Deloitte.

Model Validation

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Can you Defend your Model?

Česká Společnost Aktuárů

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Agenda

Defending your Model

Standard Formula, Internal Models and Scope of Validation		
Example IM Calibrations	Interest Rates	Equities
“Living memory” Test	Fit to Overlapping One-year Changes	
Fit to Past Data	Histogram, P-P plot, moments, KS Test	
Stability / Contra-Cyclicality	Rolling estimates, Through-Cycle Methodology	
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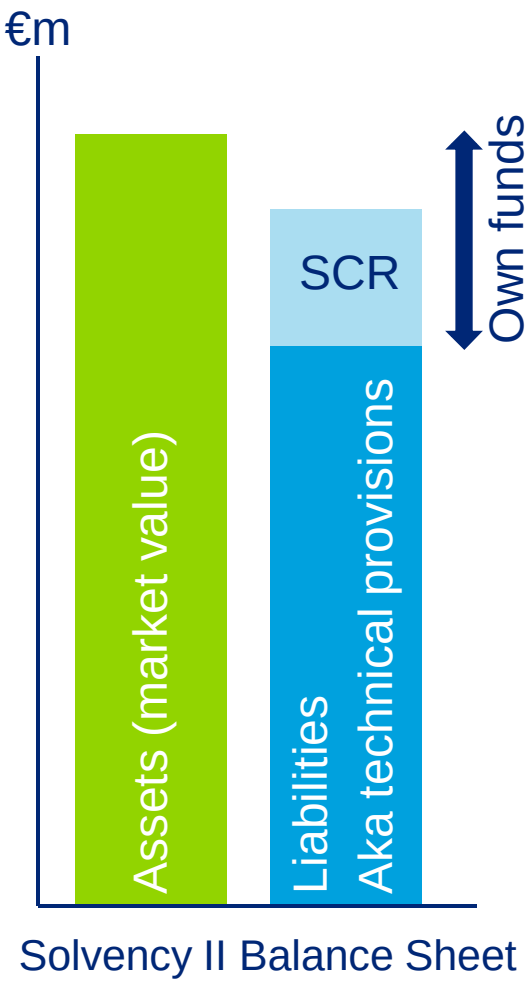
Solvency II

Quick Highlights

- Applies to (re)insurers established / wish to establish within the EU.
- Stated aim is to produce solvency capital requirements (SCR) that are reflective of the underlying business risks & ensure financial soundness:
  - ❑ SCR calculated at a 99.5% confidence level of covering those risks over a 1 year time horizon.

  with regulatory intervention if:

  - ❑ Assets < Technical Provisions + SCR.
- Requires robust governance and comprehensive risk management frameworks and processes.
- Requires increased transparency through significant documentation and disclosure requirements.
- Effective date under recent discussion. While now 1 Jan 2013, latest proposals with “transitional provisions”, many taking effect 1 Jan 2014.
- Level 2 & 3 measures not yet finalised. Expect will probably keep moving right up to the wire. 2 public consultations to be aware of:
  - ❑ QRT (October / November)
  - ❑ ORSA (November)

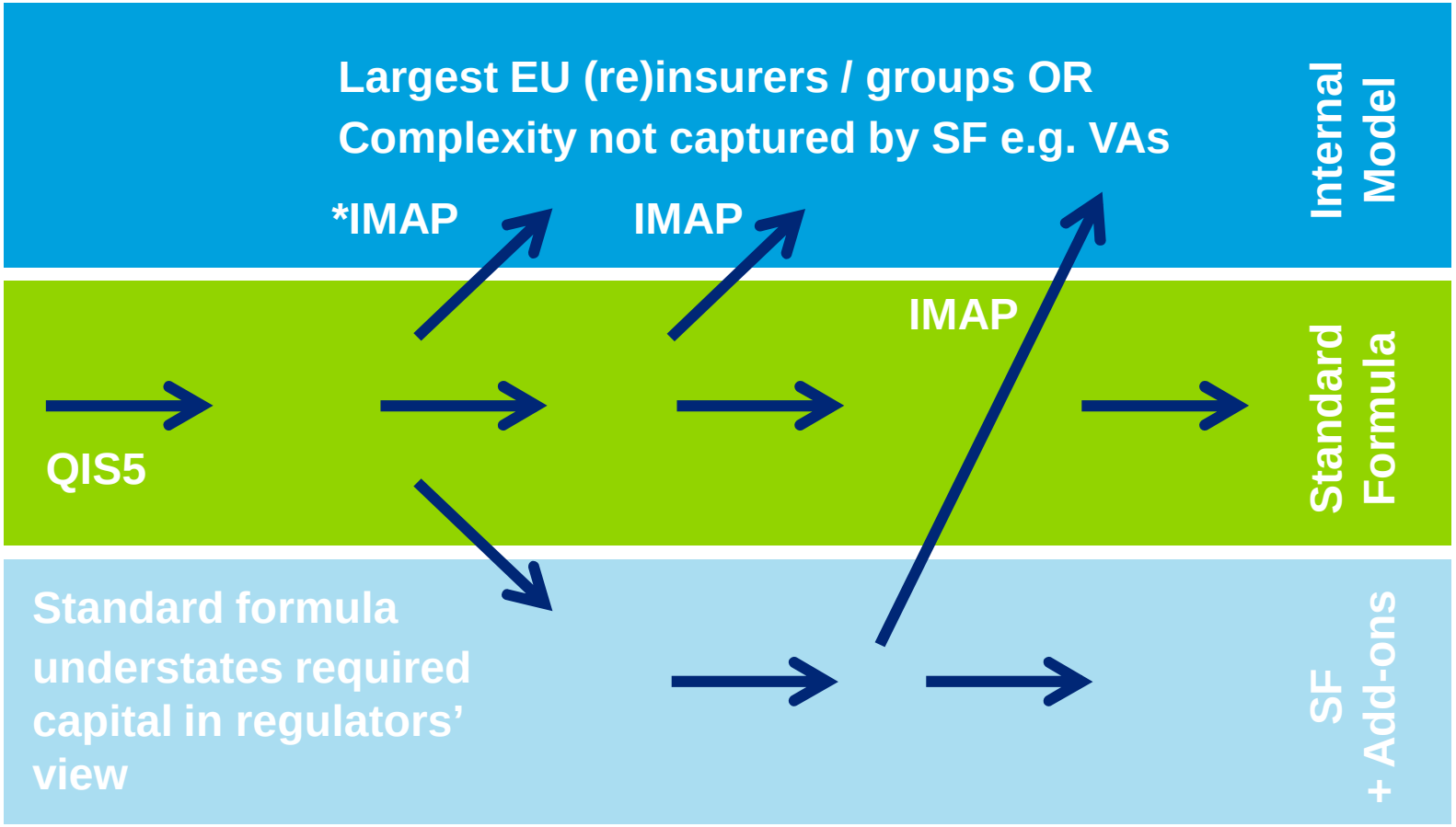


Solvency II Balance Sheet

http://ec.europa.eu/internal_market/insurance/docs/solvency/solvency2/faq_en.pdf

SCR Calculation: Standard Formula vs Internal Model

How firms and the regulator calculate the Solvency Capital Requirement



*IMAP = internal model application process

Internal Model validation

Regulatory requirements & purpose

Insurers seeking approval to use an internal model to calculate their capital requirements will have to demonstrate, as part of their IMAP, that they have had their internal model "independently validated" (Article 112 and Article 124).

Regulatory requirement

The specific "validation standards" requirements are outlined in Article 124 of the Directive and in the detailed text that supports the Level 2 and emerging Level 3 texts. The requirements include:

- having a regular cycle of model validation;
- monitoring the performance of the internal model;
- reviewing the on-going appropriateness of its specification;
- testing its results against experience;
- analysing the stability of the internal model;
- reviewing the sensitivity of the results to changes in key underlying assumptions;
- demonstrating the model's use – "use test"; and
- assessing the accuracy, completeness and appropriateness of the data used.

