Stochastic reserving and reserve risk in nonlife insurance

Actuarial seminar

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23/11/2018

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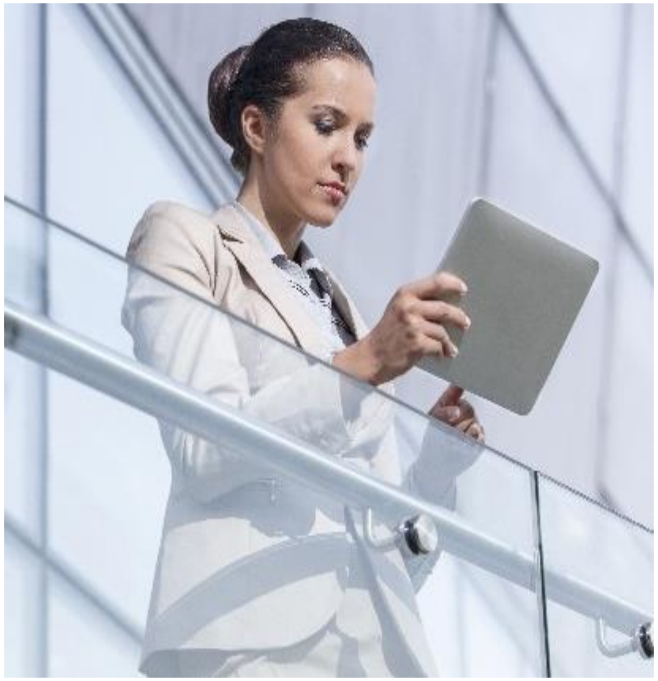
Contents

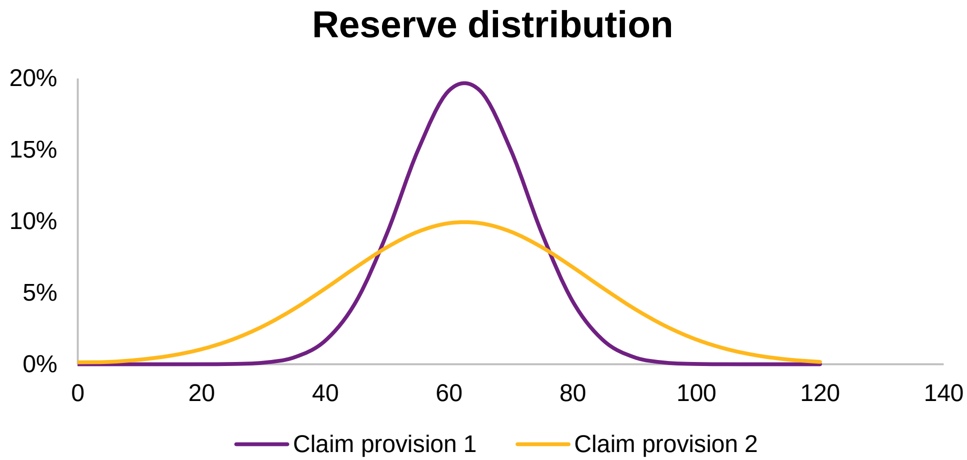


Introduction

Best estimate claim provisions

Basic reserving task – calculation of claim provisions

- Claim provisions (IBNR + RBNS) should cover insured claims incurred in the past and settled in the future and related expenses
- The value of the claim provisions is a random variable
- Concept of the best estimate of claim provisions
- Chain ladder (or in general development factor method - DFM), Bornhuetter Ferguson, ultimate claim ratio method, etc.
- Parametrization of method based on expert judgment, supported by qualitative information about portfolio development
- Uncertainty connected to the calculated claim provisions may pose considerable risk



Introduction

Ultimate vs one year view

Ultimate view

- Uncertainty related to claim provisions until run off of the claim provisions (all claims settled)

One year view

- Uncertinty related to claim provisions from the one year view
- One year view is identical to standard one year run off of claim provisions
- The sources of one year uncertainty are
  - Actual vs expected claim payments during next year
  - Recalculation of claim provisions after one year

$$\mathbf{R_0 = R_1' + C_1'}$$

$$\textbf{Run off result} = \mathbf{R_0 - R_1 - C_1 = (R_1' + C_1') - R_1 - C_1 = \underbrace{(R_1' - R_1)}_{\text{Recalculation of claim provisions after one year}} + \underbrace{(C_1' - C_1)}_{\text{Actual vs expected claim payments during next year}}}$$

Where

- R_i is claim provision at time i, i=0,1
- R_1' is claim provision at time i according to expectation at time 0
- C_1 are actual claim payments between time 0 and time 1
- C_1' are actual claim payments between time 0 and time 1 according to expectation at time 0

Introduction

Distribution of claim provisions

Understanding and quantification reserve uncertainty

Estimate of single additional characteristics

- Mack formula – estimate of standard deviation for ultimate view
- Merz Wüthrich formula – estimate of standard deviation for one year view
- Mack and Merz Wüthrich formula are distribution free, estimates of percentiles require additional distribution assumptions

Estimate of full distribution function of claim provisions

- Stochastic methods
- Mack method, ODP method, etc. for ultimate view
- Parametrization of methods based on expert judgment, supported by qualitative information about portfolio development
- The knowledge of the full distribution function allows quantification of uncertainty via different measures, for example
  - VaR (percentile)
  - TVaR
  - Standard deviation

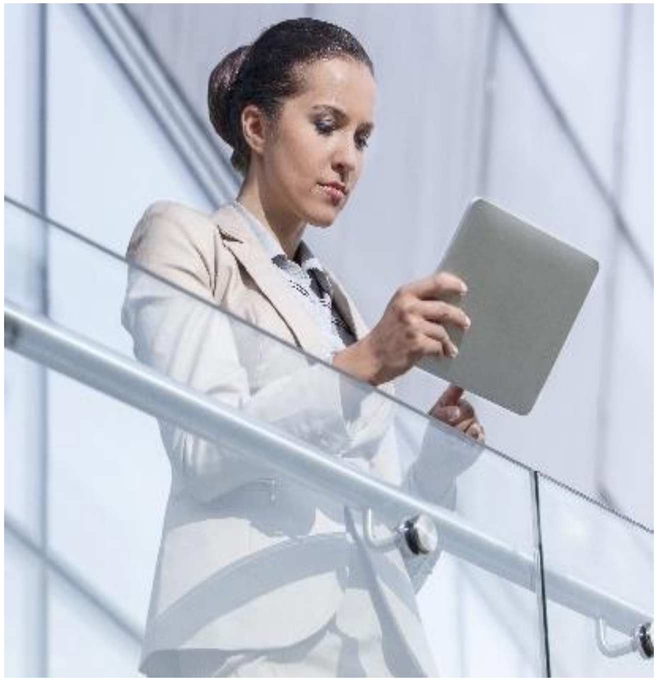
Introduction

Application of risk quantification

Examples of application of risk quantification

- Solvency II internal model
- Validation of technical provisions
- Appropriateness of standard formula
- IFRS 17 risk margin and disclosure
- Reinsurance strategy
- Asset liability matching
- Own risk management and understanding of the risk and portfolio

Contents

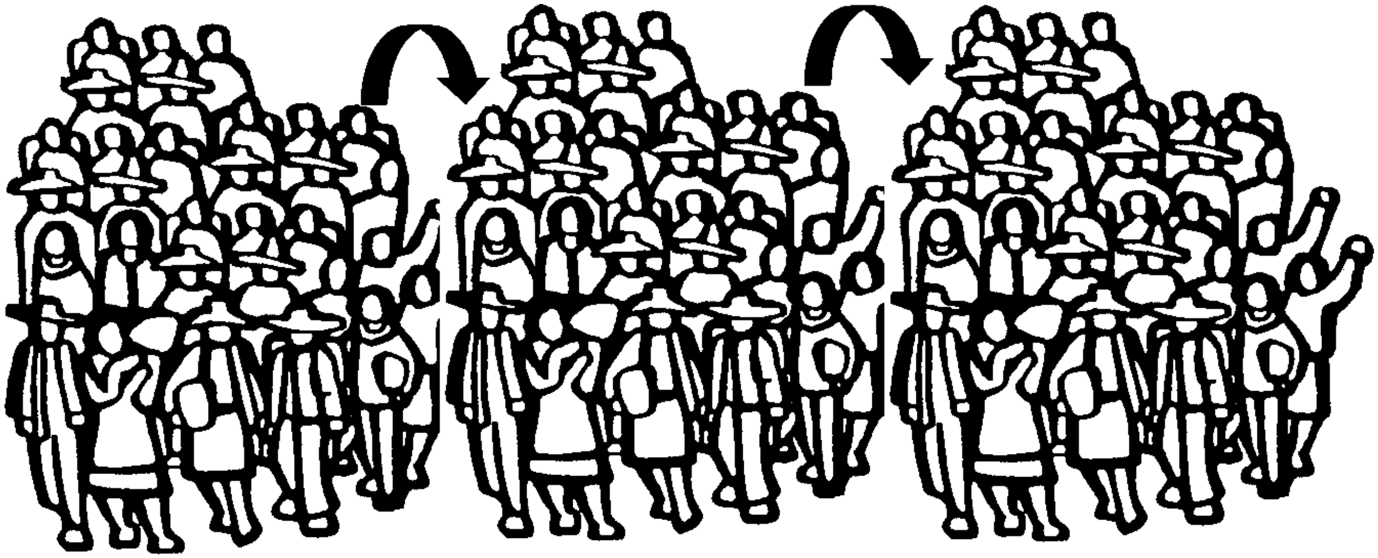


Bootstrap approach

Pseudo data

Creation of “more histories” of the random variable

- Creation of pseudo data
- The set of pseudo data should have similar characteristics as the original data
- The pseudo data is created through random permutation with repeating
- The core data should be comparable, i.e. come from the same probabilityy distribution, therefore preliminary adjustments are sometimes necessary (for example standardization)



Bootstrap approach

Pseudodata in development triangle

Pseudodata = alternative development triangles

- ODP model
  - The best estimate chain ladder development factors can be used to produce development triangle with “best estimate history” – the factors are applied to the diagonal in reverse order
  - The difference between the actual triangle and triangle with “best estimate history” provides residuals to be used for bootstrapping (permuted standardized residuals create pseudo triangles)
  - ODP model uses residuals coming from the difference of actual and “best estimate history” incremental cumulative payments in particular cells
- Mack model
  - Mack model uses residuals coming from the difference of actual and best estimate development factors for particular accident and development periods
  - Permuted standardized residuals are applied on the last diagonal to create pseudo triangle
- The residuals need to be standardized as their characteristics are different for particular triangle cells

