November 18th , 2016

EXPOSURE MODELS IN REINSURANCE

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Exposure Models in Reinsurance : Table of Contents

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Exposure Models in Reinsurance

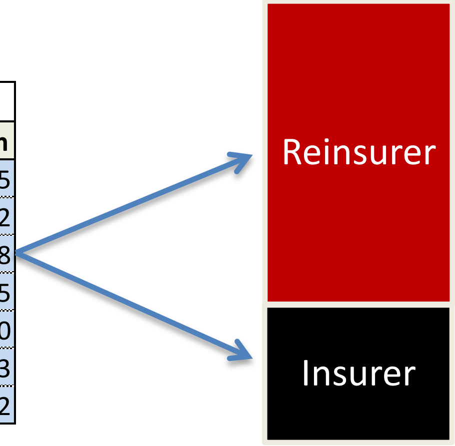
Motivation

Motivation

- Pricing of a Non-Proportional per risk reinsurance programs
  - it should not be based only on historical claims experience
  - current exposure should also be considered
  - shift in business – movements in the portfolio volumes
  - in case of insufficient claim history it is indispensable
- Historically, the need for a fast and accurate pricing process
- Aim: distribution of premium between primary insurer and reinsurer for each band/risk

Gross Risk Profile

Risk Profile Name	Band SI/PML		Nr. Of Risks	Total SI/PML	Premium
Fire - Property Small Risks	0	500 000	81 847	4 073 604 954	4 515 515
Fire - Property Small Risks	500 000	1 000 000	1 566	1 118 702 402	984 372
Fire - Property Small Risks	1 000 000	2 500 000	1 246	1 934 995 601	1 338 648
Fire - Property Small Risks	2 500 000	5 000 000	291	946 439 987	595 425
Fire - Property Small Risks	5 000 000	10 000 000	73	484 292 205	863 510
Fire - Property Small Risks	10 000 000	20 000 000	39	527 940 298	729 683
Fire - Property Small Risks	20 000 000	30 000 000	16	410 949 649	376 672



Exposure Pricing

- it uses **Risk Profiles** with the current available portfolio information
  - it contains **homogeneous risk types**
  - all risks of the same size (Sum Insured, Probable Maximum Loss, Estimated Maximum Loss) are grouped together in **Risk Bands**
  - **Total Exposure** (SI, PML, EML), **Total Premium** as well as **Number of Risks** in each band are known

- Application of a single **claim distribution** per risk band
  - Problem is that claim distribution is not known

➢ application of **Exposure Curves**

Exposure Curves

- allow direct sharing of risk premium between insurer and reinsurer
- reinsurance risk premium is a function of the deductible
- are usually in a tabular form
- constructed from claim history of large homogeneous portfolios