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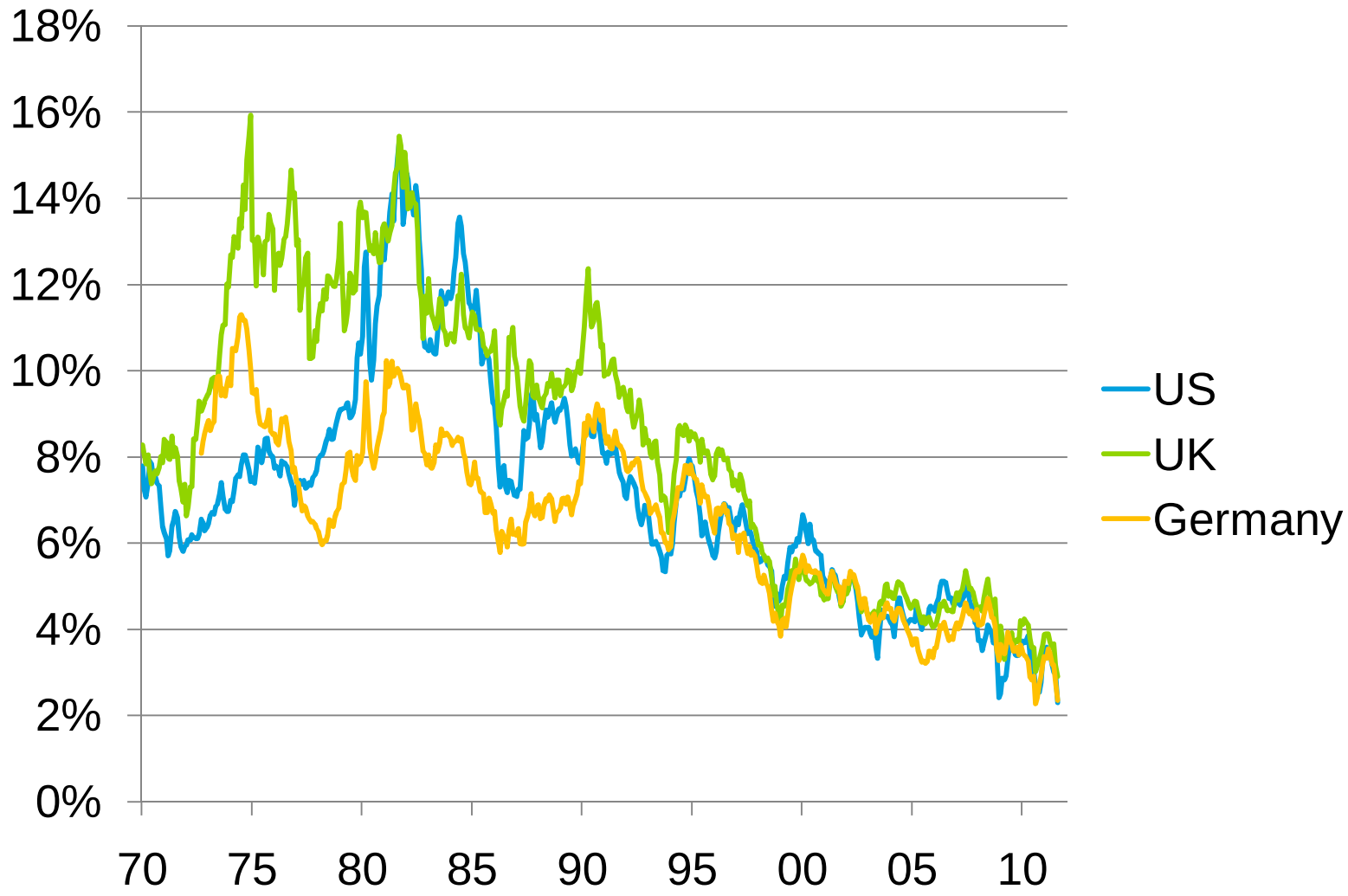
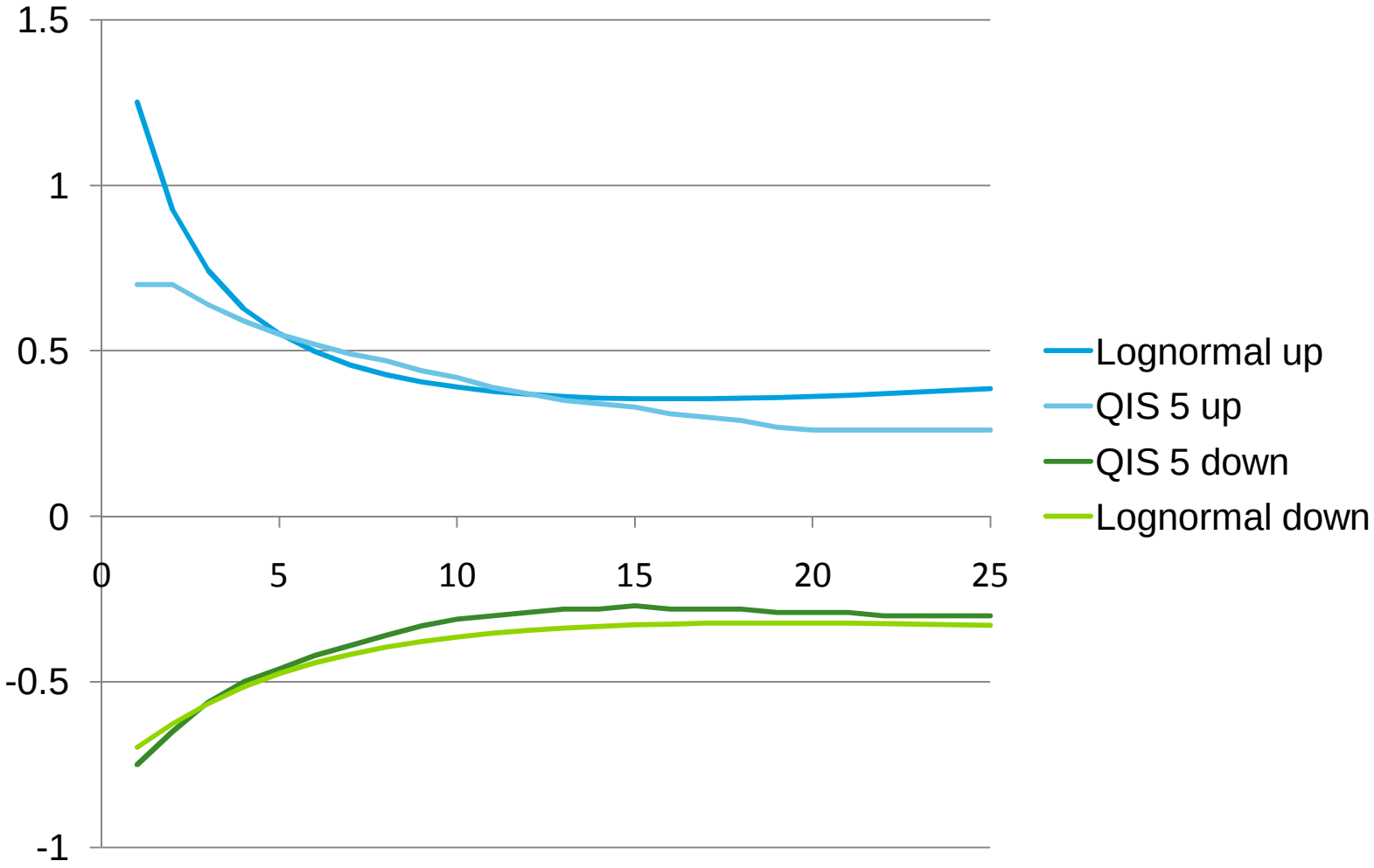
10-Year Spot Rates by Term, 1970-2011

