating the modules by use of the correlation matrix sum

$$SCR_{market} = \sqrt{\sum_{i,j} Corr_{i,j} \times SCR_i \times SCR_j}$$

- The standard formula is a simplification in several senses
- With some we will not deal – they are theoretically fixable (although it would already be an internal model)
 - The modules (risk factors) are prescribed – let's assume that we can add/remove risk drivers if we wanted to
 - The parameters (values of risk factors and correlations between modules) are prescribed – let's assume we could recalibrate the parameters to our liking



The standard formula – implicit simplifications

- Timing
 - It does not directly work with the one year time horizon – all shocks/stresses happen immediately
 - In essence it is answering the question what would happen to my own funds now if the 99.5% 1-year event happened instantaneously today, which is not the same as the original SCR question
 - In reality the development though-out the year could have been turbulent
 - The company could have reacted to the change sometime in the middle of the year
 - It is hard to judge the impact of this – for market risks it could be important, for other risks probably not so much
- Aggregation of modules
 - The correlation matrix method is used
 - This only works if two crucial assumptions are met
 - The joint distribution of the risk factors is multivariate normal (actually the family is slightly greater)
 - The own funds depend linearly on the risk factors – in practice this means
 - One calculated sensitivity to the risk drivers defines the behaviour to all stresses of the risk factor
 - There are no interactions between risk factors (the effects of 2 stresses at once are the sum of the 2 individual stresses)
 - If any of these two assumptions are not met then the method of aggregating the quantiles is only approximate

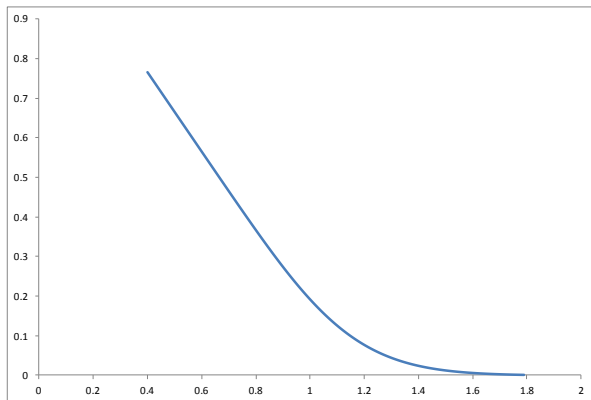
The standard formula – illustration of simplifications

- Example of distributions assumed – a lot of them are normal, but some are not – that is enough to break the standard formula

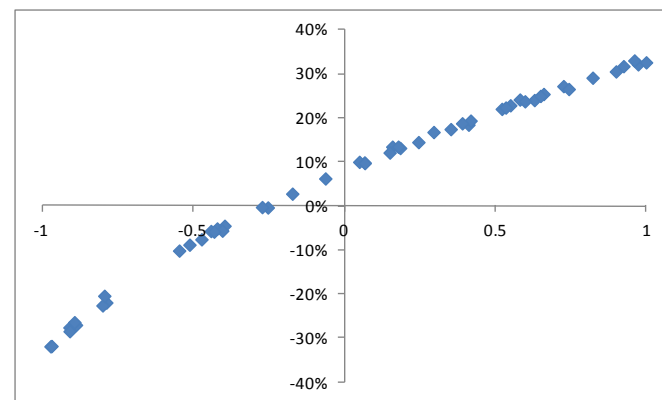
Risk Factor	Current Distribution	Risk Factor	Current Distribution
1. Equity Conventional	Student's t	13. Credit default	Lognormal
2. Equity Strategic	Normal	14. Reinsurance credit	Lognormal
3. Equity Hedges	Student's t	15. Annuitant mortality	Normal
4. Equity Volatility	Normal	16. Assured lives mortality	Normal
5. Property	Normal	17. Pandemic mortality	Lognormal
6. Interest rates-level up	Normal	18. Lapses	Normal
7. Interest rates-level down	Normal	19. Catastrophe	Lognormal
8. Interest rates-slope up	Normal	20. Claims volatility	Lognormal
9. Interest rates-slope down	Normal	21. Reserve	Lognormal
10. Interest rate volatility	Normal	22. Expenses	Normal
11. Real interest rates-level down	Normal	23. Operational	Lognormal
12. Credit spreads	Normal	24. Exchange	Lognormal