Exposure Models in Reinsurance

Construction and Interpretation of Exposure Curves

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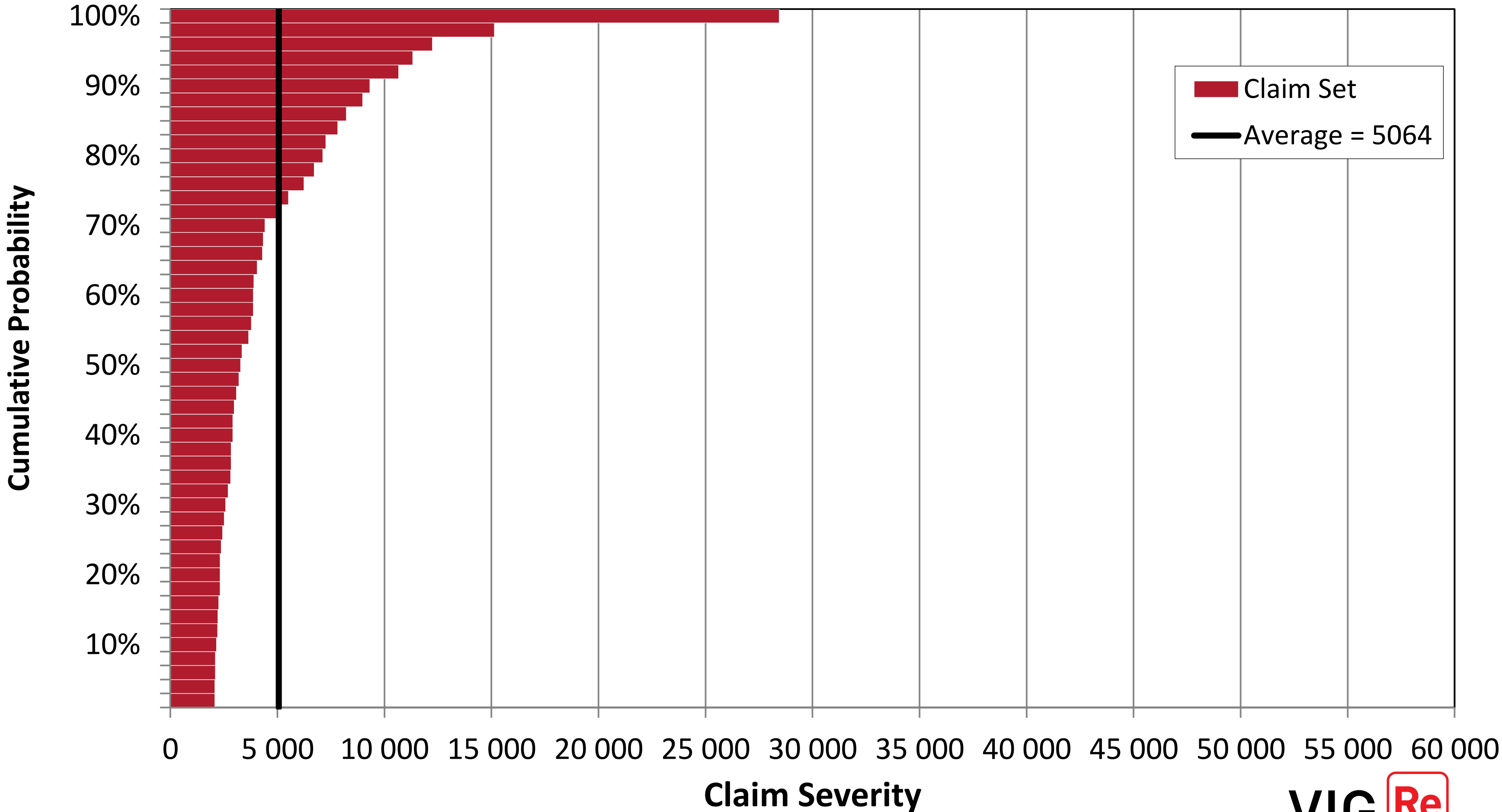
Severity Distribution of Claims