Source: Federal Reserve, Bank of England, Bundesbank



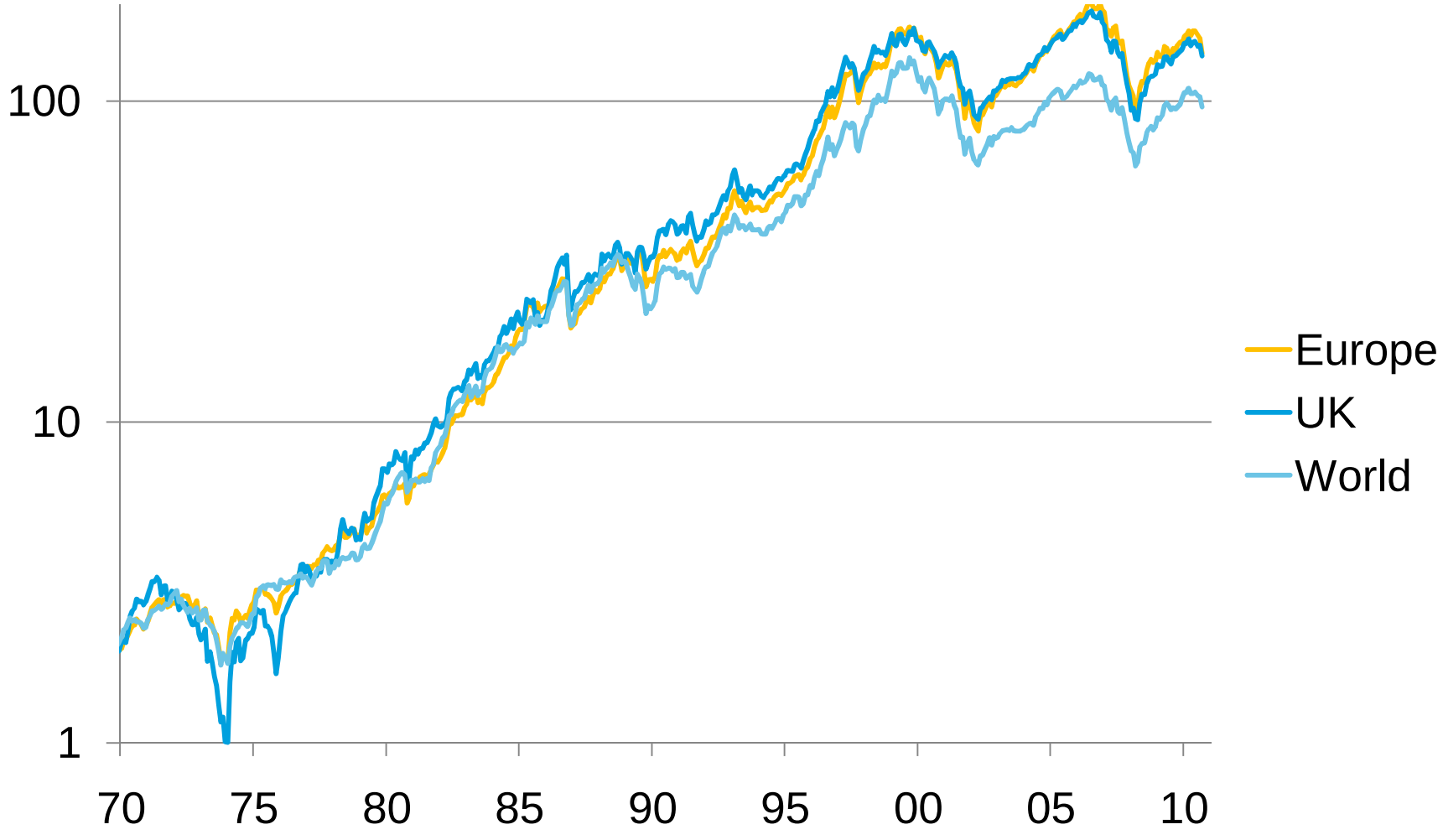
Predictive Distribution for Change in Bund Spot Curve

Comparison of Lognormal Model with QIS 5



Historic Equity Returns

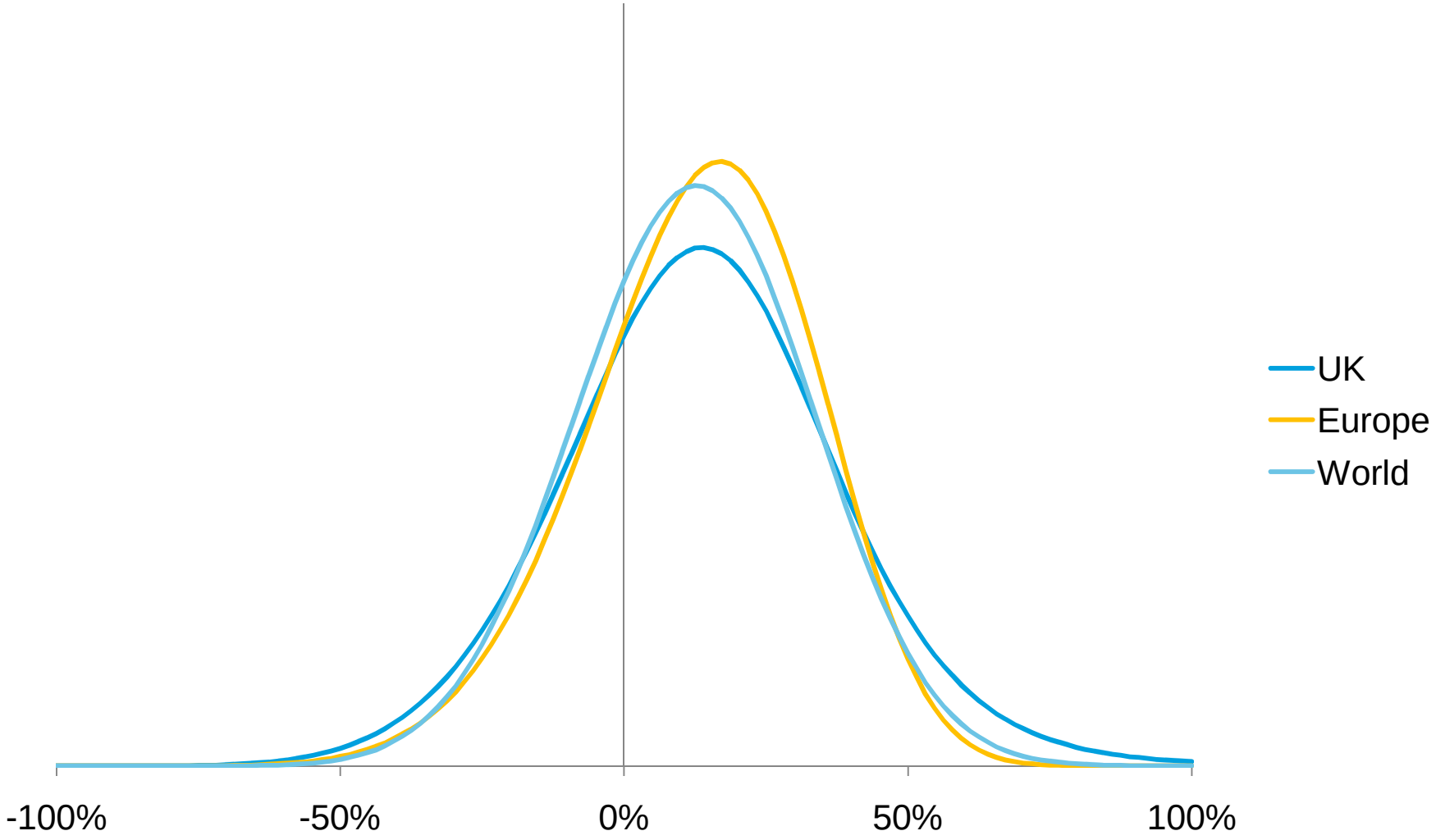
Total return MSCI in Euro (reconstructed basket prior to 1999)



Source: Datastream

Fitted Return Distributions

Based on EGB2 distribution family



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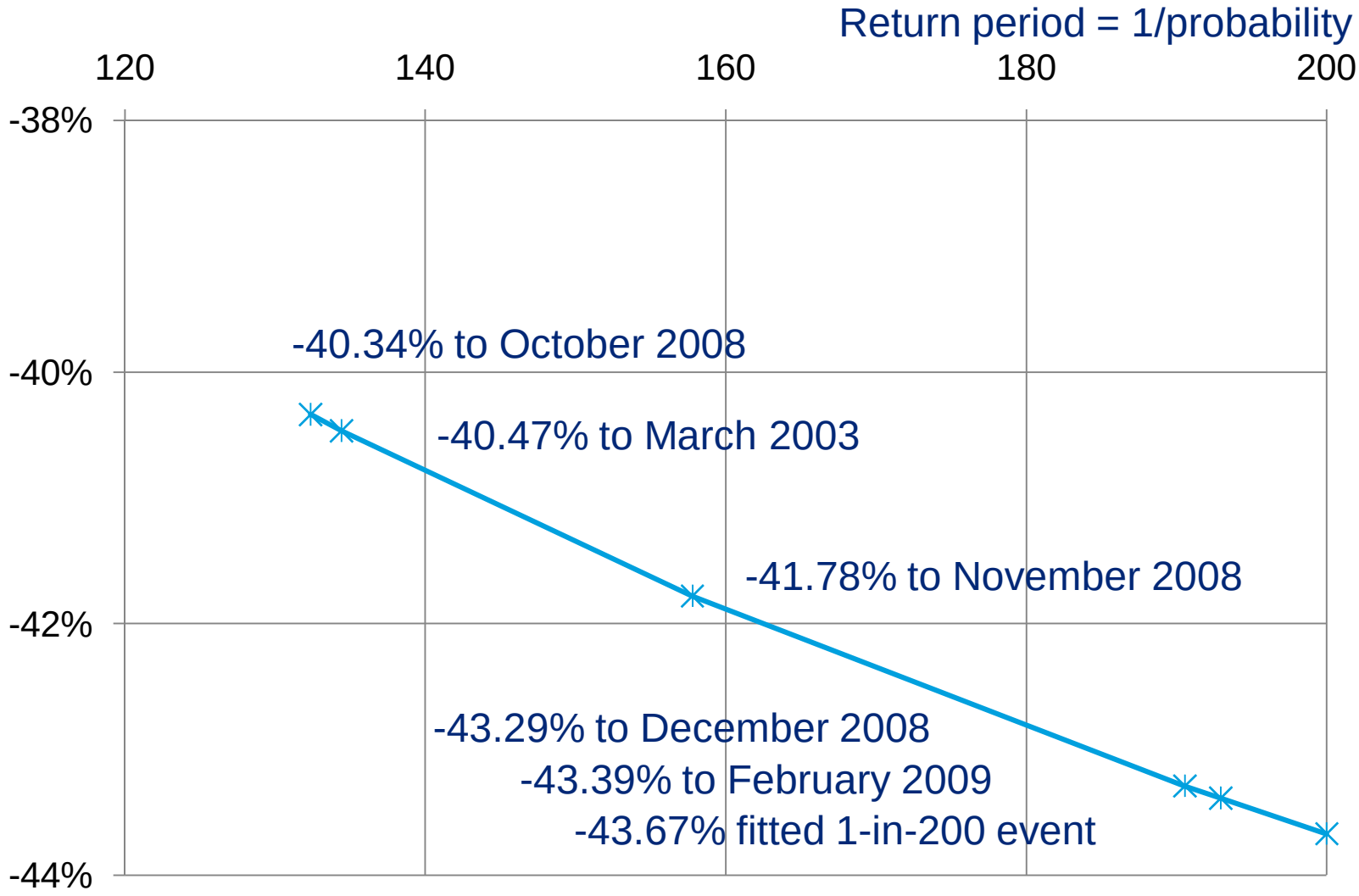
Extreme Histories: The Living Memory Test