- Non-linearity of own funds as a function of the risk drivers
 - Example – put option price – function of equity price and interest rate – 2-variate normal distribution assumed
 - Correlation matrix aggregation compared with direct Monte Carlo simulations

Put option price (as function of equity price)



Error as function of correlation



Standard formula – advantages and disadvantages

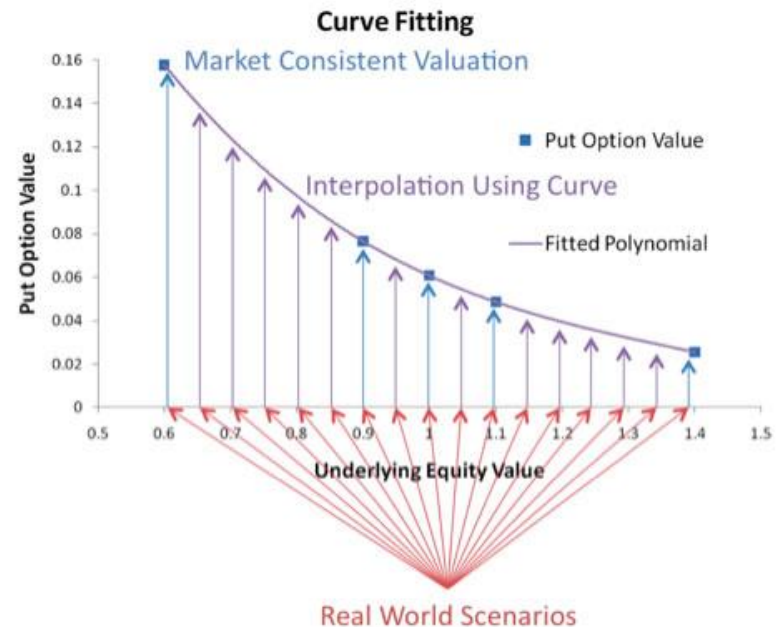
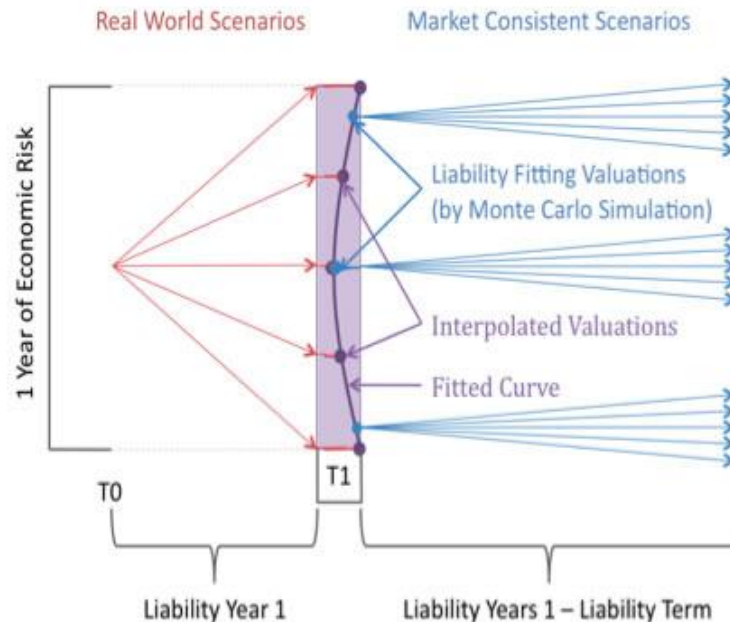
- Advantages
 - Calculation complexity is low
 - For each risk factor (module) only one full model run (with inner scenarios) is needed using the 99.5% quantile risk factor shock – tens of runs in total
 - Prescribed parameters
 - One does not have to derive the distributions of risk factors and think about inter-dependencies between risk factors
 - Theoretically one may use the standard formula calculation method while using his own calibration of parameters (distributions of risk factors and correlations) – then it would however be an internal model
- Disadvantages
 - Prescribed parameters cannot be customised
 - The simplified method has a high error if
 - The distributions are not normal in reality
 - The dependency of the liability value on the risk factors is highly non-linear
 - The method often underestimates the risk