- Empirical Distribution Function



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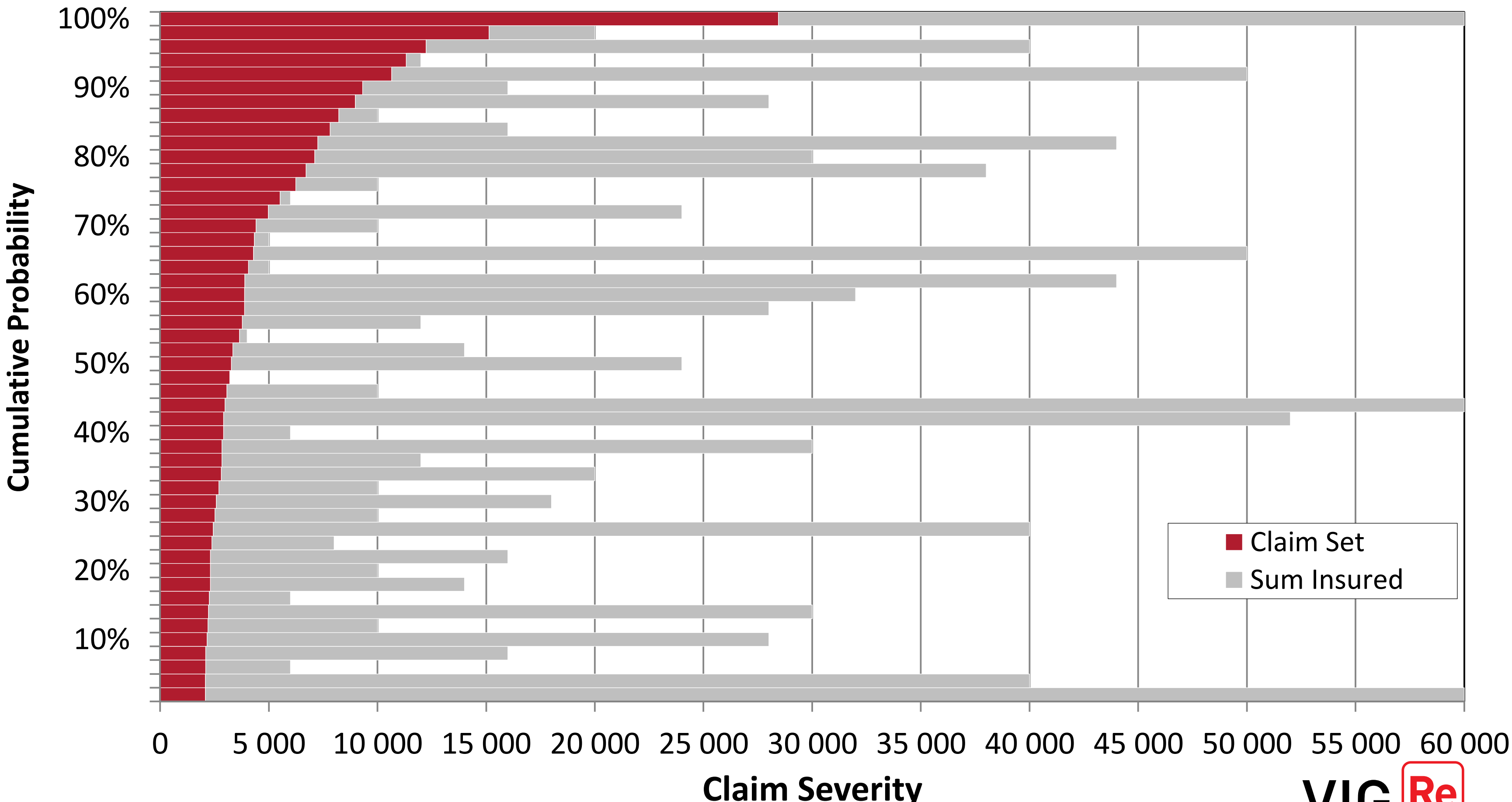
Severity Distribution of Claims