Largest Falls in MSCI Europe € Index since 1970



Fitting to Overlapping Intervals

Monthly time series analysis may fail the “Living Memory” test

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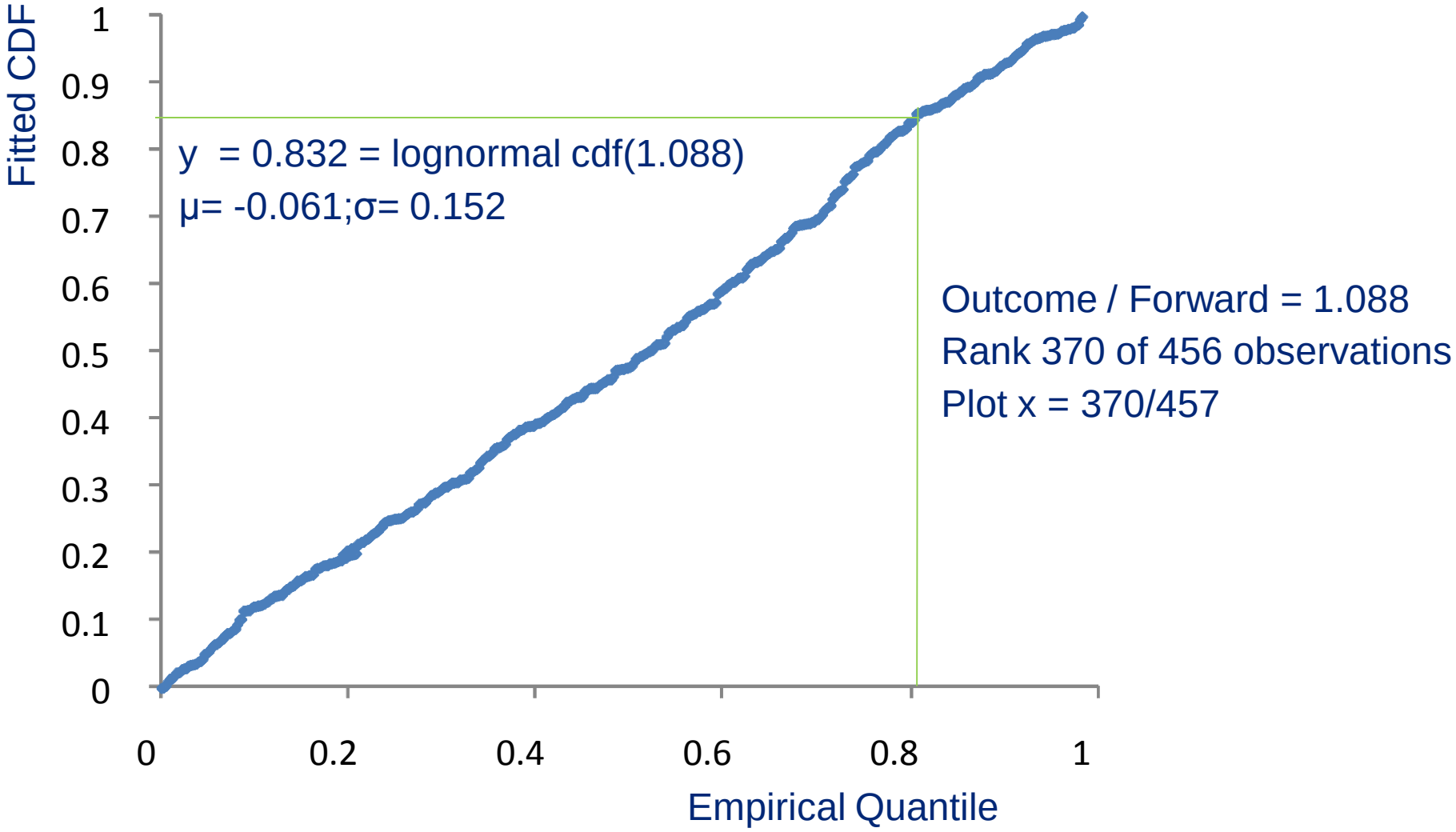
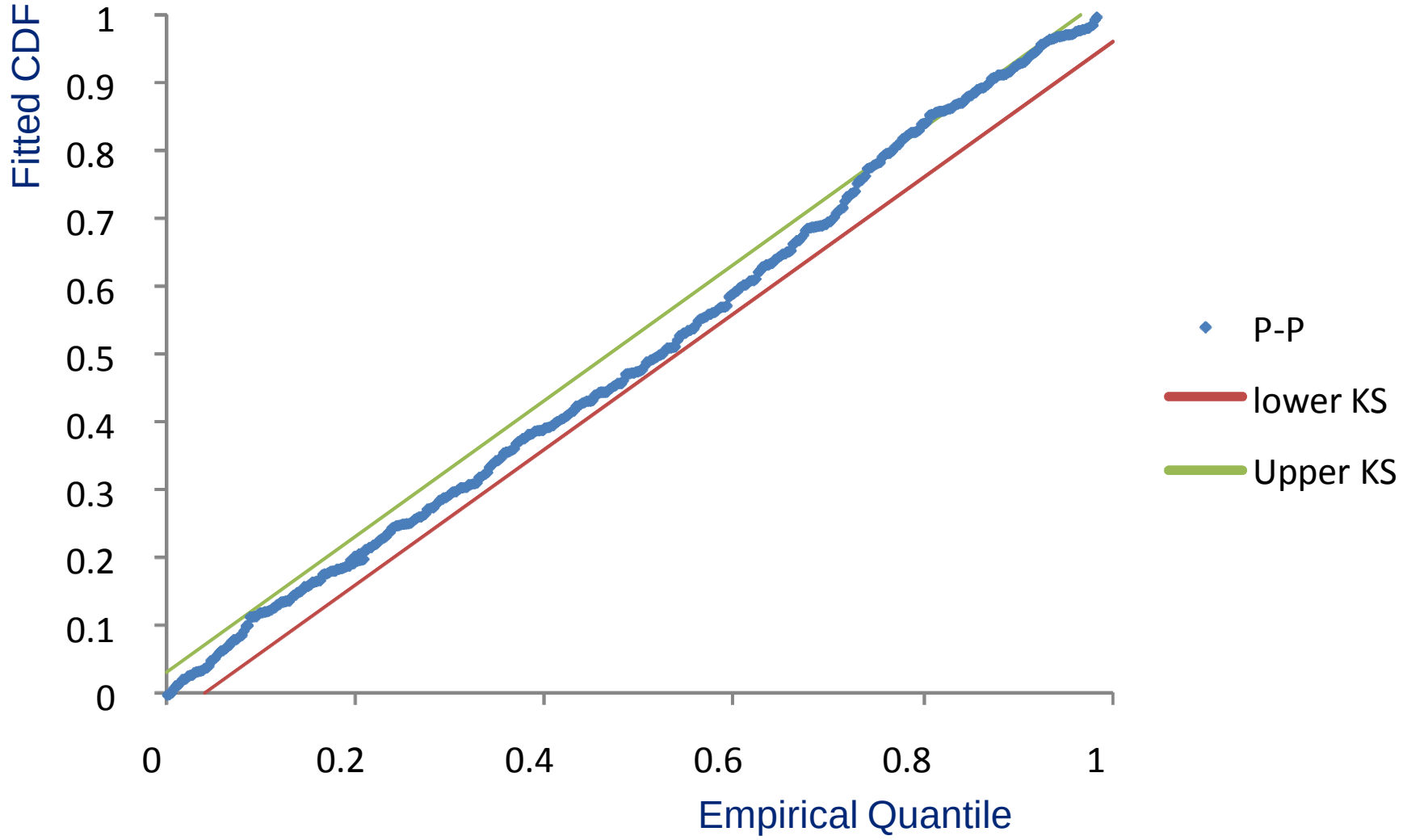
P-P Plot (10 year interest rate)