Looking for something in between nested stochastic and the standard formula

- What is the most annoying about the nested stochastic method?
 - That you have to calculate many inner scenarios for each outer scenario – i.e. calculating the value of liabilities in each scenario is complex
- Can we go around this somehow?
 - Instead of doing the full liability valuation run in each outer scenario let's do something more simple – proxy methods to liability valuation
 - Let's calculate the full valuation of the liabilities in a few outer scenarios and interpolate the behaviour = Curve Fitting
 - Let's calculate only a few random inner scenarios for each outer scenario = least squares Monte Carlo (LSMC)
 - Let's replace the liabilities by an easy to value combination of assets, which produce the same cash-flows = Replicating Portfolios
 - The rest stays the same as in nested stochastic
 - We have marginal distributions of risk factors and dependencies based on copulas
 - We create a large set (e.g. 100 000) of outer scenarios (vectors of risk factors)
 - In each of the scenarios we value the liability by use of the proxy method
 - We get a distribution of own funds and from that we find the quantile

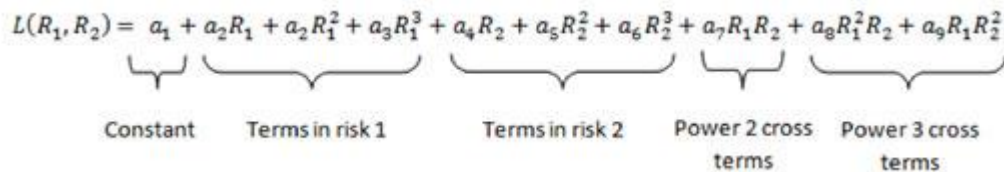
Curve fitting - principles

- The stochastic liability model (e.g. Prophet ALS) can be viewed as a black box which has as inputs the value of the risk factors (outer scenario) and as an output the value of the liabilities in this scenario
- The idea of curve fitting is to replace the slow black box (e.g. Prophet model) by a fitted function of the risk factors – the loss function
- In that case it's enough to run the liability model only in a few risk factor scenarios and interpolate between them
- Then whatever combination of risk factors is brought (outer scenario) the liability valuation consists only of evaluating the interpolated function in the given points – this is very quick and easy



Curve fitting - practice

- Choice of interpolation is judgemental
 - The dependency of the liability value on the risk factor is often quite complex (non-linear) and it is not clear how many and what points to use as a basis for the interpolation – the choice of points is judgemental and impacts the result
 - It is not clear what function to use for the interpolation (piecewise linear, polynomial, splines, goniometric functions, exponentials, ...)
- Number of points (runs) needed can be high
 - If there is a lot of risk factors than the number of points needed grows very quickly – especially due to cross terms – in practice it may be hundreds which is not far from nested stochastic
 - In practice higher order cross terms are often assumed to be zero – only first order cross terms are considered – runs with changes in pairs of risk factors are needed