- Empirical Distribution Function with Sum Insured



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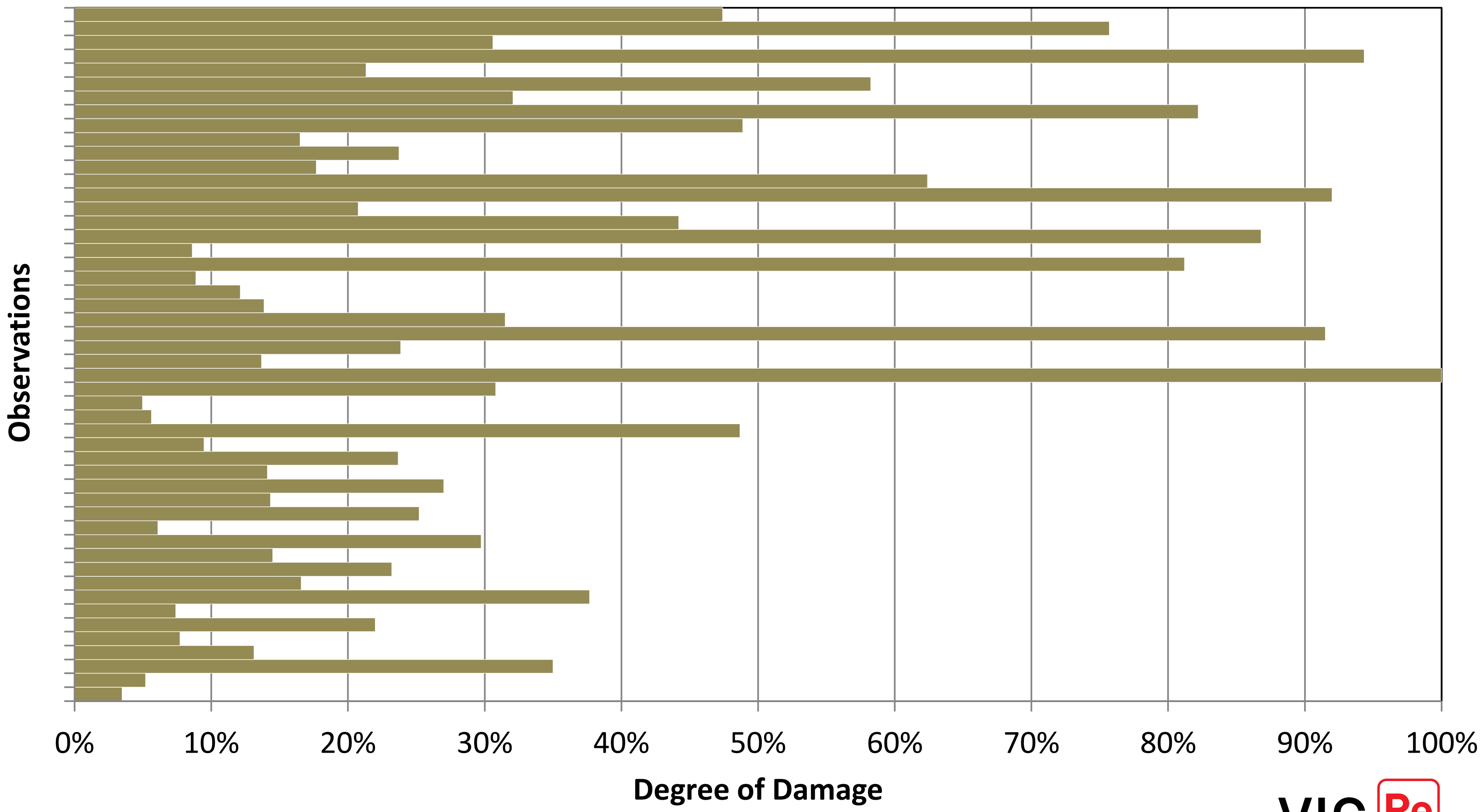
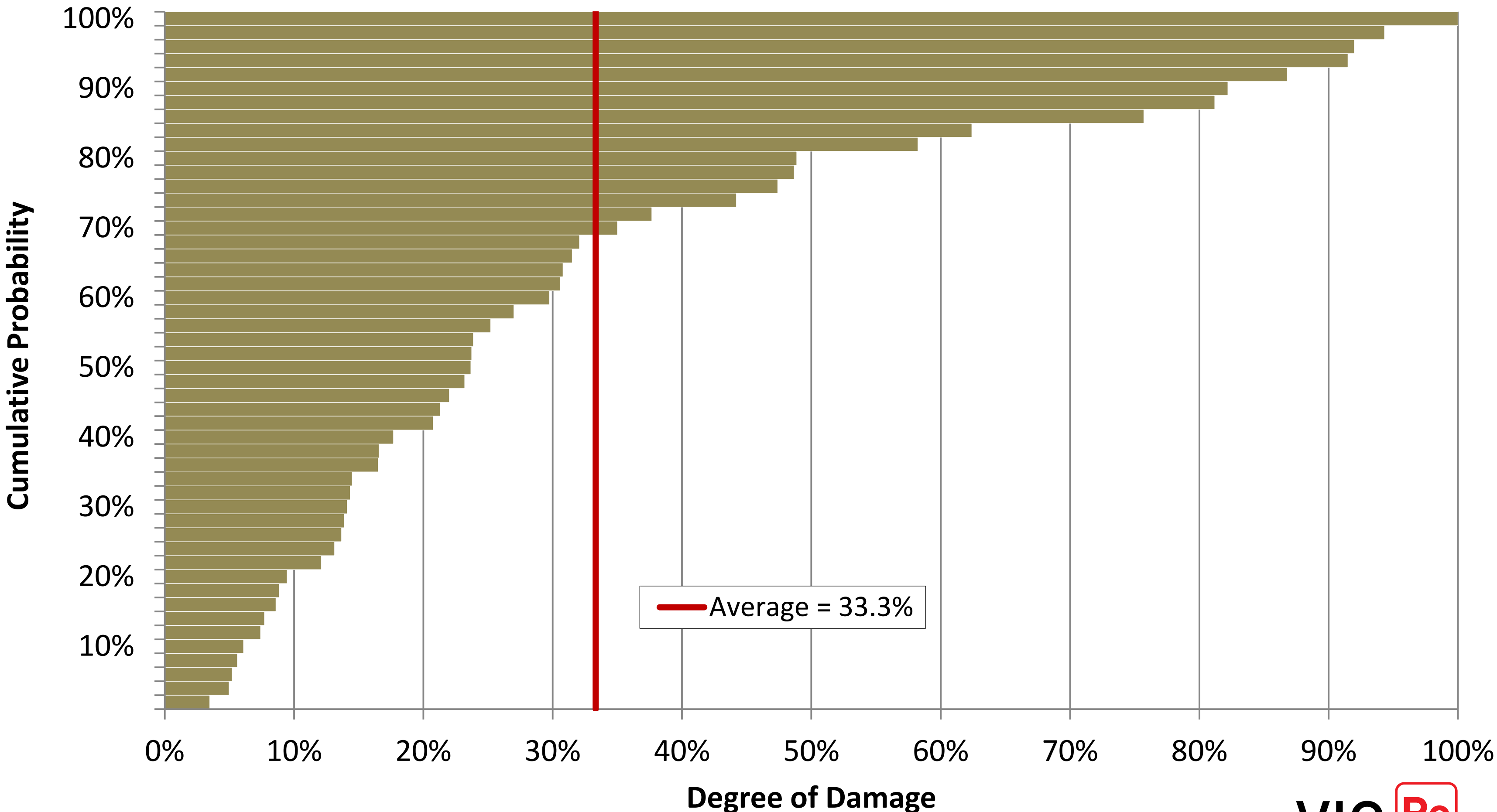
Degree of Damage