Visualising Goodness of Fit



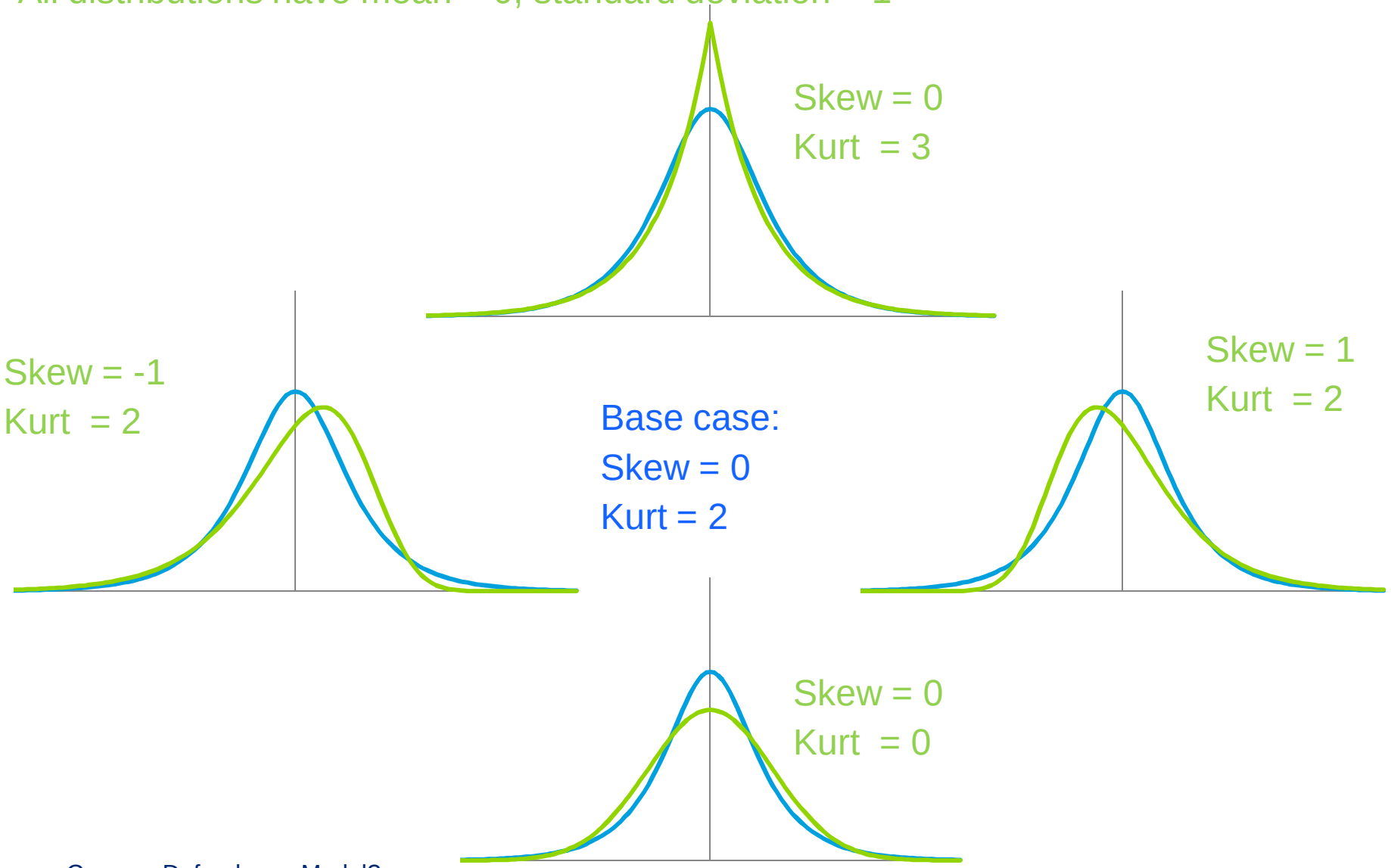
Kolmogorov-Smirnov Goodness of Fit Test

Look at largest deviation in P-P Plot



Skew & Kurtosis to fit Equity Returns

All distributions have mean = 0; standard deviation = 1



Can you Defend your Model?

References

Some textbooks to read in your spare time!

Continuous Univariate Distributions by Johnson, Kotz and Balakrishnan (JKB), published by Wiley in two volumes (Norman Johnson was an actuary).

Volume 1 of Kendall Advanced Theory of Statistics (revised by Stuart & Ord).

Quantitative Risk Management by McNeil, Frey and Embrechts (Princeton).

Pearson Type IV (covered in JKB, vol 1 p15 et seq and in Kendall p221). Also look at http://www-cdf.fnal.gov/physics/statistics/notes/cdf6820_pearson4.pdf

Johnson's SU distributions. Covered in JKB vol 1 p33 and Kendall p240.

EGB2 distributions. Covered in JKB, vol 2 p141

Generalised hyperbolic distributions. Treated by McNeil et al p78. See also http://cran.r-project.org/web/packages/ghyp/vignettes/Generalized_Hyperbolic_Distribution.pdf

MULE distributions. You won't find these in the literature because they are my invention. Unlike the other classes, the MULE (mixed exponential uniform logistic) permits distributions with negative kurtosis, with uniform logistic and exponential distributions as special cases. The inverse CDF is a linear combination of {1,x, ln(x), ln(1-x)}.

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Pro and Contra-Cyclical Tests

Arguments to avoid pro-cyclicality

Unconditional estimates = “Through the cycle”

- Average over states of the world
- Estimate through historic distributions, as in this presentation
- Can satisfy “1-in-200” test
- Capital requirements can increase following large market moves

.

Conditional estimate = “Point in Time”

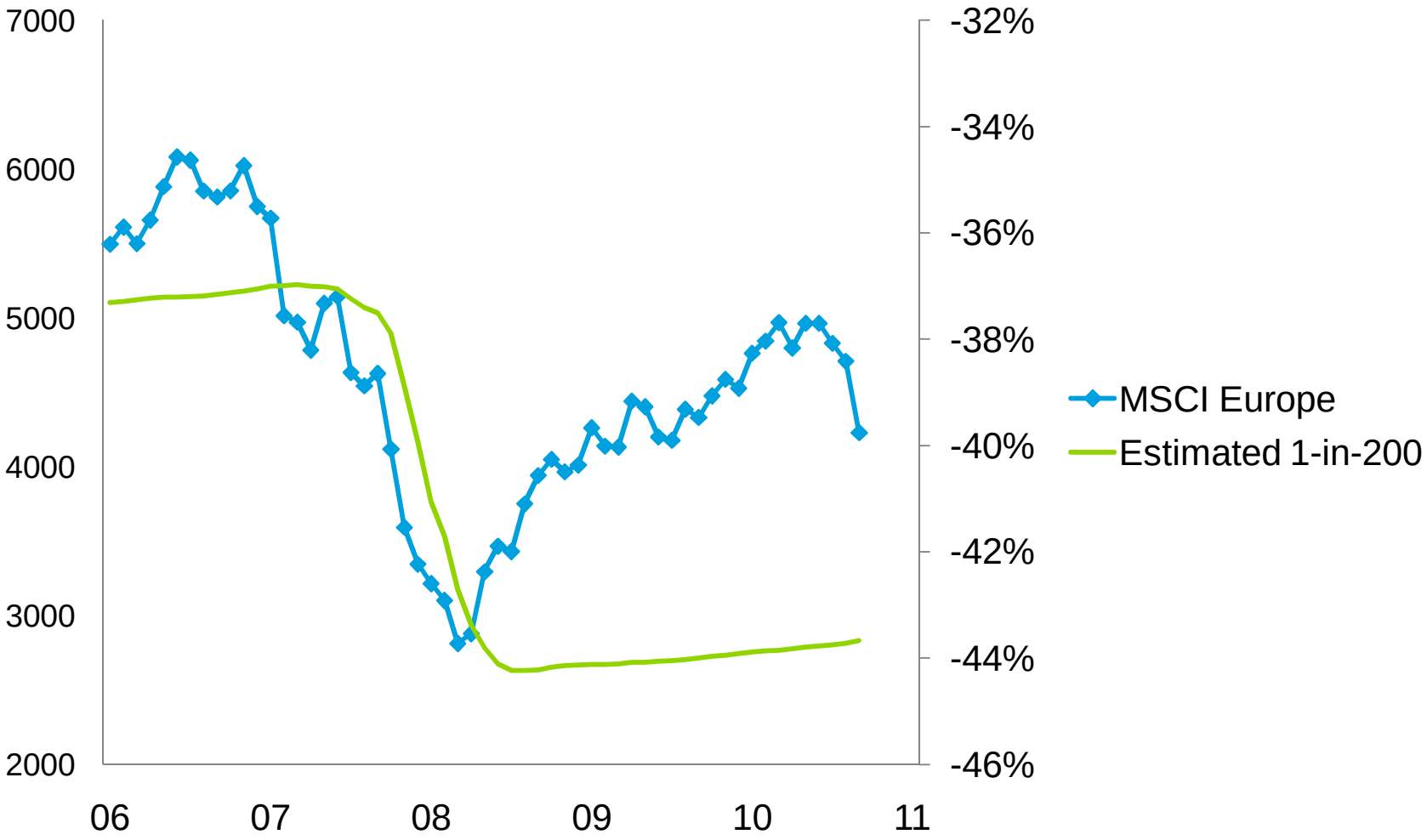
- Given current state of the world
- Empirical validation by comparing to historic periods with the same starting point
- Sensitive to time series model formulation
- Capital requirements may rise suddenly from small market moves, making this approach commercially unattractive
- Arguments against this approach based on fear of “pro-cyclicality”