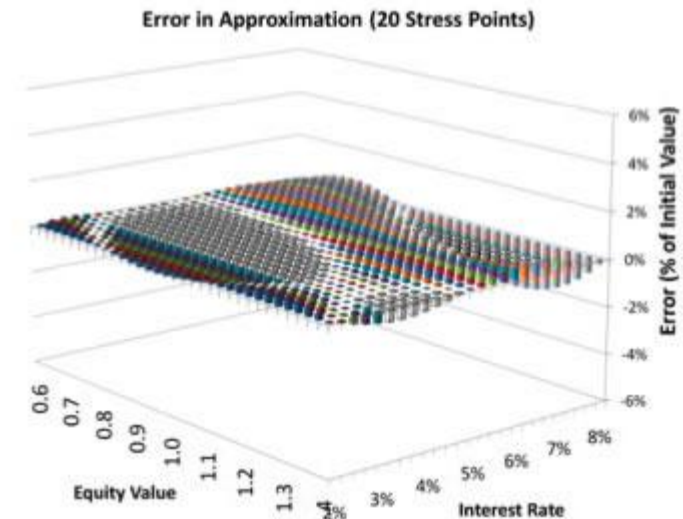
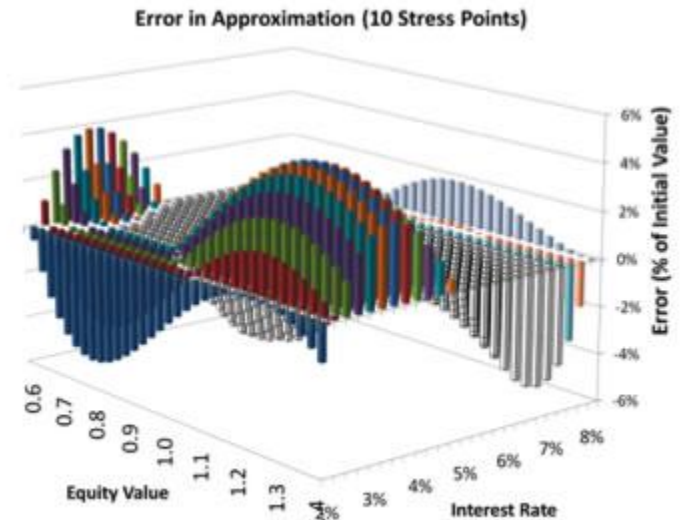
$$L(R_1, R_2) = a_1 + a_2R_1 + a_2R_1^2 + a_3R_1^3 + a_4R_2 + a_5R_2^2 + a_6R_2^3 + a_7R_1R_2 + a_8R_1^2R_2 + a_9R_1R_2^2$$


Constant Terms in risk 1 Terms in risk 2 Power 2 cross terms Power 3 cross terms

- Timing
 - As it is difficult to roll the liability model a year forward, in practise the loss function are calculated as of time 0 instead of time 1 and the impact of the change in the risk factor is assumed to be instantaneous
 - This is the same simplification as in the standard formula
 - Does not allow to accurately model risk mitigation

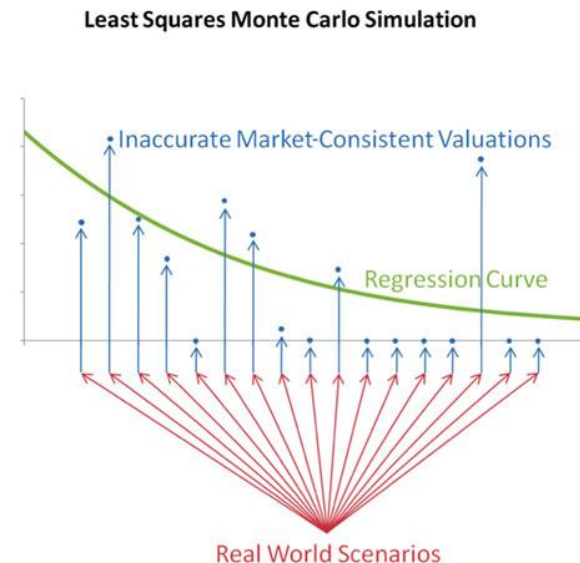
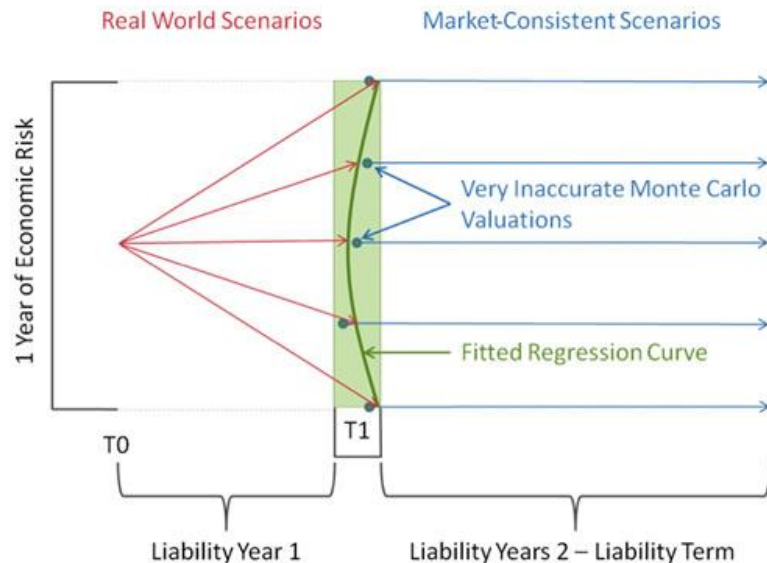
Curve fitting – advantages and disadvantages