	1	2	3	4	5	6	7	8	9	10
1	208,0	0,0	-139,1	-386,0	209,6	643,2	-291,9	-122,7	0,0	0,0
2	-49,2	68,1	-59,8	160,9	-220,5	-168,2	310,7	30,3	90,3	
3	-166,5	94,8	-57,3	-27,6	285,1	-499,8	255,3	71,9		
4	-115,3	251,3	-228,8	658,2	-483,9	-89,5	-324,5			
5	227,0	-195,8	150,6	-216,1	-26,2	217,0				
6	87,2	72,9	-97,8	-227,2	265,4					
7	96,0	-161,3	132,8	-36,1						
8	-199,5	-124,3	242,9							
9	-28,2	16,6								
10	0,0									

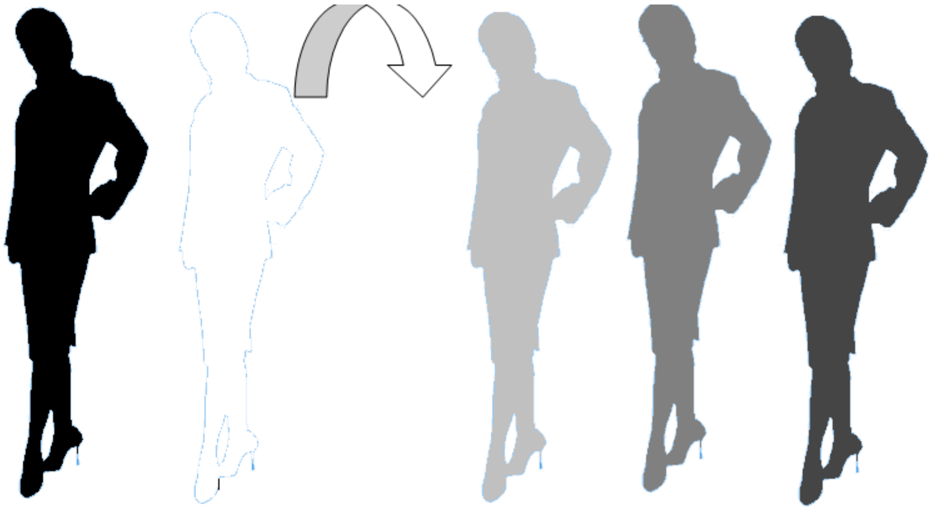
Bootstrap approach

Process error

- Pseudo data is a limited set and can produce only a limited set of results with in advance limited cardinality
- The prediction variance can be expressed as:

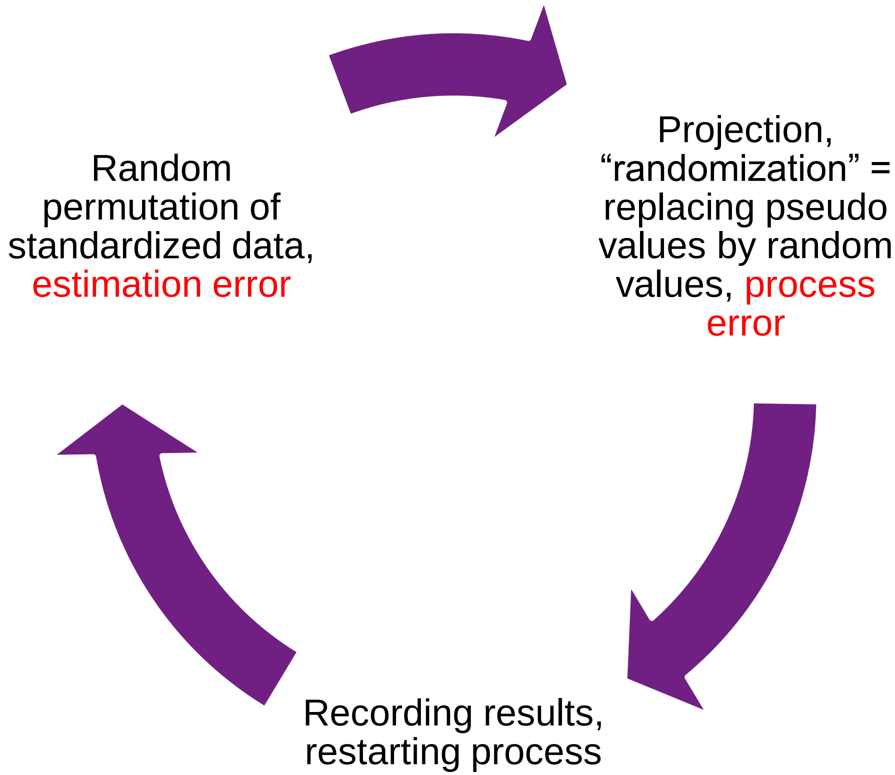
prediction variance = estimation variance + process variance

- To capture also the process error, the bootstrapping can be enriched through “randomization”– replacing values in an appropriate calculation step by a generated random value, this incorporates also process error
- This requires additional assumption regarding the distribution function of data
- Correctness of assumptions, especially correctness of assumed model need to be tested to minimize the model risk



Bootstrap approach

Outline of algorithm



ODP model

Assumptions

Assumptions of ODP model

- Over dispersed Poisson distribution of incremental values
- We assume that each incremental value $I_{i,j}$ in origin period *i* and development period *j* consists of a systemic and random component and

$$E[I_{i,j}] = m_{i,j} \qquad Var[I_{i,j}] = \phi m_{i,j}$$

$$log(m_{i,j}) = \eta_{i,j} \qquad \eta_{i,j} = c + \alpha_i + \beta_j \qquad \alpha_1 = \beta_1 = 0$$

Where

- Parameter ϕ is called scale parameter. It can be either constant for whole triangle or different for particular development periods
- Coefficients α_i, β_i can be found through the method of maximum likelihood. This can be short calculated by chain ladder method.

ODP model

Algorithm

1 Initial model fit

- The starting point is fitting chain ladder method.
- The chain ladder model is used to derive incremental fitted values $m_{i,j}$ based on the observed cumulative latest diagonal.

2 Calculation of residuals

- Calculate unscaled Poisson residuals

$$u_{i,j} = \frac{I_{i,j} - m_{i,j}}{\sqrt{m_{i,j}}}$$

- Calculate scale parameter

$$\phi = \frac{\sum_{i,j} u_{i,j}^2}{n - p}$$

Where

- n is number of fitted values (cells of underlying development triangle)
- p is number of parameters in chain ladder model

ODP model

Algorithm

- Calculate standardized residuals and apply bias adjustment

$$r_{i,j} = \frac{u_{i,j}}{\sqrt{\phi}}\sqrt{\frac{n}{n-p}}$$

3 For each simulation

- Create triangle of pseudo data – based on random permutation with repeating of residuals $r_{i,j}$ from those derived at step 2 above to calculate a pseudo triangle data value
- Re-apply chain ladder method based on the pseudo data to generate new projections and reserves
- Add process variance to projected future development – replace projected values by random values from selected distribution

4 Calculate selected measure

distributions, percentiles, means and standard deviations for reserves and other items of interest

Mack model

Assumptions

Statistical model underlying chain ladder, assumptions of Mack model same as chain ladder

- Expected value of cumulative value of claims

$$E\left[C_{i,j+1} \middle| C_{i,1}, \ldots, C_{i,j}\right] = f_j C_{i,j}$$

- Variance of cumulative value of claims

$$Var\left[C_{i,j+1} \middle| C_{i,1}, \ldots, C_{i,j}\right] = \sigma_j^2 C_{i,j}$$

- Origin periods $[C_{i,1},\ldots,C_{i,j}]$ are independent

Where

- C_{ij} is cumulative value of claims in the origin period *i* and development period *j*
- f_j is the development factor for the development period *j*
- σ_j is the squared sigma for the development period *j*

Properties

- Pseudo data originate in values of development factors $f_{i,j}$
- The method is robust against negative incremental values (more suitable for incurred development triangles than ODP)

Mack model

Algorithm

1 Initial model fit

- The starting point is fitting chain ladder method.
- Subsequently Mack's alpha squared is calculated as follows:

$$\alpha_j^2 = \frac{1}{n_j}\sum_i [C_{i,j}(f_{i,j} - f_j \ \ )^2]\frac{n}{n-p}$$

Where

- n_j is the number of included development factors at development j
- n is the sum of the n_j
- p is the number of parameters in the chain ladder model
- $f_{i,j}$ are the observed ratios of $C_{i,j+1}/C_{i,j}$

Mack model

Algorithm

2 Calculation residuals

- Calculate residuals and apply bias adjustment

$$r_{i,j} = \frac{\sqrt{C_{i,j}}(f_{i,j} - f_j)}{\alpha_j} \sqrt{\frac{n}{n-p}}$$

3 For each simulation

- Create triangle of pseudo data – based on random permutation with repeating of residuals $r_{i,j}$ from those derived at step 2 above to calculate a pseudo triangle data value
- Re-apply chain ladder method based on the pseudo data to generate new projections and reserves
- Add process variance to projected future development – replace projected value by random value from selected distribution

4 Calculate selected measure

distributions, percentiles, means and standard deviations for reserves and other items of interest

Practical stochastic

- The Practical stochastic method provides an alternative to the Bootstrap method for use in segments where it is not possible to produce a sensible bootstrap method result, typically because of poor or erratic data.
- It requires the following inputs
  - A vector of ultimate claims and a triangle basis to enable the expected reserves to be calculated
  - A payment pattern vector to enable cash flows to be generated (if required)
  - Coefficients of variation for each origin period
  - Assumptions regarding the correlation between reserves by origin period
- In each simulation
  - The ultimate claims are simulated for each origin period. Typically lognormal, normal or gamma distributions are chosen
  - The claim provisions are calculated as the difference of simulated ultimate and the last diagonal of underlying cummulative development triangle
  - The results per origin period are aggregated to total based on correlation matrix
- This method provides simulation results of claim provisions, but not stochastic cash flows for each simulation (only constant distribution of unpaid claims based on input pattern vector)