Fitting Methods: In Search of Stability

Method	Feature Replication	Fit Optimisation
Examples	Method of moments Modal fit	Maximum likelihood Minimax cdf difference (minimise Kolmogorov-Smirnov)
Pros	Can prove it has worked	Most powerful for large n Parameter stderr known for large n
Cons	Need Plan B outside feasible set	Solution may not exist May not converge Difficult to demonstrate method has worked

Rolling Estimates

Effect of 2008 Crisis on Estimated Stress Tests (fitted to data since 1970)



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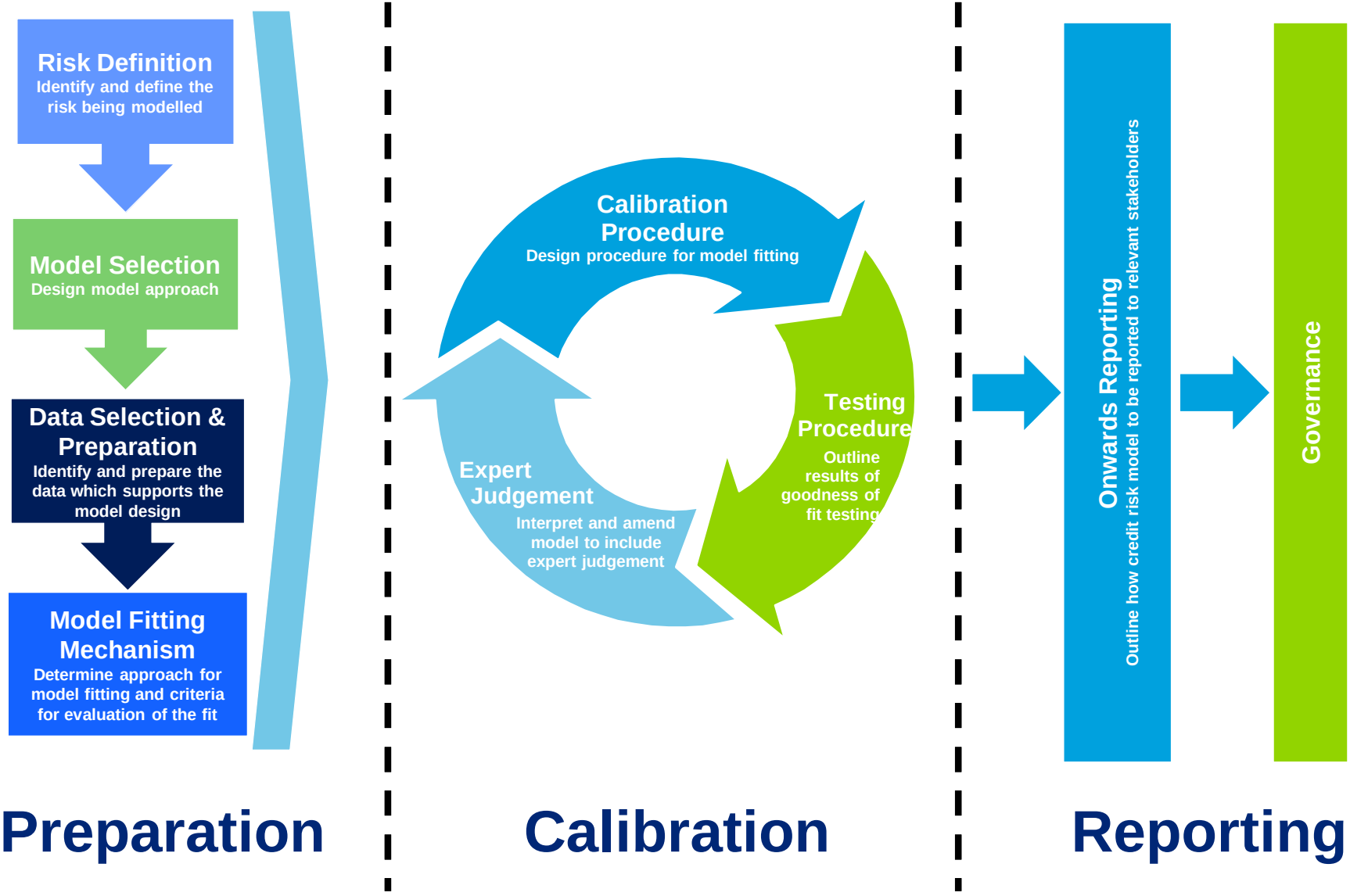
Many firms have different specialists to model each risk

Roles and Responsibilities

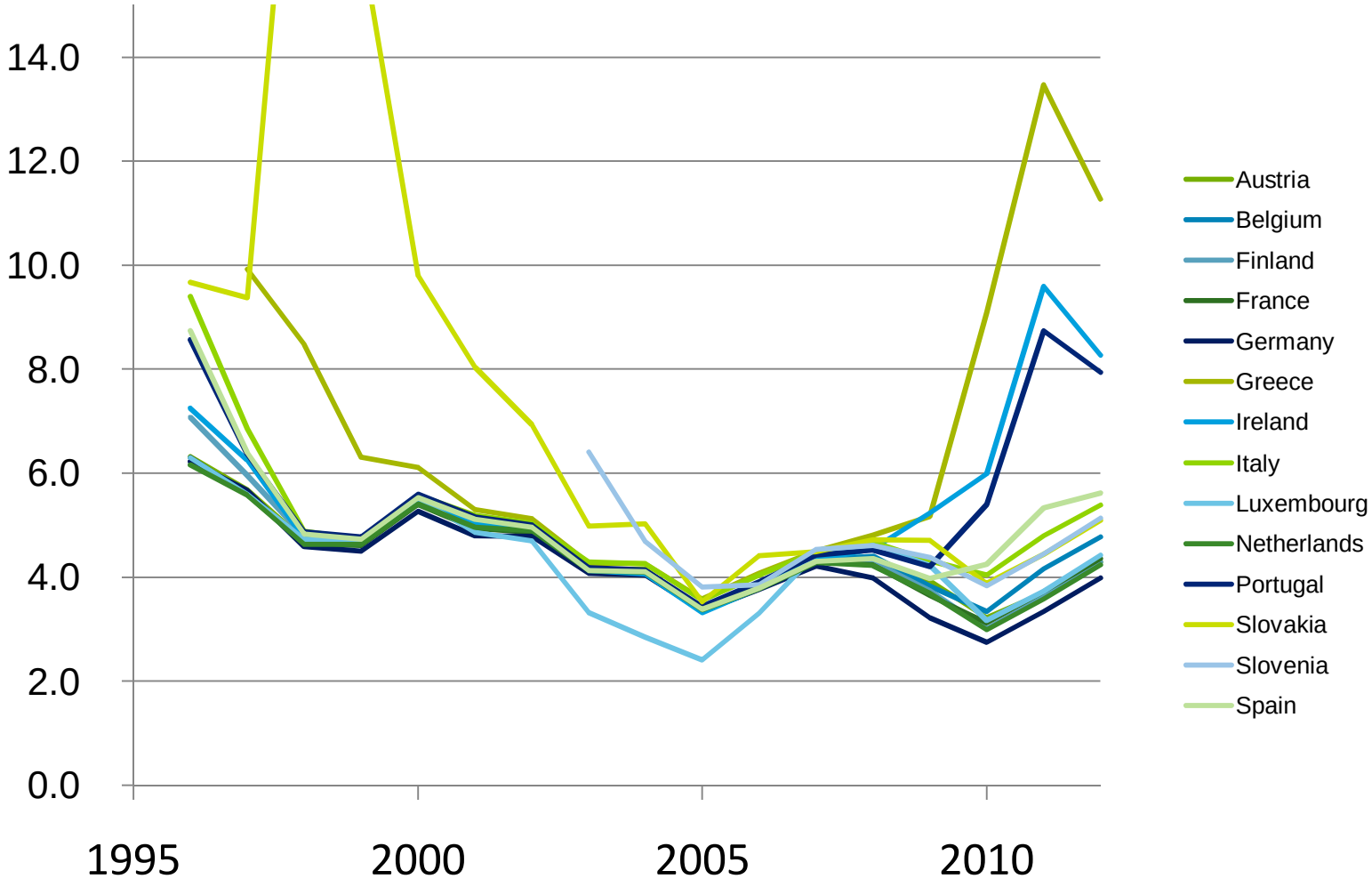
Data	Calibration	Simulation	Documentation	Validation
Equity risk: Bill Brewer PhD (GARCH models)				
Interest Rate Risk: Jan Stewer PhD (Principal Components Analysis)				
FX Risk: Peter Gurney, PhD (Modern Languages)				
Liquidity Risk: Peter Davy PhD (Fluid Dynamics)				
Counterparty Credit Risk: Daniel Whiddon PhD (Gibbs Sampler)				
Lapse Risk: Harry Hawk PhD (Markov processes)				
Operational Risk: Tom Cobley PhD(Modern dance)				

What a modelling framework for all risks might look like...