- Advantages
 - Allows to assume any probability distributions for the risk factors along with any dependency structure
 - Takes into account the non-linearity of the dependency of the liabilities on the risk factors
 - If reasonable number of points is used then number of calculations needed is significantly smaller than in the case of nested stochastic
- Disadvantages
 - Usually loss function is approximated as at time 0 and risk factor impacts are assumed to be instantaneous as in the standard formula – risk mitigation is not taken into account entirely accurately
 - The choice of points and interpolation functions is judgemental and the results depend on this choice
 - In case of many risk factors and complex interpolation functions the numbers of runs necessary can still be quite high
- References
 - In UK – 30% use curve fitting, 5% replicating portfolios, 35% both, 30% undecided
 - Aviva, L&G, Prudential



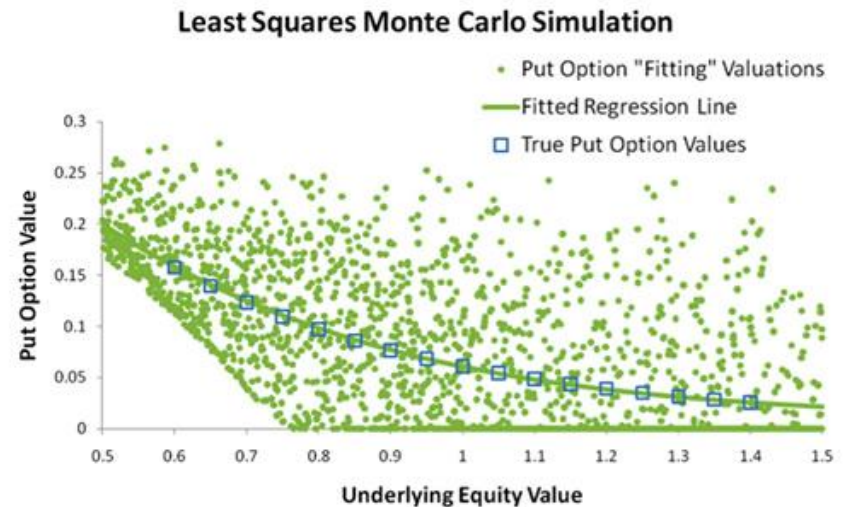
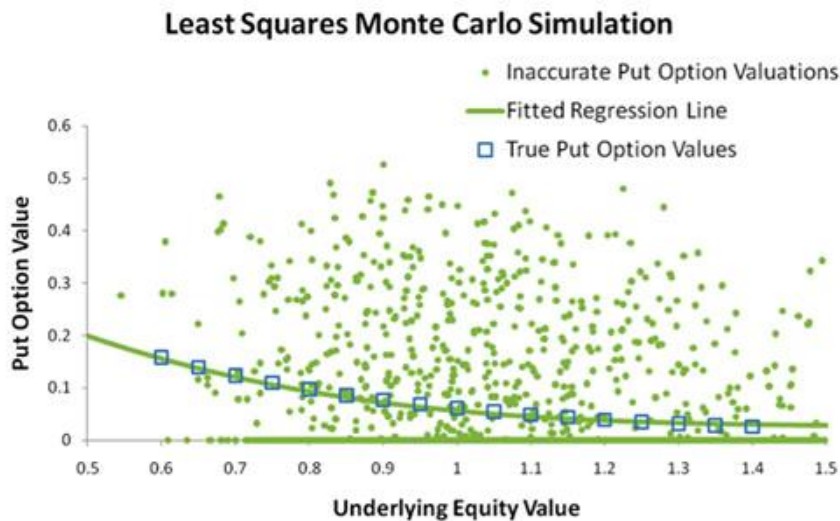
Least squares Monte Carlo - principles