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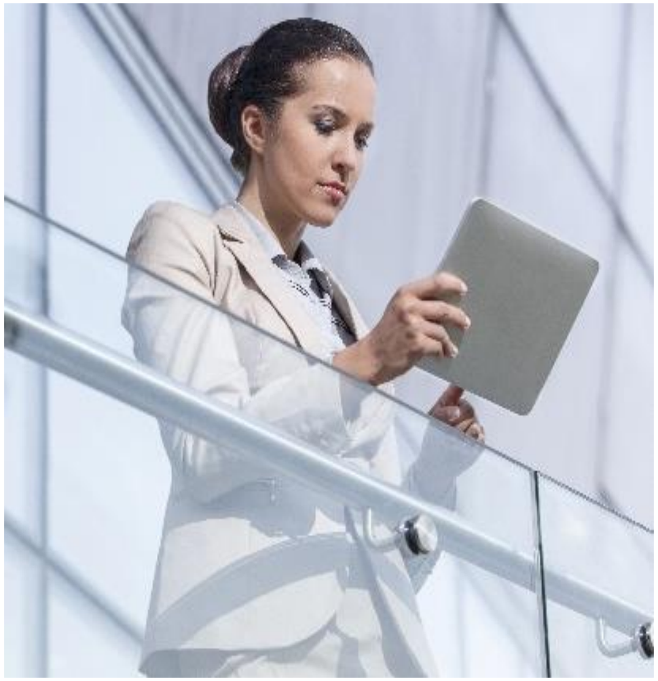


Stochastic run off

Re-reserving approach

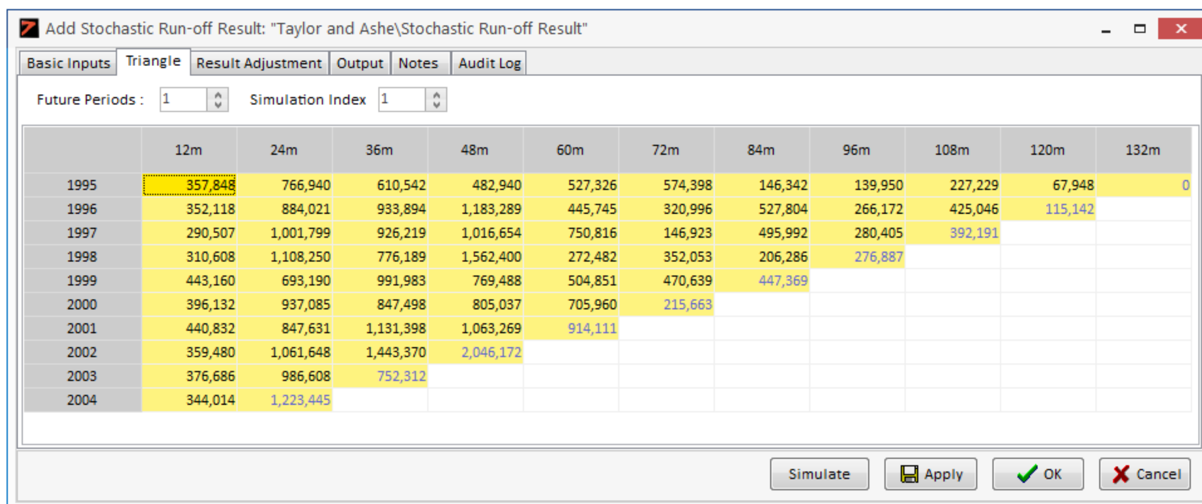
1 Projection of next calendar period(s)

- The projection of next calendar period(s) is created for example by bootstrapping method. The projections of the bootstrapping methods are a suitable input for the stochastic run off

2 Re-reserving based on triangle enhanced by next development period(s)

- “Actuary in a box” approach to re-reserving
- The re-reserving can be done based on several reserving methods, most frequently chain ladder or Bornhuettter Ferguson and different parametrization of these methods
- A realistic approach to re-reserving must be chosen

3 Calculate selected measure (distributions, percentiles, means and standard deviations for reserves and other items of interest)



	12m	24m	36m	48m	60m	72m	84m	96m	108m	120m	132m
1995	357,848	766,940	610,542	482,940	527,326	574,398	146,342	139,950	227,229	67,948	0
1996	352,118	884,021	933,894	1,183,289	445,745	320,996	527,804	266,172	425,046	115,142	
1997	290,507	1,001,799	926,219	1,016,654	750,816	146,923	495,992	280,405	392,191		
1998	310,608	1,108,250	776,189	1,562,400	272,482	352,053	206,286	276,887			
1999	443,160	693,190	991,983	769,488	504,851	470,639	447,369				
2000	396,132	937,085	847,498	805,037	705,960	215,663					
2001	440,832	847,631	1,131,398	1,063,269	914,111						
2002	359,480	1,061,648	1,443,370	2,046,172							
2003	376,686	986,608	752,312								
2004	344,014	1,223,445									

Stochastic run off

Emergence pattern approach

Emergence pattern approach

- Alternative to re-reserving, suitable especially if stochastic cash flows are not available, for example in the case of Practical stochastic method (re-reserving cannot be used)
- The emergence pattern defines how the reserve uncertainty decreases over time. There are various methods for calculation of the pattern

$$BaseReserve_{i,s} = SimulatedReserve_{i,s} - SimulatedCashflow_{i,s}$$

$$AverageReserve_i = \frac{\left(\sum_{s=1}^{SimulationCount} BaseReserve_{i,s}\right)}{SimulationCount}$$

$$RunoffReserve_{i,s} = SelectedPattern_i * BaseReserve_{i,s} + (1 - SelectedPattern_i) * AverageReserve_i$$

Where

- *SimulatedReserve*$_{i,s}$ is the simulated reserve for origin period *i* and simulation *s* in the input method
- *SimulatedCashFlow*$_{i,s}$ is the simulated cash flow for origin period *i* and simulation *s* in the input method (based on average pattern)
- *SelectedPattern*$_i$ is item of the selected emergence pattern for origin period *i*

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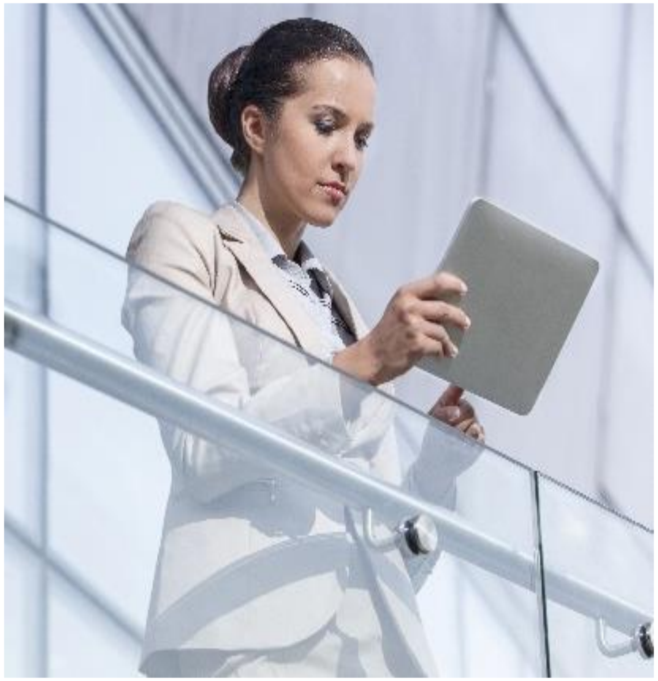


Practical example

Task definition

Task: Calculation of reserve risk SCR for segment based on the following development triangle of claims paid

Edit Triangle "Taylor and Ashe\Paid Claims"

Details | Data | Graph | Notes | Audit Log

Cumulative : ☐ Transposed : ☐ Origin Length : 12 Stored at : 12 Max

◉ Development ○ Calendar Development Length : 12 Stored at : 12 Decimal Places : 0

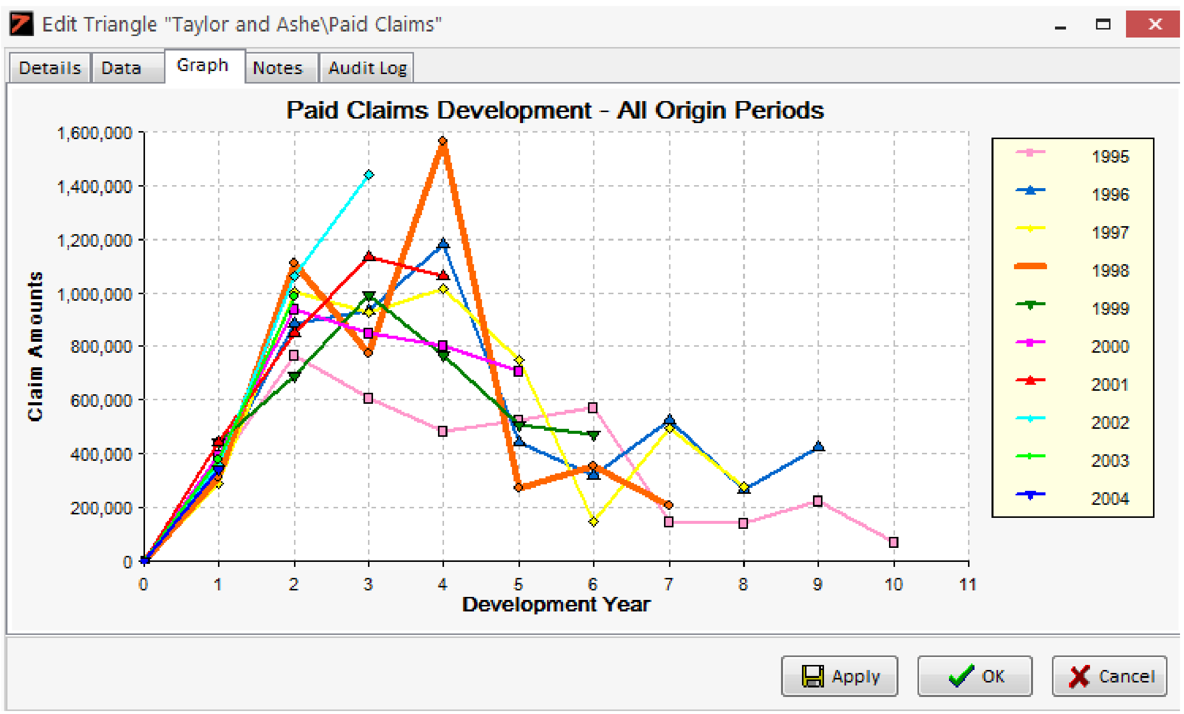
Accident Year	12m	24m	36m	48m	60m	72m	84m	96m	108m	120m
1995	357,848	766,940	610,542	482,940	527,326	574,398	146,342	139,950	227,229	67,948
1996	352,118	884,021	933,894	1,183,289	445,745	320,996	527,804	266,172	425,046	
1997	290,507	1,001,799	926,219	1,016,654	750,816	146,923	495,992	280,405		
1998	310,608	1,108,250	776,189	1,562,400	272,482	352,053	206,286			
1999	443,160	693,190	991,983	769,488	504,851	470,639				
2000	396,132	937,085	847,498	805,037	705,960					
2001	440,832	847,631	1,131,398	1,063,269						
2002	359,480	1,061,648	1,443,370							
2003	376,686	986,608								
2004	344,014									
Total	3,671,385	8,287,172	7,661,093	6,883,077	3,207,180	1,865,009	1,376,424	686,527	652,275	67,948