- Ratio of Claim Severity and Sum Insured



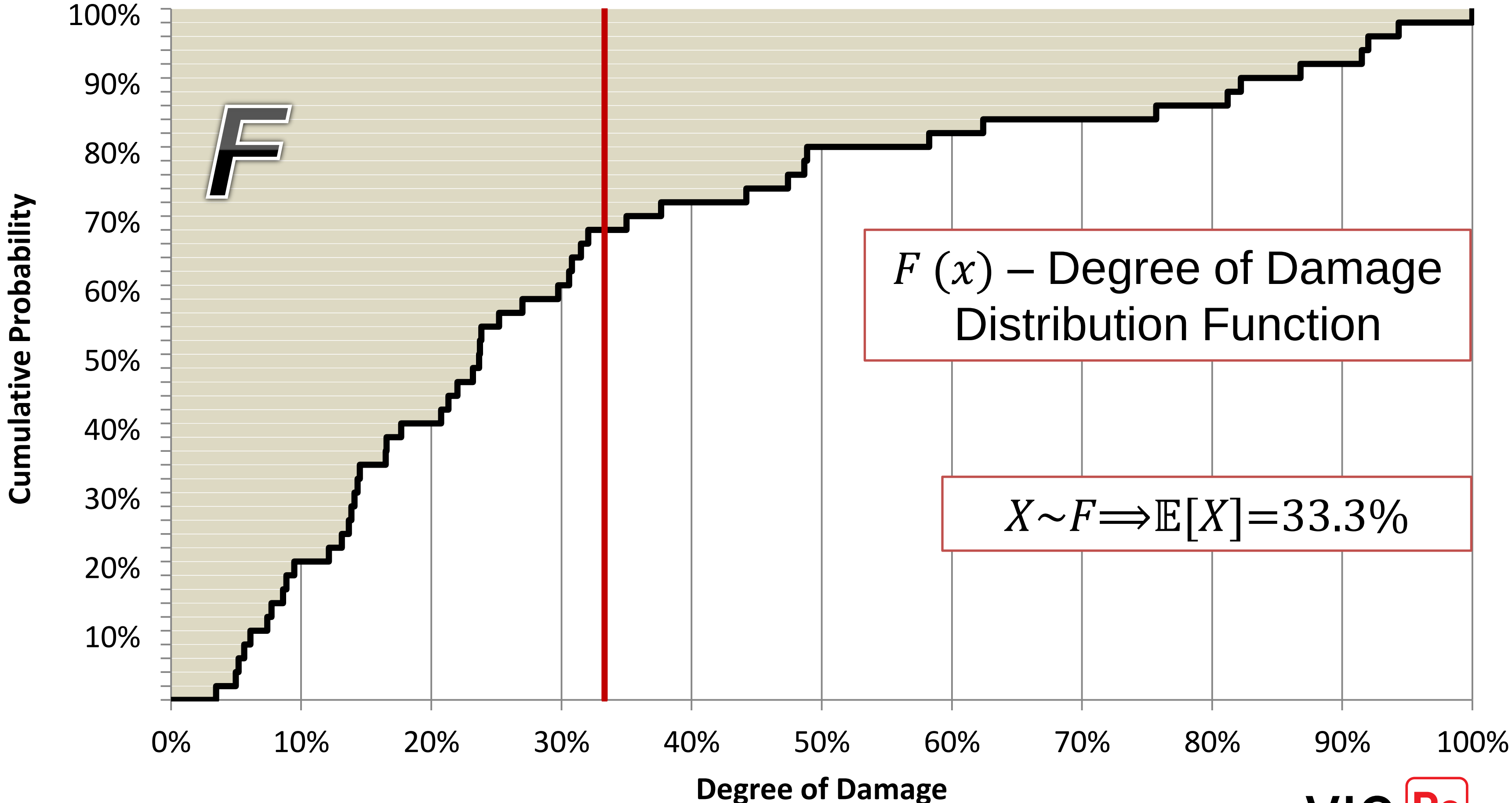
Degree of Damage Distribution

- Empirical Distribution Function



Degree of Damage Distribution F