Application of Expert Judgement

10-year rate (%) for Euro-zone governments (source OECD)



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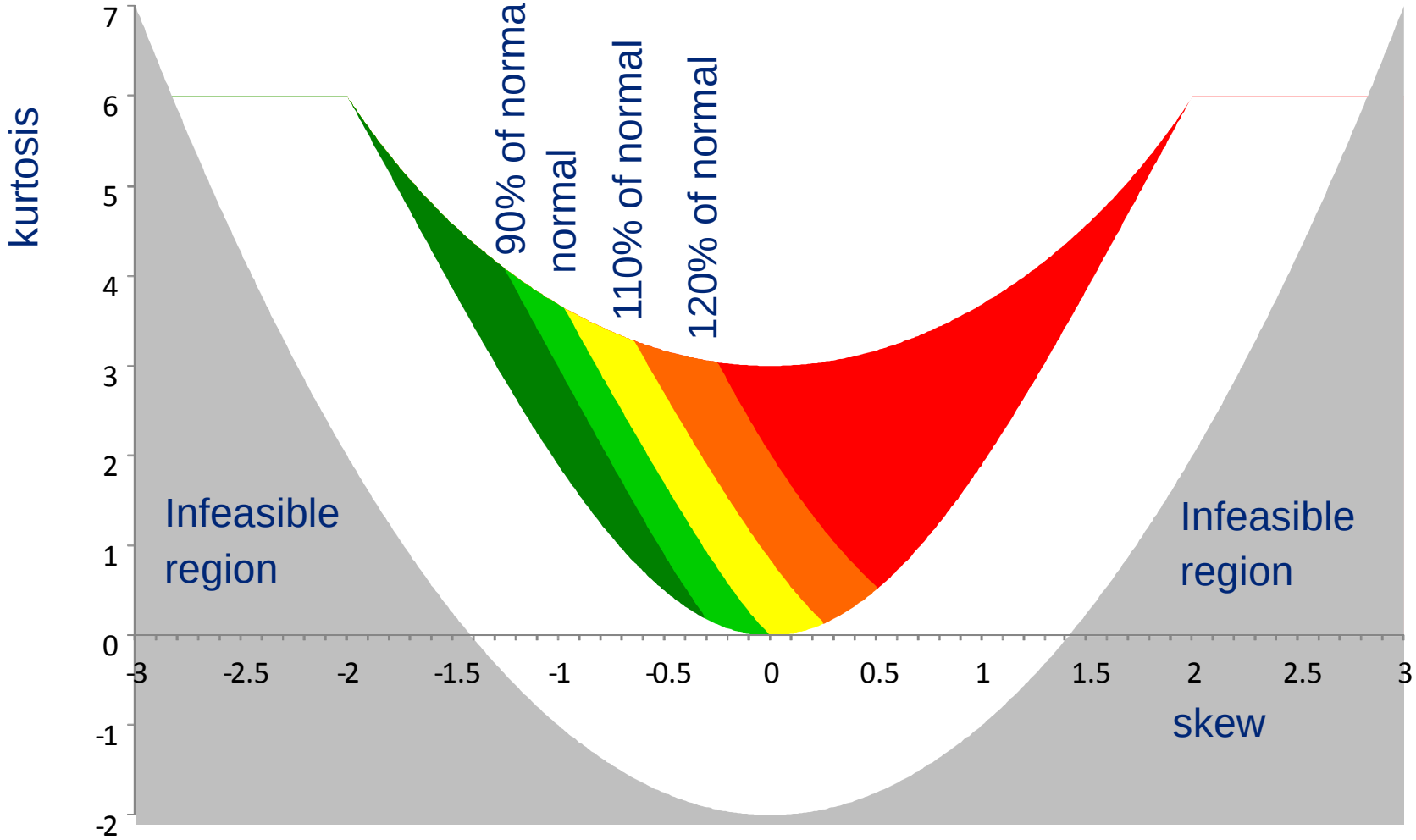
Upper Stress: Effect of Skewness and Kurtosis on EGB2

Contours of 99.5%-ile expressed as number of standard deviations

To consider a 0.5%-ile (eg equity fall), reflect this diagram left to right.



Example Equity Stress Test Calculation

Fitted Gamma Power Distribution to MSCI Europe Returns

Parameter	Fitted Parameters
α	1.346608
β	1.999722
power	0.185368

Quantile q	Gamma inverse G = gammainv(q,α,β,true)	Return $R = G^{power}-1$
0.5%	0.045219	-43.67%
10.0%	0.455908	-13.55%
50.0%	2.063342	14.37%
90.0%	5.761522	38.35%
99.5%	12.17842	58.94%

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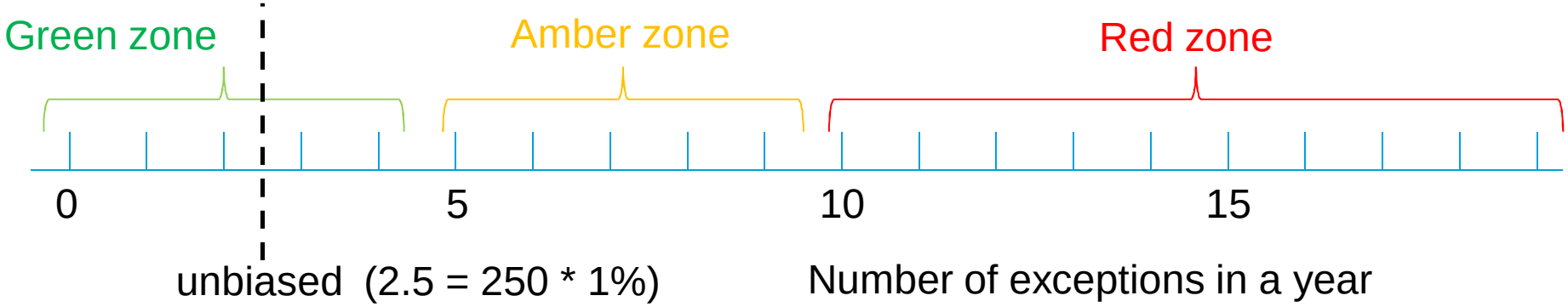
Validation under Basel - Banks

Banks have different rules:10 day VaR at 99% Confidence

- Look back over last year (250 trading days, overlapping periods each looking 10 days back) in which both VaR and profit are updated



What does this process test?

- The “back test” includes implicit tests of model and parameter error as well as outcomes
- Although it won’t test risks that didn’t materialise in the last year

Possible Approaches to Out-of-Sample Review

and their limitations

A. Check the documentation and formulas against best statistical practice.

But documentation is very lengthy and there's a shortage of real experts to conduct in-depth reviews.