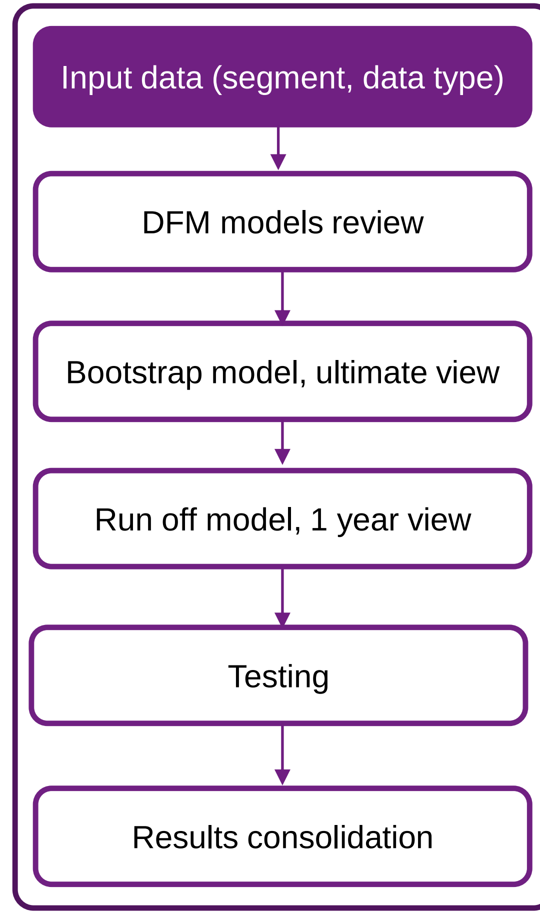
Apply | OK | Cancel

Practical example

Input data

- Approach to large claims
- Interpretation of differences between accident periods
- Choice of data type and analysis of input data





Practical example

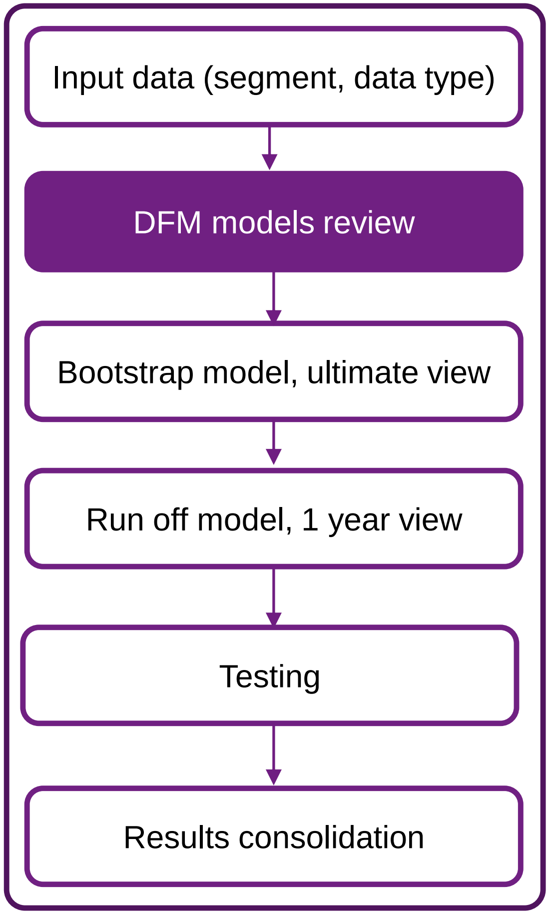
DFM models review, analysis of residuals

- Parametrization of DFM method to be used by the bootstrap model.
- Expert judgments (exclusion of development factors, curve fitting for the development factors, choice of final DFM method, tail factors)
- Assumption checks of underlying DFM method
- The result of DFM must be reasonably close to the BE CP

Validation | Sensitivity | Residuals Data | Residuals Graph

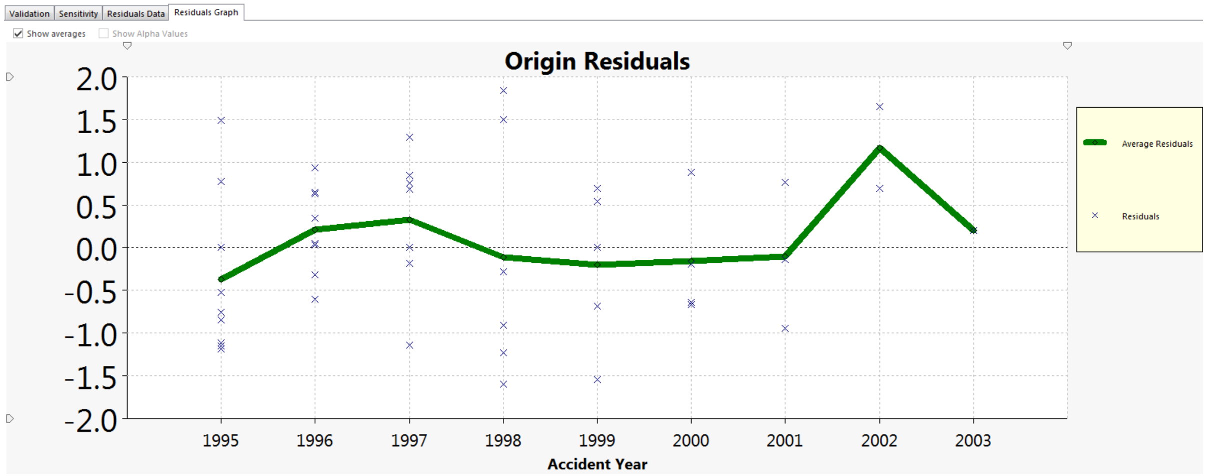
Show: ☐ Reserves ☐ Reserves % Change ☑ Std Errors ☑ Coefficients of Variation

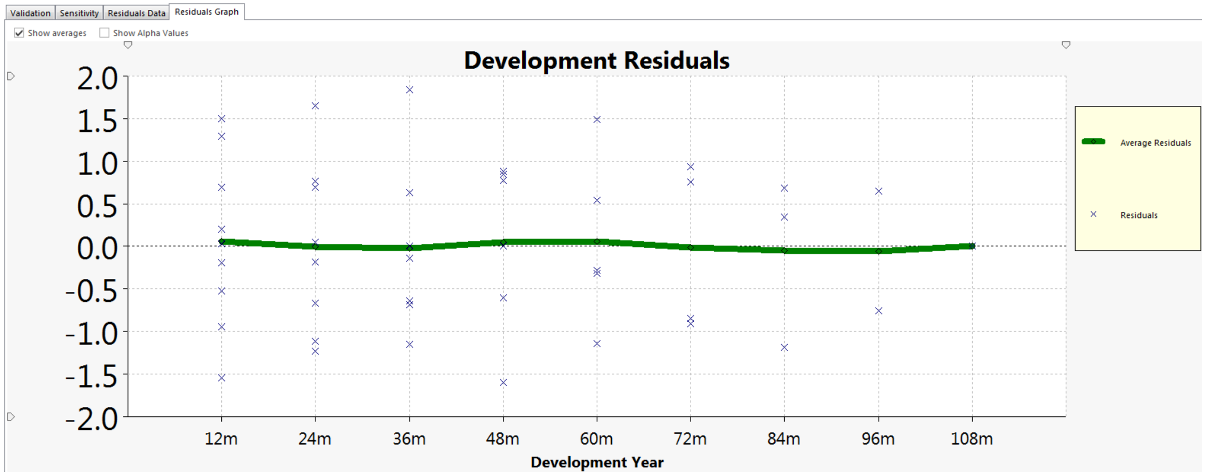
Accident Year	(1) 12-24	(2) 24-36	(3) 36-48	(4) 48-60	(5) 60-72	(6) 72-84	(7) 84-96	(8) 96-108	(9) 108-120
1995 Ratio	3.14320	1.54281	1.27830	1.23772	1.20921	1.04408	1.04037	1.06301	1.01772
Standard Error	2,474,822	2,455,101	2,452,348	2,455,835	2,316,785	2,501,016	2,388,708	2,447,959	2,344,884
Coeff. of Variation	13.21%	13.04%	12.93%	13.27%	12.73%	13.15%	12.66%	12.83%	13.15%
1996 Ratio	3.51058	1.75549	1.54529	1.13293	1.08449	1.12811	1.05727	1.08650	
Standard Error	2,479,738	2,473,816	2,475,624	2,496,905	2,544,560	2,435,696	2,510,837	2,408,220	
Coeff. of Variation	13.28%	13.25%	13.38%	13.24%	13.53%	13.36%	13.49%	13.28%	
1997 Ratio	4.44845	1.71672	1.45826	1.23208	1.03686	1.12001	1.06058		
Standard Error	2,414,818	2,476,285	2,512,448	2,447,723	2,459,038	2,479,831	2,464,839		
Coeff. of Variation	13.02%	13.24%	13.45%	13.28%	12.83%	13.53%	13.29%		
1998 Ratio	4.56800	1.54705	1.71178	1.07252	1.08736	1.04708			
Standard Error	2,392,469	2,451,643	2,261,617	2,411,253	2,548,542	2,498,620			
Coeff. of Variation	12.92%	12.99%	12.45%	12.57%	13.55%	13.06%			
1999 Ratio	2.56420	1.87296	1.36154	1.17422	1.13831				
Standard Error	2,409,911	2,452,884	2,497,617	2,498,872	2,502,371				
Coeff. of Variation	12.76%	13.20%	13.24%	13.38%	13.54%				
2000 Ratio	3.36559	1.63568	1.36916	1.23644					
Standard Error	2,480,630	2,472,177	2,500,871	2,445,103					
Coeff. of Variation	13.26%	13.16%	13.26%	13.26%					
2001 Ratio	2.92280	1.87810	1.43939						
Standard Error	2,457,740	2,448,556	2,516,136						
Coeff. of Variation	13.07%	13.19%	13.44%						
2002 Ratio	3.95329	2.01565							
Standard Error	2,458,337	2,375,433							
Coeff. of Variation	13.22%	12.90%							
2003 Ratio	3.61918								
Standard Error	2,477,320								
Coeff. of Variation	13.28%								