- Empirical Distribution Function



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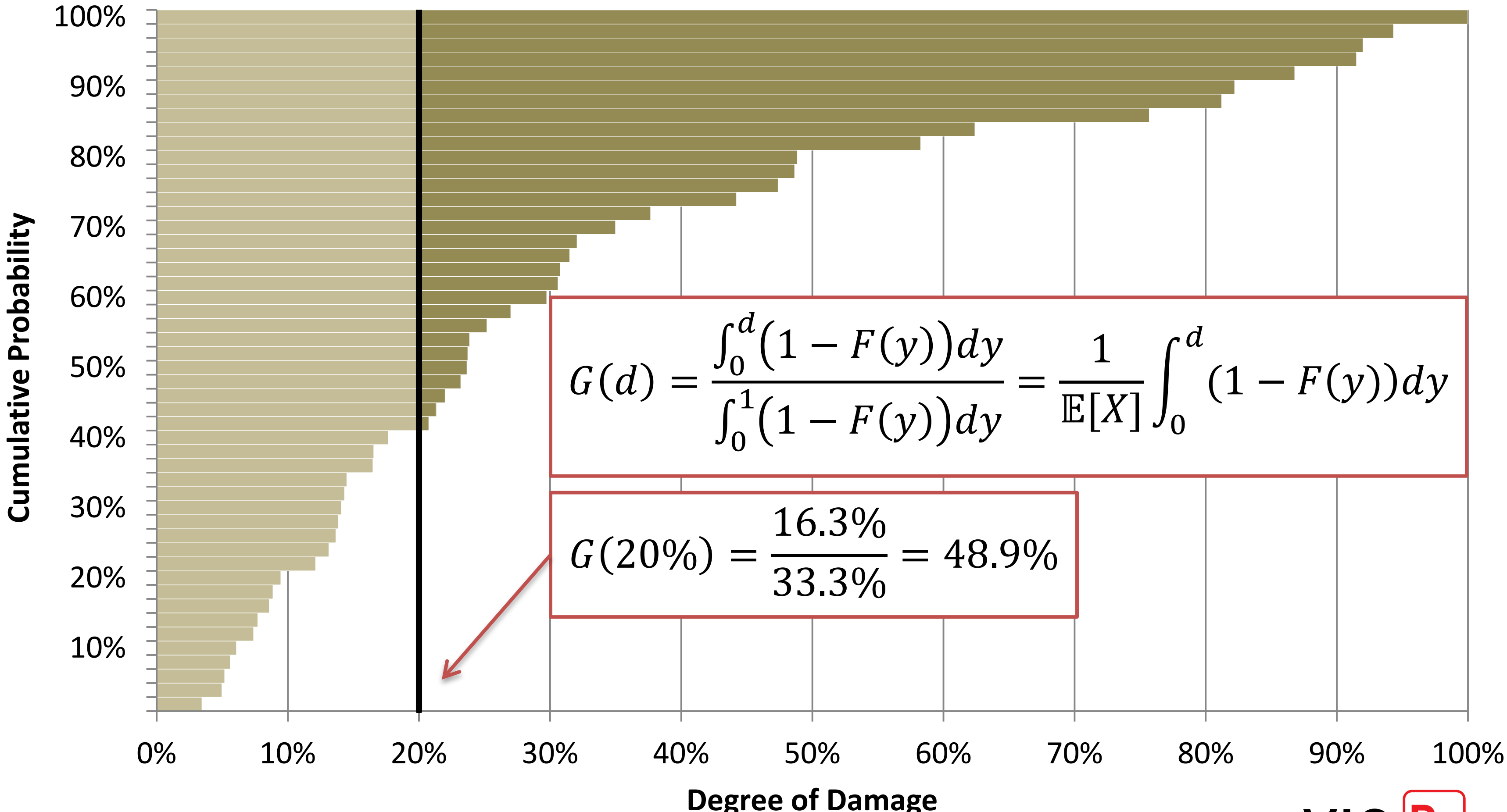
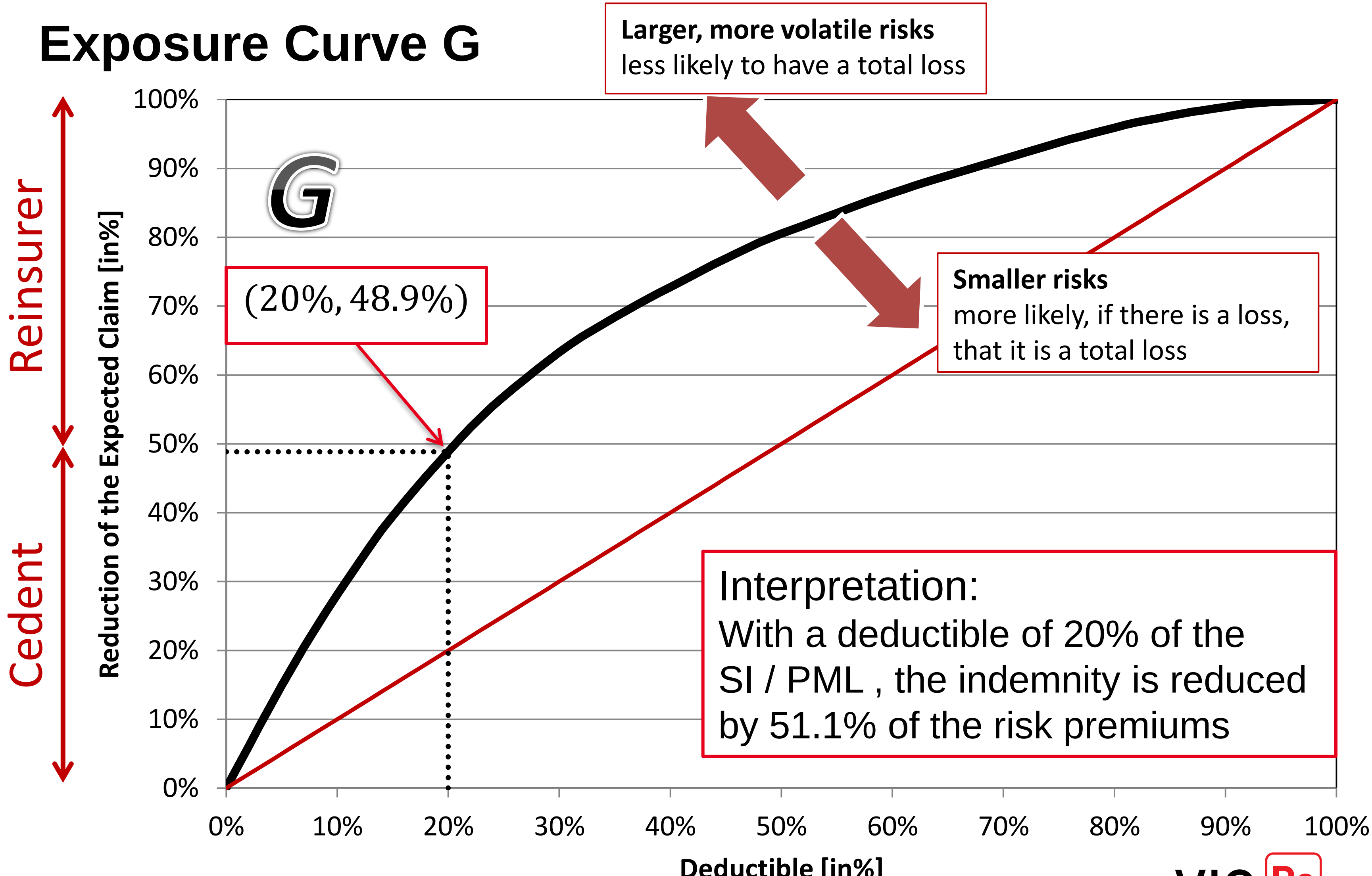
Degree of Damage Distribution