- The idea is close to nested stochastic, we have many outer scenarios
- But instead of many inner scenarios in each outer scenario, only one inner scenario is used
- The liability value based on one inner scenario is therefore very inaccurate
- But if this is done many times, we can make a regression curve through all of these inaccurate valuations – minimization of least squares
 - The risk factors are the explanatory variables
 - The liability value is the response variable
- Once we have the curve it's the same as curve fitting



Least squares Monte Carlo - practice

- In practice, a lot less simulation runs are needed for good accuracy than in curve fitting (or nested stochastic) – especially if there are many risk factors
 - In curve fitting each inner simulation is used only for the setting of one point of the loss function, in LSMC each inner simulation is used for fitting the whole function
 - Same accuracy as in curve fitting gained with significantly less simulations (outer x inner)
- It is not necessary to use the same outer scenarios for the regression as for the actual run
 - It may be actually better to use a different set of scenarios for the regression – with more details in the tails – if the actual distribution of scenarios is used than the tails that interest us the most are not very well covered
- More than one inner scenarios can be used – i.e. 2, 3 – this helps accuracy dramatically



Least squares Monte Carlo – advantages and disadvantages

- Advantages

- A significantly lower number of simulations is needed to get an accurate result
- This allows to capture several effects, which would not be realistic in curve fitting due to calculation time
 - Modelling of first year – if liability value depends on the path of the risk factor over the first year an additional variable characterizing the path can be added – adding variables is not such a big deal in LSMC compared to curve fitting
 - Projections of the SCR – can be calculated in the future as well
- Quantification of sampling error is easier

- Disadvantages

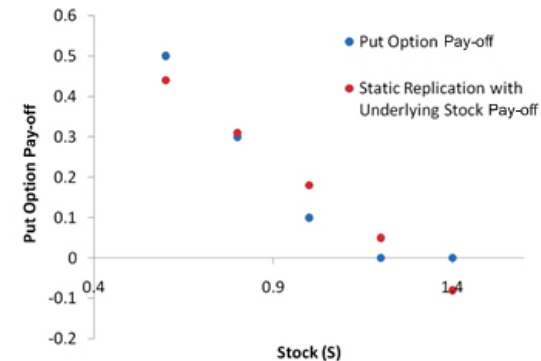
- Has to be embedded in the ALM model (not just working with the results as in curve fitting)
- The theory is more complicated – harder to explain and believe

- References

- It still very new, most insurers started developing either curve fitting or replicating portfolios before so they did not change to LSMC
- We have a experimental ALS workspace from our colleagues in France, where it is implemented
- To my knowledge colleagues in Germany are also using it, probably in UK and Switzerland as well

Replicating portfolios - principles

- If a portfolio of assets has the same cash-flows as another asset or liability, then their price has to be the same for there to be no arbitrage
- If we find an asset portfolio which replicates the cash-flows of our insurance liabilities under any possible future state than we can use this asset portfolio instead of the original liabilities
- Revaluing these assets under different (outer) scenarios is simple compared to revaluing the liabilities (lots of inner scenarios needed)
- For simplicity let's assume that we are replicating uncertain cash-flows in one future time period
 - Dynamic replication – replicating portfolio of assets changes in time – can always be done exactly using simple assets – not used in practice
 - Static replication – replicating portfolio held until maturity (or pre-defined selling point) – can only be done approximately using simple assets, for exact replication more complex assets are needed (options, etc.) – we will only talk about static from now on
- In case of one risk factor any set of cash-flows can be replicated using options
 - But this fit will not hold under stressed scenarios – the fitting needs to be done on a wide range of scenario sets (combination of outer scenario and inner scenarios)
- In case of multiple risk factors it is not possible to replicate using single risk assets – multiple risk assets are needed (basket options, etc.)
- Path dependency – if the cash flows are path dependent the assets also need to be (Asian options, etc.)



Replicating portfolios - practice

- Process

- Choose a set of candidate assets for the replicating portfolio
- Run the liability model on several economic scenarios
- Find the weights of candidate assets that best replicate the liability cash-flows over all scenarios
- Test quality of replication

$$\sum_s \left\| \sum_i w_i * CA_i(s) - L(s) \right\|$$

- Candidate assets

- Best to use asset whose price can easily be observed on the market – if we need Monte Carlo simulations to value them we don't save much
- Exposed to same risk factors as liabilities we want to replicate (i.e. If there is an equity guarantee we need equities and equity options, if path dependency we need path dependant, etc.)