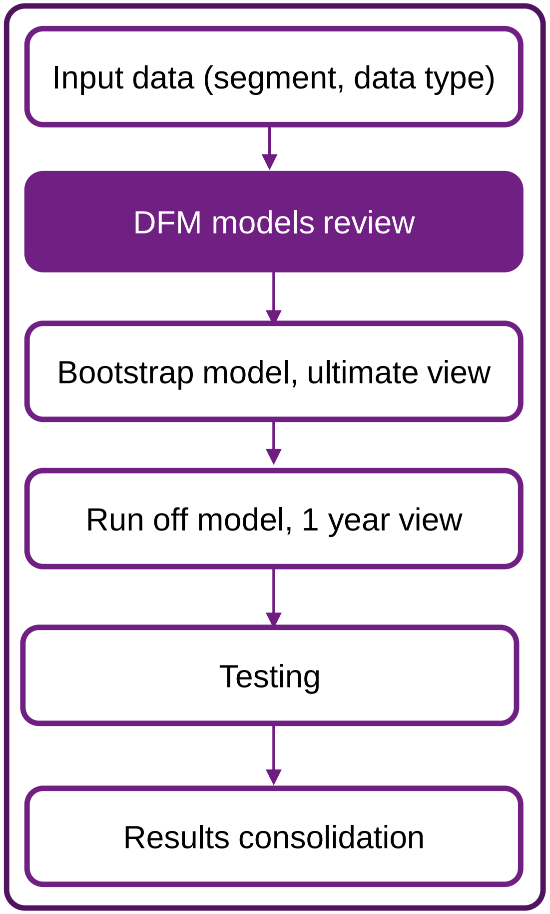


Practical example

DFM models review, analysis of residuals

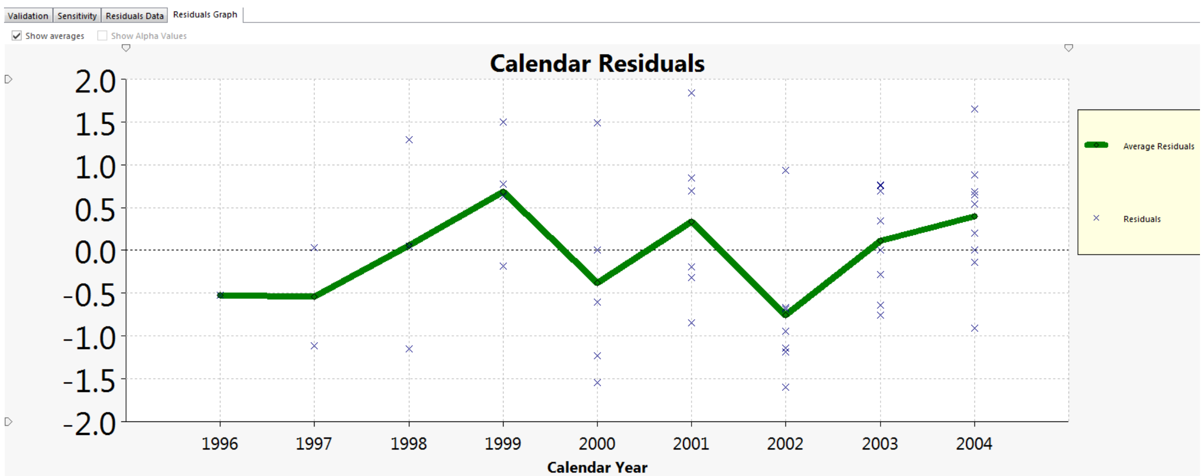






Practical example

DFM models review, analysis of residuals



Validation | Sensitivity | Residuals Data | Residuals Graph

Calculation Basis

DFM Basis | Bootstrap - Modified Mack | Bootstrap - Default

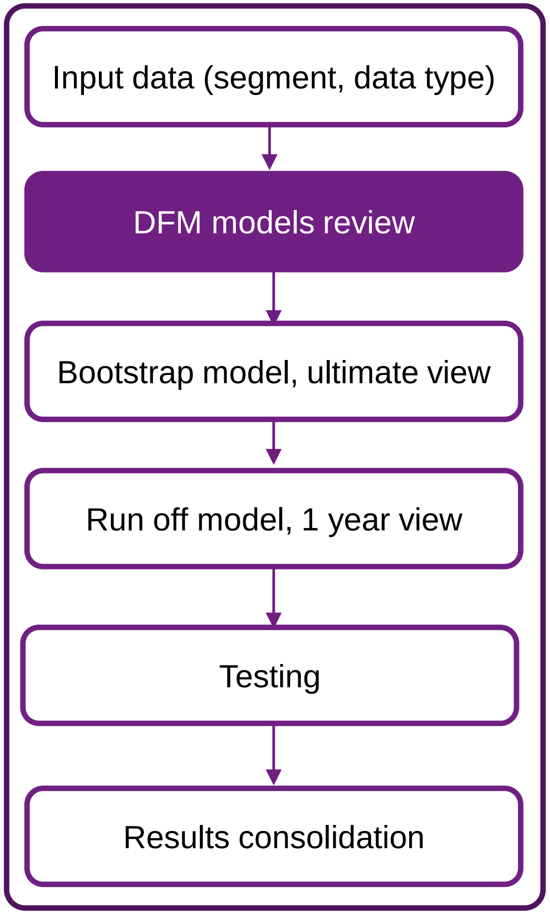
Significant origin periods:

Significant development periods:

Significant calendar periods: 2002

Total reserve : 18,680,856

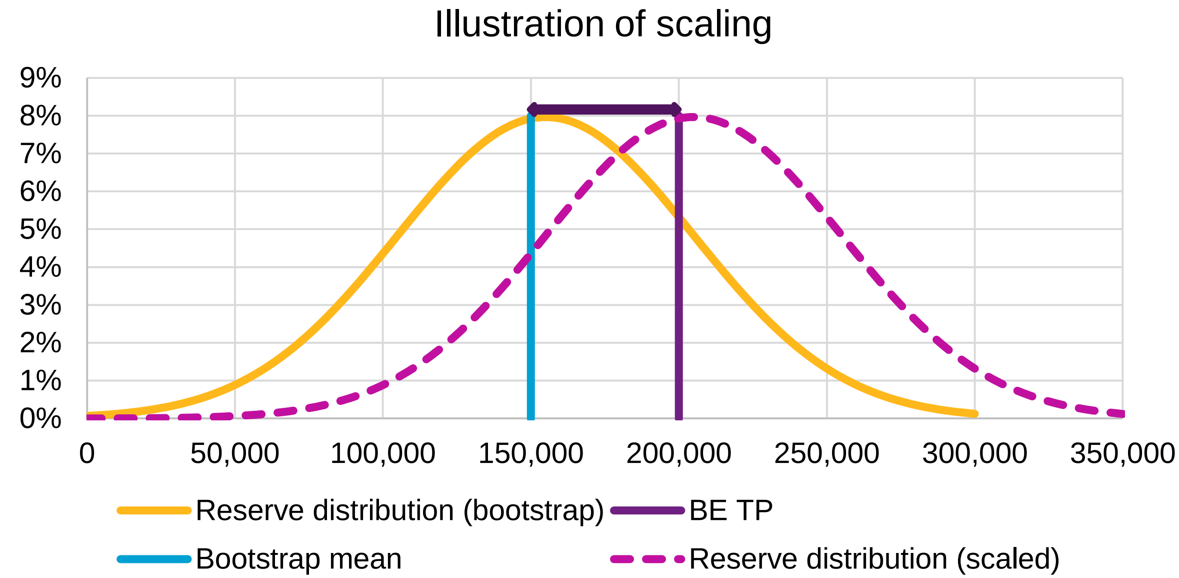
Accident Year	(1) 12-24	(2) 24-36	(3) 36-48	(4) 48-60	(5) 60-72	(6) 72-84	(7) 84-96	(8) 96-108	(9) 108-120
1995	-0.56405	-1.20717	-1.25755	0.83221	1.65259	-0.99458	-1.47012	-1.08747	
1996	0.01794	0.03646	0.66906	-0.67972	-0.37294	1.07033	0.41048	0.90660	
1997	1.35429	-0.20499	-0.00684	0.91761	-1.28904	0.86197	0.82225		
1998	1.57734	-1.32634	1.97363	-1.75975	-0.32881	-1.06057			
1999	-1.64734	0.72348	-0.75090	-0.00793	0.59357				
2000	-0.22193	-0.72294	-0.70061	0.94804					
2001	-1.01226	0.80339	-0.16127						
2002	0.72148	1.74681							
2003	0.19560								
2004									
Alpha Value	422.00622	203.16172	212.04417	125.75979	117.18073	87.60229	19.29198	26.77879	26.77879

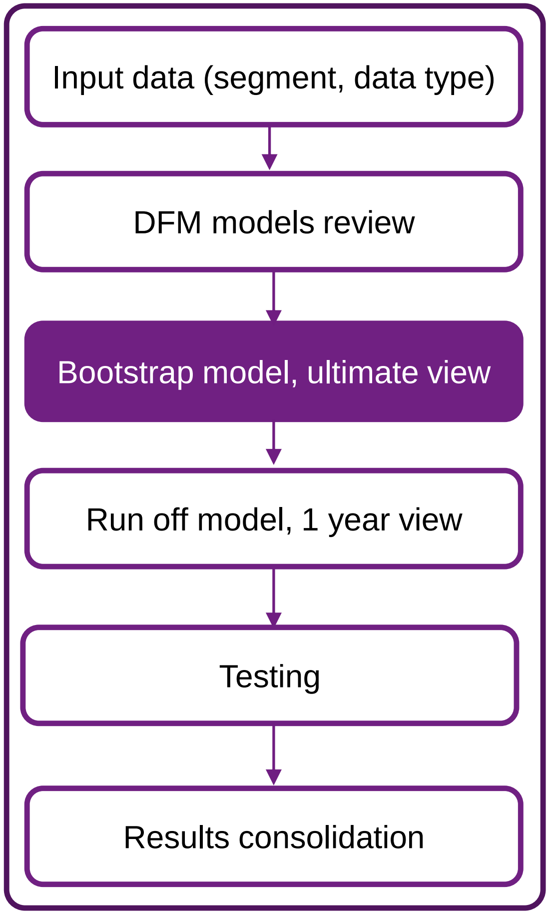


Practical example

Bootstrap model, ultimate view

- The Bootstrap method is used to produce the scenarios of development of the ultimate claims
- The following choices must be made
  - Choice of model (Mack, ODP)
  - Adjustments to residuals and scale parameter (smoothing, zero average residuals, etc)
  - Scaling of the result (multiplicative, additive, user defined)





Practical example

Bootstrap model, ultimate view

Details | Residuals | Simulation | Results | Output | Notes | Audit Log

Tiled Tabs...