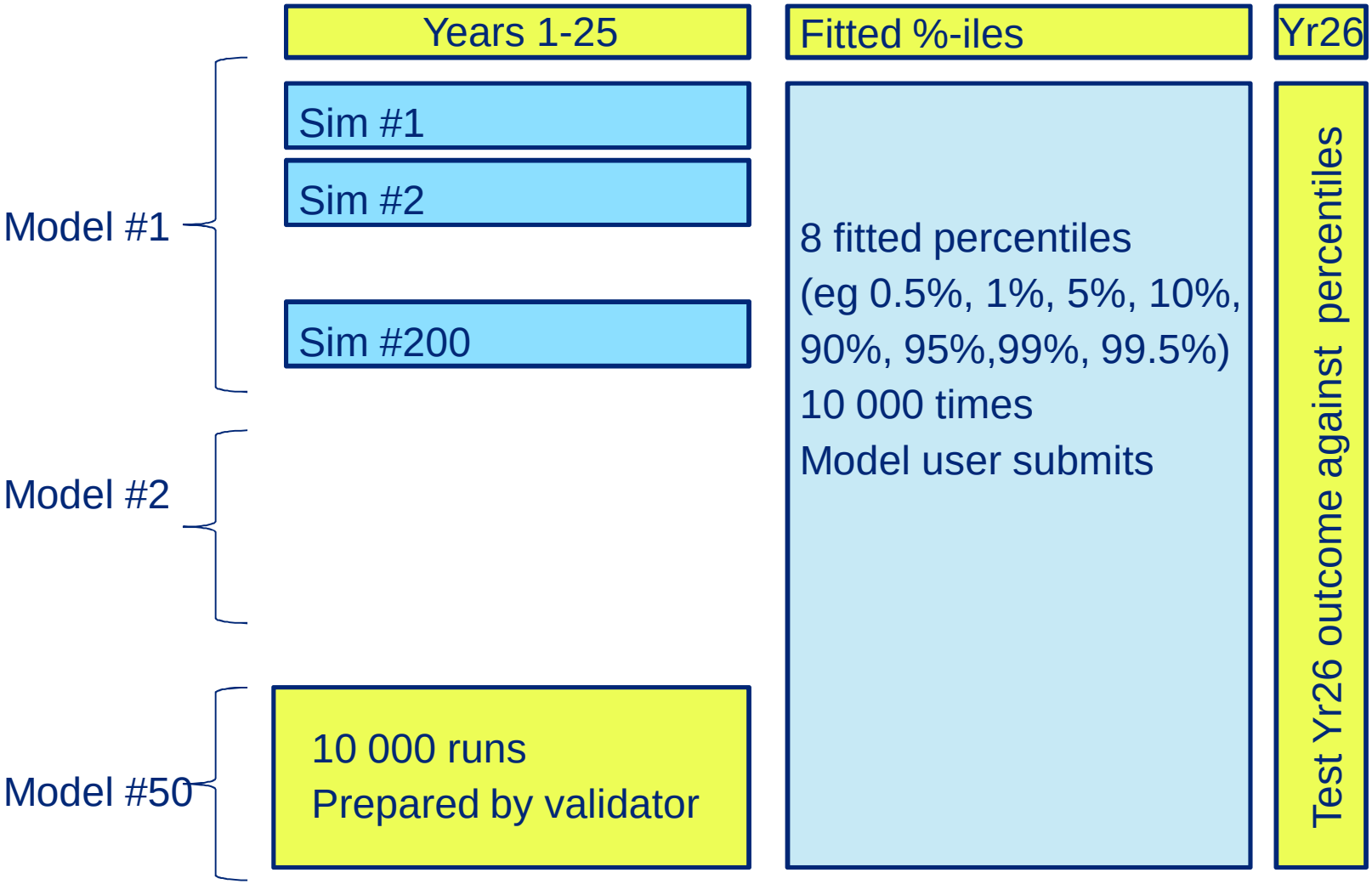
B. Compare insurers, and, (as with ICAS in UK) invite insurers with the most aggressive assumptions to reconsider them on a risk by risk basis.

But now more difficult because you are comparing probability distribution forecasts rather than stress tests so its not clear who is being most prudent. This is only a test of relative numerical conformity rather than confirmation of the 1-in-200 standard.

C. Require back-testing as with VaR models under Basel.

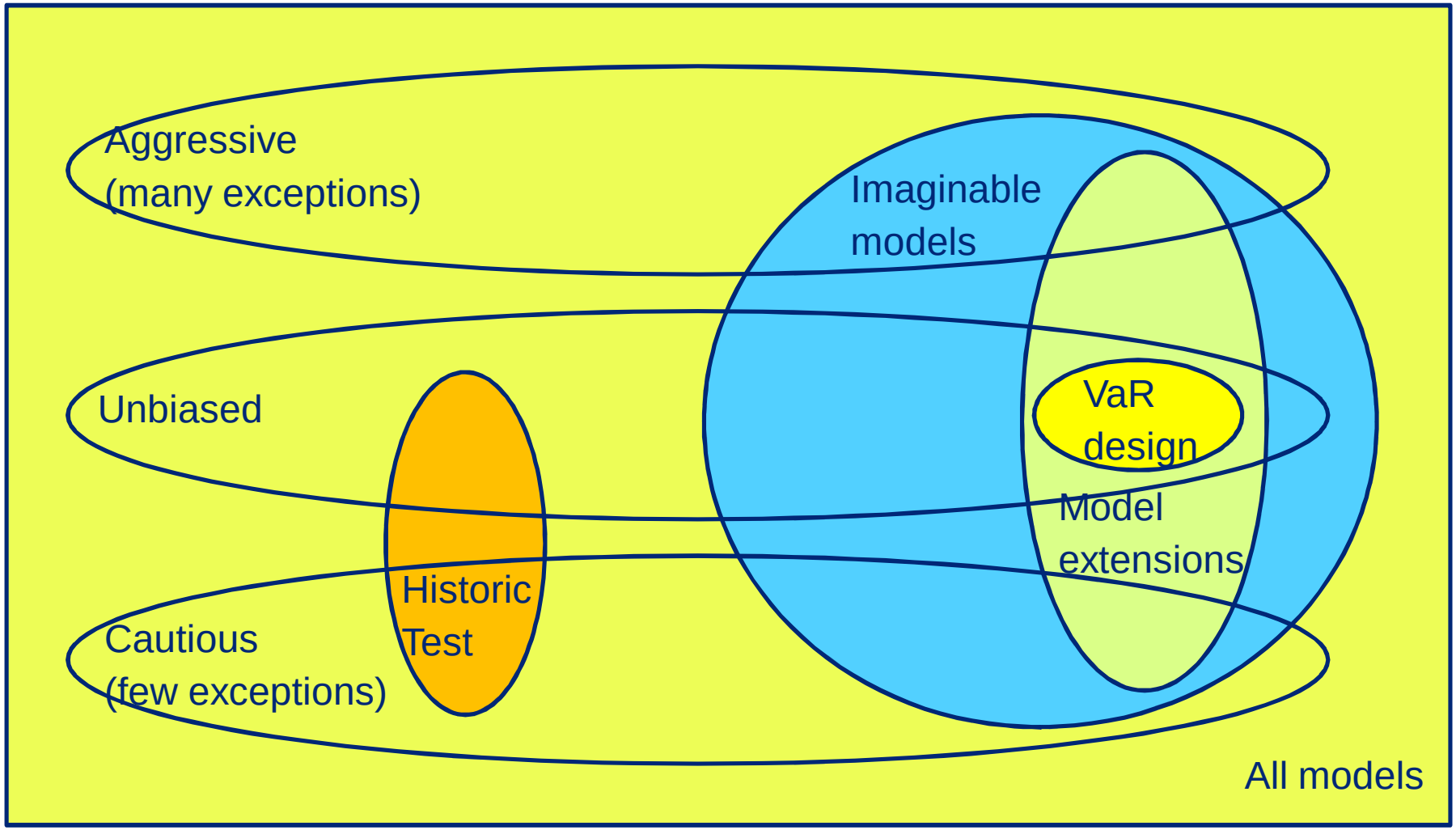
Under Basel II, VaR is calculated at 99% confidence over 10 days. Allowing overlapping intervals and 250 trading days over a year, a correct model should produce 2.5 exceptions. Based on 1-year 99.5% VaR, you would need 500 years of test data for insurers

Monte Carlo Calibration Test



Validating VaR

Understand what makes a method prudent or aggressive



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Internal Model validation

Practical challenges

Timing	• The Solvency II Directive requires an insurer's internal model to be independently validated at regular intervals once Solvency II is fully implemented. However, validation is also important before Solvency II implementation:
	  • Integrating validation modules into the process of developing, building and testing the model provides greater confidence in the model and reduces the risk that late stage validation identifies major re-working of the model.
	  • A complete independent validation must be provided to the Board as part of the evidence to support their approval of the model before it is submitted for review by the CBI.

Board involvement	• The validation policy and report will be used by the Board when reporting to the CBI. The validation report will need to be accessible to all members of the Board, taking into account their varying experience and familiarity with Solvency II. • The validation report should addresses the scope of the validation, the strengths and weaknesses of the model and the data and tools used in the validation process.
Documentation	• Detailed and complete validation documentation will help facilitate internal model approval. • Validation documentation should address model theory, model implementation and model governance.
Risk assessment	• "Expert judgement" and "data" are likely to be high risk areas given the subjectivity and regulatory scrutiny respectively around these inputs.

Internal Model validation

Practical challenges

Independence	• An effective internal model validation process requires independent and objective challenge. The use test should provide evidence that the model has been challenged.
	
	• Independence is a strict requirement but can be achieved in a number of ways, or through a combination of:
	  • Existing resources
	  • An internal audit team with specialist skills
	  • External resources / auditors
	
	• Care should be taken to ensure that model validators are independent from those who have been involved in designing and building the model . Reporting lines should also be independent.

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