- Features which are hard to replicate using simple assets

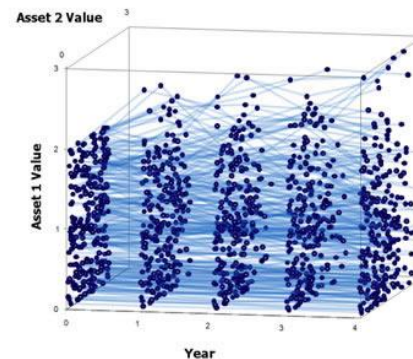
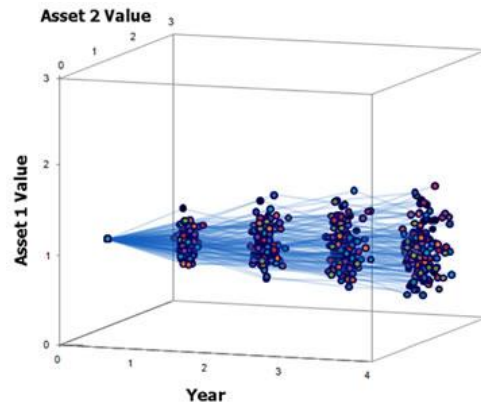
- Book value accounting, Special bonus reserves (German, French, etc.), Management actions, Policyholder behaviour

- Solution could be to produce exotic assets with the same guarantees, rules, etc.

- You will need Monte Carlo simulations to value them – back where we started
- In the end it could lead just to selecting representative model points – this is exactly what the MP grouping tool does
- Simple assets are therefore usually preferred although the accuracy is not great

Replicating portfolios - practice

- Finding weights
 - Least squares regression
 - Is it not enough just to fit on base (inner scenarios), we especially need the fit to be good under stress – fitting under many sets of inner scenarios
 - Problems
 - Many candidate assets – candidate asset selection to avoid over-determining the linear system
 - Many similar assets are included – the linear system has very bad numerical properties (it is close to singular)
- Testing
 - Split the scenarios in fitting ones and validation one
- Biggest problem – works only for market risks – no way to incorporate non-market risks



Replicating portfolios – advantages and disadvantages

- Advantages
 - Elegant idea
 - If a good replicating portfolio is found it is very easy to revalue (if no complex assets are included)
- Disadvantages
 - Only for market risks
 - Difficulty in capturing multi-asset dependence, path-dependence, etc.
 - Difficult to capture book value accounting, special reserves, policy-holder behaviour, etc.
 - Selection of assets is difficult, can cause numeric problems
 - Only works reliably on those outer scenarios that have been used for the fitting
 - The fitting procedure is hard to automate – judgement and manual adjustments are often necessary
- References
 - ING, Axa

How to use this?

- Directly – implementation of internal models for Solvency II
- Validation that the correlation matrix aggregation methodology does not introduce too much error
 - Can be considered part of validation/ORSA
 - Relevant for companies using the standard formula but also companies having an internal model which uses the correlation matrix aggregation method
 - What is needed?
 - Construct risk factor distributions as in the standard formula calibrations – including dependency – based on calibration reports from EIOPA
 - Process
 - Run a set of stresses (quite a few)
 - Fit the loss functions and calculate the capital using curve fitting
 - Compare it to results (standard formula or correlation matrix method internal model) and assess error

References

Adam Koursaris

Calculating the solvency capital requirement, part 1 – 4

<http://www.insuranceerm.com/analysis/calculating-the-solvency-capital-requirement-part-1.html>

Contact

Petr Bednařík

Manager, Czech Republic

Actuarial & Insurance Solutions

Tel: +420 246 042 327

Email: pbednarik@deloitteCE.com

Deloitte.

Deloitte refers to one or more of Deloitte Touche Tohmatsu Limited, a UK private company limited by guarantee, and its network of member firms, each of which is a legally separate and independent entity. Please see www.deloitte.com/cz/about for a detailed description of the legal structure of Deloitte Touche Tohmatsu Limited and its member firms.

© 2012 Deloitte Czech Republic