☐ Modified Mack Bias Adjustment

Residual Type: Scaled, Bias Adjusted, Zero-Average

☐ Show Residual Scale Values

☐ Tile Grid and Graph

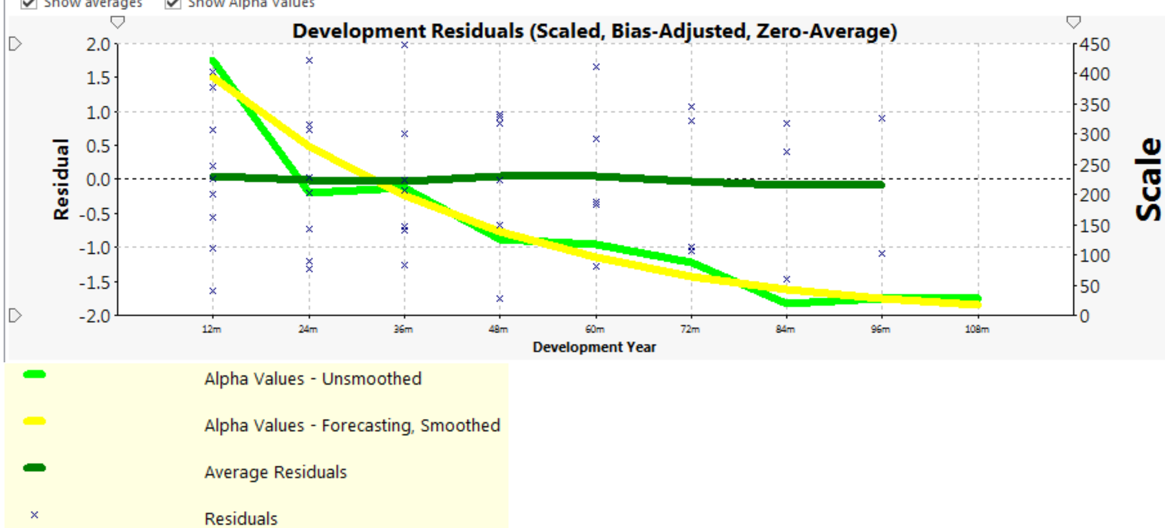
Residual Scale Values Smoother — Smoothing Parameter: 0

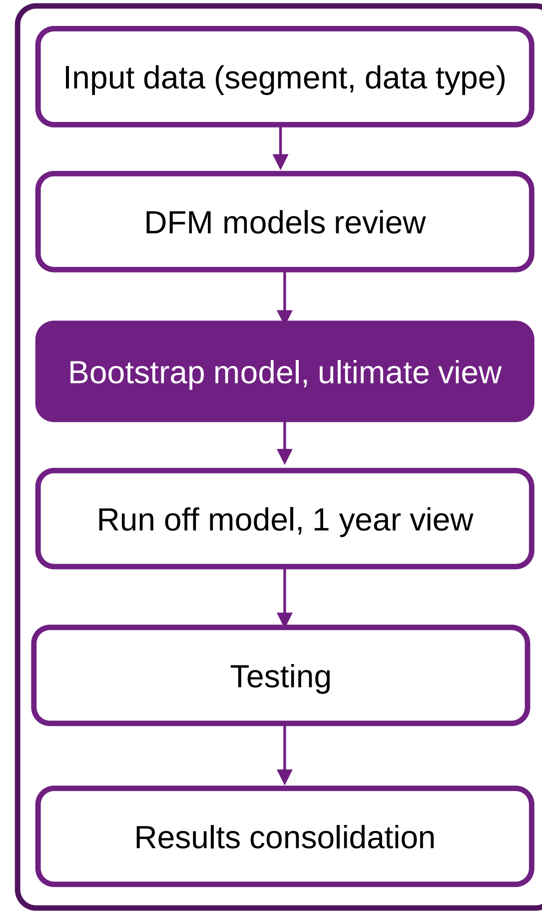
Forecast Scale Values Smoother — Smoothing Parameter: 80

Residuals adjusted by +0.013466 to make mean zero

Accident Year	(1) 12-24	(2) 24-36	(3) 36-48	(4) 48-60	(5) 60-72	(6) 72-84	(7) 84-96	(8) 96-108	(9) 108-120	(10) 120-132
1995	-0.564	-1.207	-1.258	0.832	1.653	-0.995	-1.470	-1.087		
1996	0.018	0.036	0.669	-0.680	-0.373	1.070	0.410	0.907		
1997	1.354	-0.205	-0.007	0.918	-1.289	0.862	0.822			
1998	1.577	-1.326	1.974	-1.760	-0.329	-1.061				
1999	-1.647	0.723	-0.751	-0.008	0.594					
2000	-0.222	-0.723	-0.701	0.948						
2001	-1.012	0.803	-0.161							
2002	0.721	1.747								
2003	0.196									

☑ Show averages ☑ Show Alpha Values

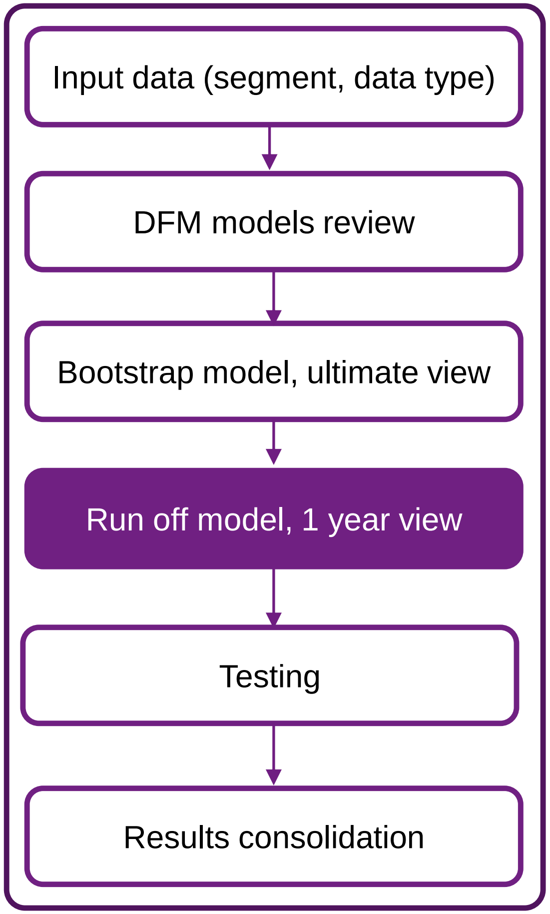




Practical example

Run off model, 1 year view

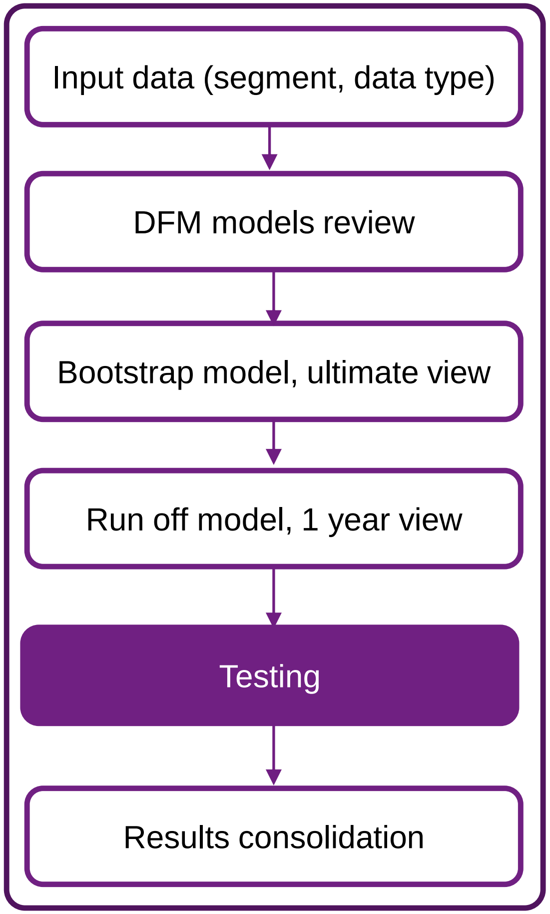
- Choice of approach
- Re-reserving approach
  - Chain ladder
  - Bornhuetter Ferguson with adjusted a priori ultimate claims and pay out pattern
  - Bornhuetter Ferguson with adjusted pay out pattern
  - Bornhuetter Ferguson with no adjustments
  - No re-reserving
- Emergence pattern approach
- Potential scaling of result, but average run off result should be close to zero in principle



Practical example

Testing

- The stability can be tested with respect to all inputs and choices made:
  - Input data type
  - Number of diagonals
  - Parametrization of DFM method
  - Random seed
  - Number of simulations
  - Bootstrap model used
  - Choice of forecast distributions for process error
  - Scaling approach
  - The re-reserving method used
- Comparison with benchmarks
- Strategy and automation of testing convenient due to large amount of possible tests



Practical example

Comparison of results – ultimate view

Model	Parametrization	Prediction error %	99.5% VaR %
Mack formula*		17.1%	51.3%
Mack	Resampled f.d.	17.4%	48.5%
Mack	Normal f.d.	17.6%	48.4%
Mack	Gamma f.d.	17.5%	49.4%
ODP	Resampled f.d.	14.5%	40.7%
ODP	Normal f.d.	14.4%	39.8%
ODP	Gamma f.d.	14.4%	41.4%
Practical stochastic	Lognormal distribution, 0% correlation between origin periods, CoV suggested by Mack formula	14.5%	45.7%
Practical stochastic	Normal distribution, 0% correlation between origin periods, CoV suggested by Mack formula	14.5%	37.3%
Mack formula*	Highest residual excluded	13.0%	39.0%
Mack	Resampled, highest residual excluded	13.2%	35.1%
ODP	Resampled f.d., highest residual excluded	12.3%	33.3%
Practical stochastic	Lognormal distribution, 0% correlation between origin periods, CoV suggested by Mack formula with highest residual excluded	10.6%	31.6%

**99.5% VaR selected as 3 times prediction error (in line with standard formula)*

Practical example

Comparison of results – one year view

Model	Parametrization	Prediction error %	99.5% VaR %
Merz Wuthrich formula*		12.1%	36.4%
Stochastic run off, CHL re-reserving	Mack model, resampled f.d.	12.5%	29.5%
Stochastic run off, BF1 re-reserving	Mack model, resampled f.d.	5.3%	12.5%
Stochastic run off, BF2 re-reserving	Mack model, resampled f.d.	6.7%	16.6%
Stochastic run off, BF3 re-reserving	Mack model, resampled f.d.	8.6%	20.7%
Stochastic run off, CHL re-reserving	ODP model, resampled f.d.	12.0%	27.9%
Stochastic run off, CHL re-reserving	Practical stochastic, lognormal d.	10.8%	22.5%
Merz Wuthrich formula*	Highest residual excluded	9.2%	27.5%
Stochastic run off, CHL re-reserving	Mack model, resampled f.d., highest residual excluded	9.6%	23.4%
Stochastic run off, CHL re-reserving	ODP model, resampled f.d., highest residual excluded	9.7%	22.9%
Stochastic run off, CHL re-reserving	Practical stochastic, lognormal d., highest residual excluded	8.1%	17.9%

**99.5% VaR selected as 3 times prediction error (in line with standard formula)*

CHL – chain ladder

BF1 – Bornhuetter Ferguson with unchanged both pay out pattern and a priori ultimate claims

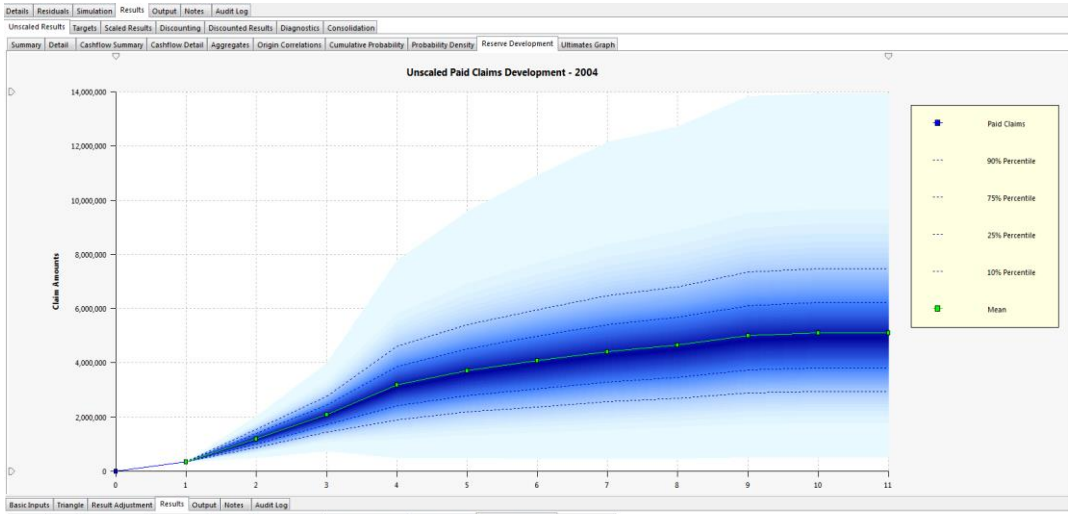
BF2 – Bornhuetter Ferguson with recalculation of pay out pattern and unchanged a priori ultimate claims

BF3 – Bornhuetter Ferguson with recalculation of both pay out pattern and a priori ultimate claims

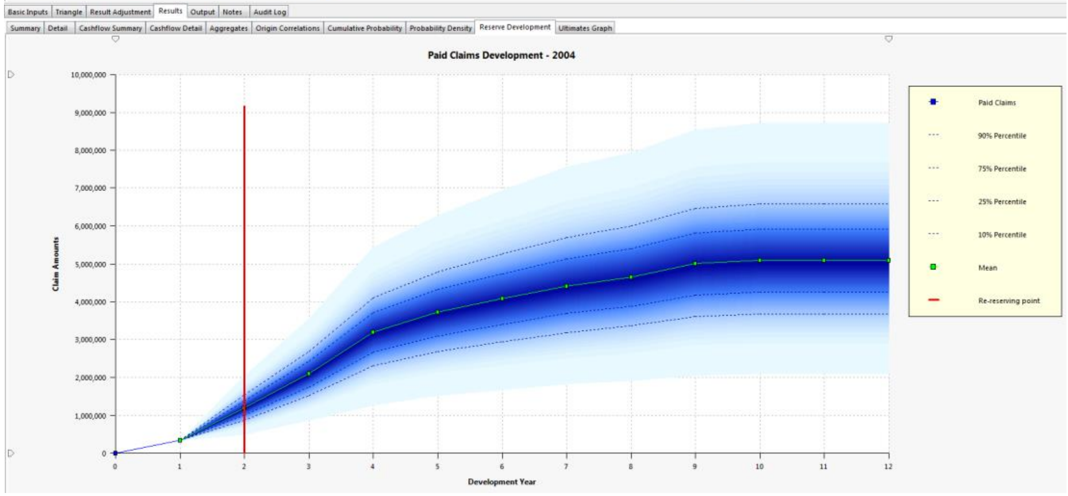
Practical example

Claim payments development

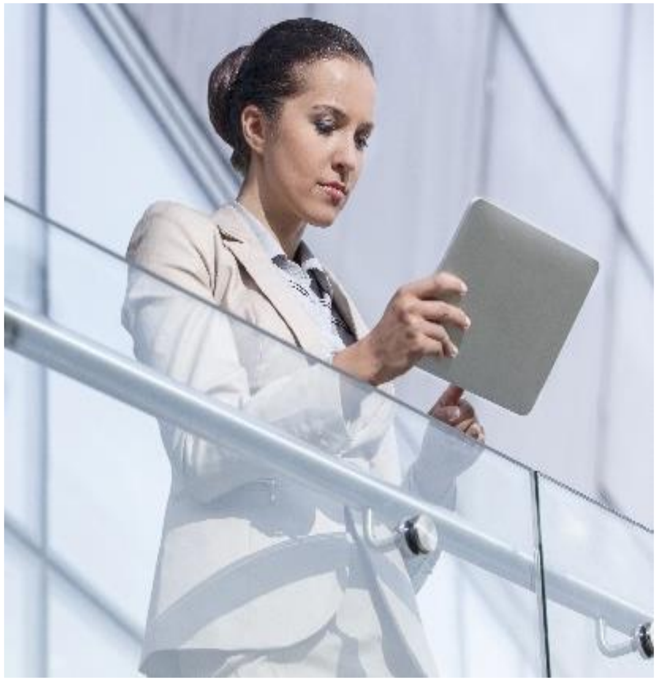
Development of total claim payments – ultimate view simulation



Development of total claim payments – one year view simulation



Contents



GLM

Model definition

- We assume that each incremental value $I_{i,j}$ in origin period i and development period j consists of a systemic and random component and

$$E[I_{i,j}] = m_{i,j} \qquad Var[I_{i,j}] = \phi V(m_{i,j})$$

$$m_{i,j} = g^{-1}(\eta_{i,j}) \qquad \eta_{i,j} = c + \sum_{r=2}^{i} \alpha_r + \sum_{s=2}^{i} \beta_s + \sum_{t=2}^{i+j-1} \gamma_t$$

Where

- Parameter ϕ is called scale parameter. It can be either constant for whole triangle or different for particular development periods
- Coefficients α_r β_s,γ_t can be found through the method of maximum likelihood. These are origin, development and calendar year parameters.
- V is variance function
  - 1 for Normal random component
  - $m_{i,j}$ for Poisson random component
  - $m_{i,j}^2$ for Gamma random component
- G is link function (identity or log link is often used in reserving)
- This model is overparametrized, all parameters cannot be independent
- Further, exposures can be assigned to each triangle cells
- The linear predictor is piecewise linear function

GLM

Selection of parameters

Example of approach to parameters

- A parameter at a given period can be set to be independent, then its value needs to be estimated. There will be a change of slope in the linear predictor (i.e. a new straight line segment).
- A parameter at a given period can be set to be the same as previous. The linear predictor continues with the same slope as the previous period (for as many periods as parameter is unchanged).
- A parameter at a given period can be set to 0. In this case the linear predictor is flat.

The main task is to select the approach described above for each parameter

- This can be done through automated optimization, which includes parameters that are statistically significant and exclude parameters that are not.
- There is no guarantee that a model that is a good fit to the observed data will be useful for forecasting into the future, so care should be taken.

Choice and analysis of residuals (residuals also used for estimation of scale parameter)

- Pearson residuals
- Deviance residuals

GLM

Prediction error error

The prediction error is the square root of the mean squared error of prediction (MSEP) where

$$MSEP = \sqrt{E[(I - \hat{I})^2]} = \sqrt{Var(I) + Var(\hat{I})}$$

For a single cell, the MSEP is given by

$$MSEP_{i,j} = \sqrt{Var\left(I_{i,j}\right) + Var(\widehat{m}_{i,j})}$$

and for an origin period reserve or total reserve, the MSEP is given by:

$$MSEP \quad = \sqrt{Var\left(\sum I_{i,j}\right) + Var(\sum \widehat{m}_{i,j})}$$

GLM

Special cases of GLM model

The general definition of model covers several model types, for example:

- ODP chain ladder – origin and development parameters independent, calendar parameters zero, log link function, scale free Poisson error distribution, single scale parameter
- De Vylder model – origin and development parameters independent, calendar parameters zero, log link function, normal error distribution, single scale parameter
- Separation model – calendar and development parameters independent, origin parameters zero, log link function, normal error distribution, single scale parameter
- Mack's additive –development parameters independent, origin and calendar parameters zero, scale parameter independent for each development period, identity link function, normal error distribution

GLM

Modelling topics

Negative incremental values

- For the Poisson and Gamma error structures with a log link, some negative and zero incremental values may be permitted (depending on the parameterization), although all fitted values will be positive. Where a model fails, the presence of negative and zero values should be investigated, and some values excluded to ensure a good fit. In general, the identity link function should be avoided with the Poisson and Gamma error structures, which will fail if a fitted linear predictor is negative in any cell of the triangle.
- Where there are many negative incremental values, the normal error structure with an identity link function is the only one that may be suitable, since it offers support for negative values and allows negative fitted values (a log link will give positive fitted values). However, there is no guarantee that the fit or forecast values will be reasonable.

Scale parameter

- Usually with GLMs, the scale parameter is assumed to be constant. It is also possible to allow for heteroscedasticity by allowing the scale parameter to vary by development period. This is achieved by modelling the squared bias adjusted unscaled residuals using a second GLM with a log link and a Gamma error structure. Model fitting is via an iterative procedure known as joint modelling: the model for the mean provides the residuals used in the model for the scale parameters. The reciprocal of the fitted values from the model for the scale parameters is then used as weights in the model for the mean, and the process iterates until convergence.

GLM

Model comparison

Evaluation of model fit

- T test if the difference of two successive parameters is significantly different from zero
- Model comparison
  - Chi squared and F test for nested models with constant scale parameters - test of the significance of the difference in the parameters.
  - AIC (Akaike's information criterion)
  - BIC (Schwarz's Bayesian information criterion)
  - Estimate of error

GLM

Markov Chain Monte Carlo – probability distribution

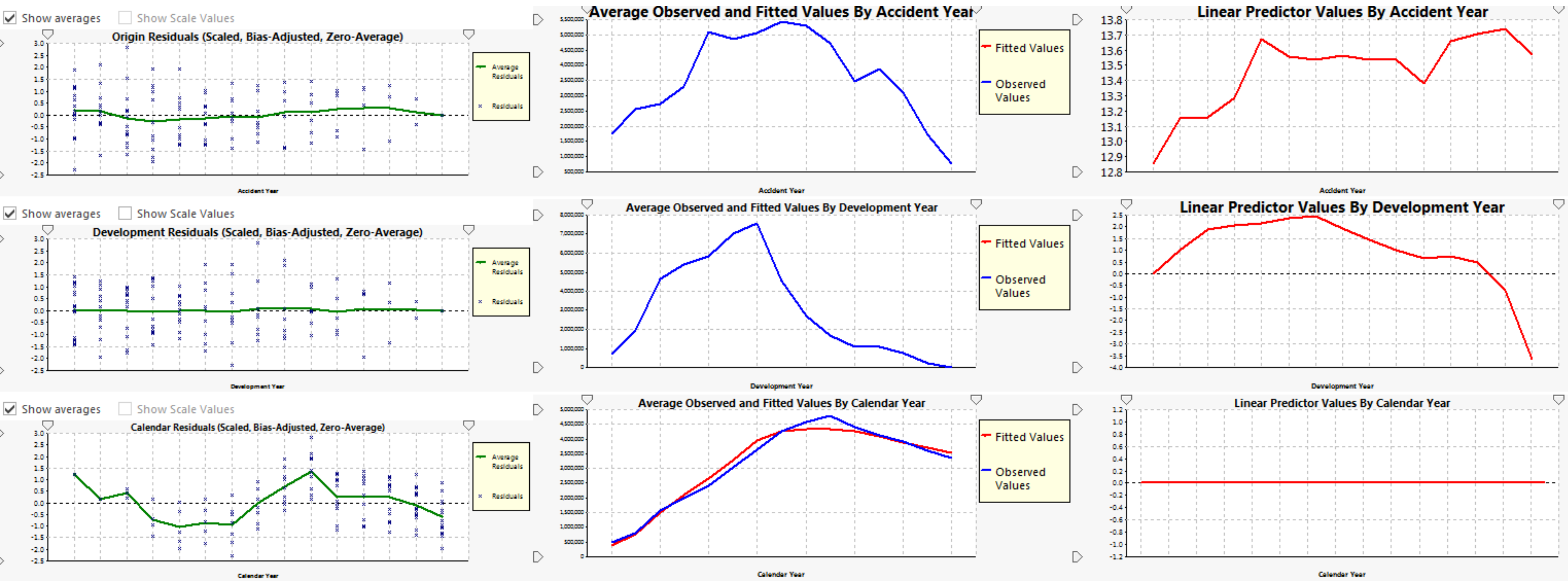
- The aim of the Markov Chain Monte Carlo (MCMC) method is to derive an estimate of the full distribution of claim outcomes based on the results of a GLM model
- Whereas the GLM method provides maximum likelihood parameter estimates, MCMC methods provide a simulated distribution of parameters given the log likelihood specified by the underlying GLM model
- A distribution of forecasts is then generated by simulating from the assumed process distribution conditional on the parameters
- There are several MCMC algorithms: Gibbs with Adaptive Rejection Sampling, Single Component Adaptive Metropolis Hastings, Block Updating Metropolis Hastings, etc.

GLM

Example

Model 1 - ODP

- Diagnostics:
  - Residuals, average observed vs fitted values and linear predictor of the model per:
  - Origin, development and calendar period.

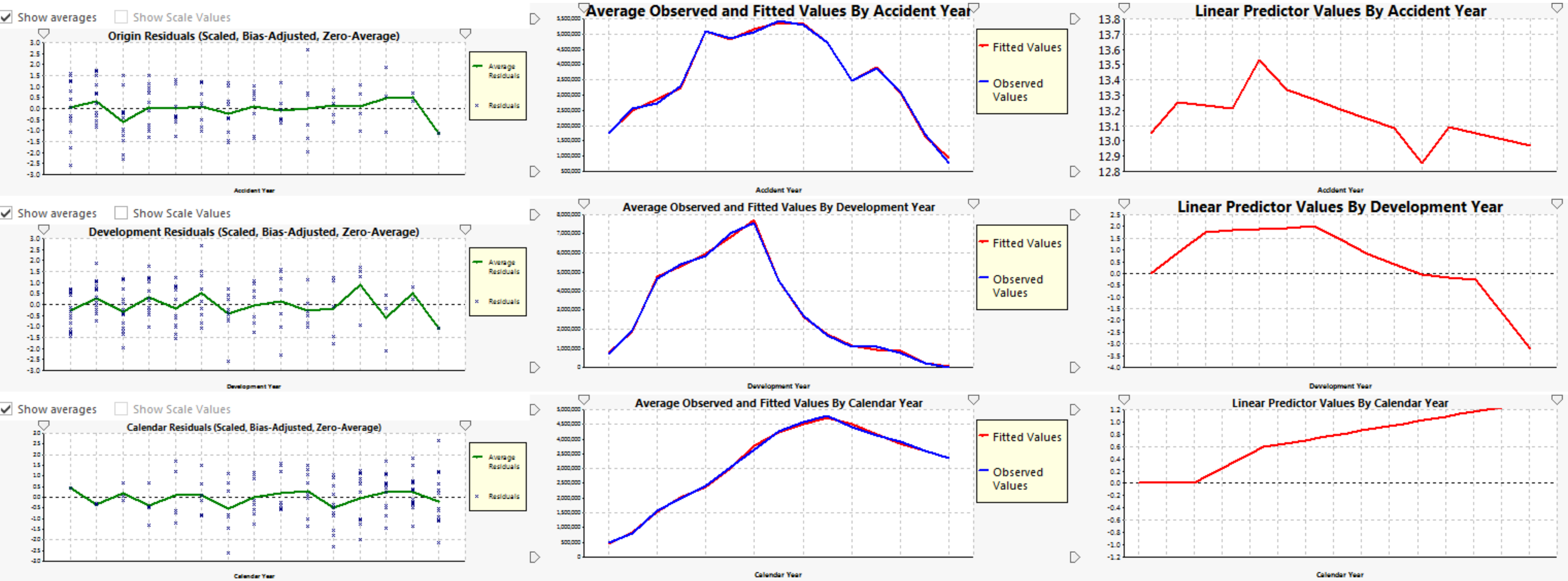


GLM

Example

Model 2 – also with independent calendar year parameters

- Diagnostics:
  - Residuals, average observed vs fitted values and linear predictor of the model per:
  - Origin, development and calendar period.



GLM

Model comparison

Comparison of model 2 with model 1:

- Model 2 has a lower number of parameters
- Model 2 has a lower value of AIC and BIC
- Model 2 significantly lower total reserve
- Model 2 has significantly lower total reserve prediction error

Model	Model 2 (with calendar parameters)	Model 1 (ODP-chain ladder)
Triangle	Paid Claims	Paid Claims
Exposure	(none)	(none)
Link Function	Log	Log
Error Distribution	Poisson - Scale Free	Poisson - Scale Free
Number of Parameters in Mean	17	29
Log Likelihood	-1,652	-1,670
Deviance Scale	23,646	35,702
Pearson Scale	23,338	35,571
AIC	3,340	3,401
BIC	3,390	3,484
Total Ultimate	709,750,158	746,820,579
Total Reserve	266,430,798	303,501,219
Total Reserve Prediction Error	11,637,982	15,785,124

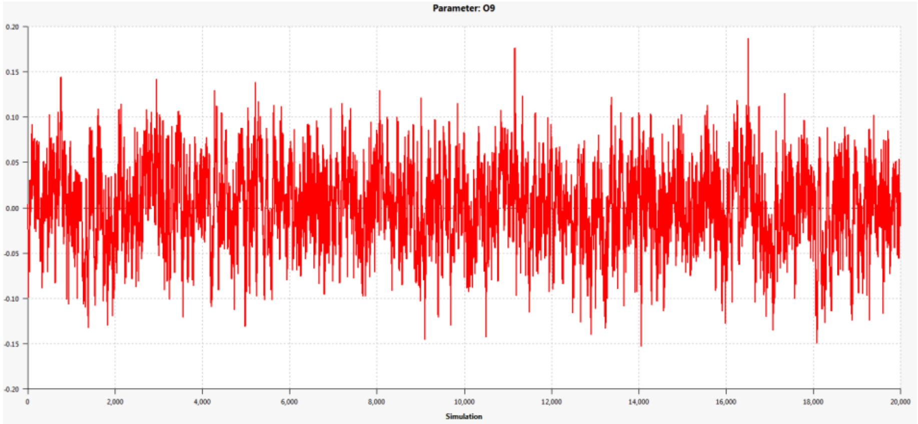
GLM

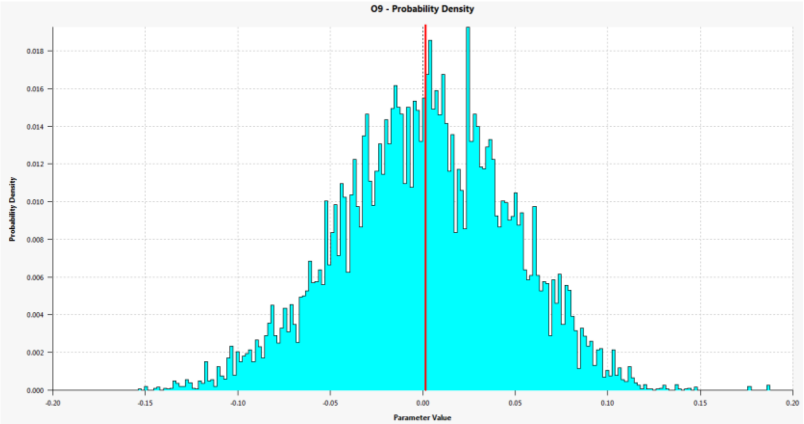
MCMC

Comparison of MCMC results are presented in the table below (Metropolis Hastings – Single Component algorithm)

	Model 2 (with calendar parameters)	Model1 (ODP - chain ladder)
GLM Total Reserve	266,430,798	303,501,219
GLM prediction error	4.4%	5.2%
MCMC Total Reserve	266,077,051	304,204,641
MCMC prediction error %	4.4%	5.2%
MCMC 99.5% VaR % - ultimate view	12.1%	15.1%

The trace plot and probability distribution of selected parameter are presented below





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