- Transition to the Exposure Curve



Construction and Interpretation of Exposure Curves

Exposure Curve G



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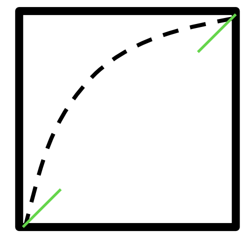
Properties of the Exposure Curve G

$$G(d) = \frac{\int_0^d (1-F(y))dy}{\int_0^1 (1-F(y))dy} = \frac{1}{\mathbb{E}[X]} \underbrace{\int_0^d (1-F(y))dy}_{LAS(d)} = \frac{LAS(d)}{LAS(1)}$$

$LAS(d)$ - Limited Average Severity

- By definition it holds that:
  - $G(0) = 0$
  - $G(1) = 1$

$$G'(0) \geq 1$$
$$G'(1) \geq 0$$

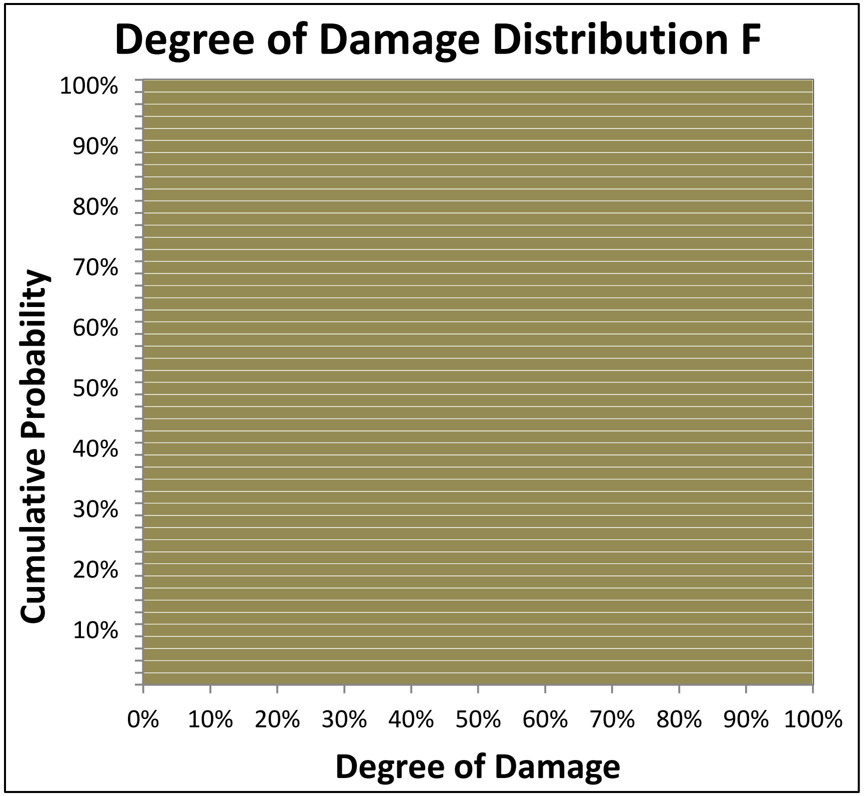


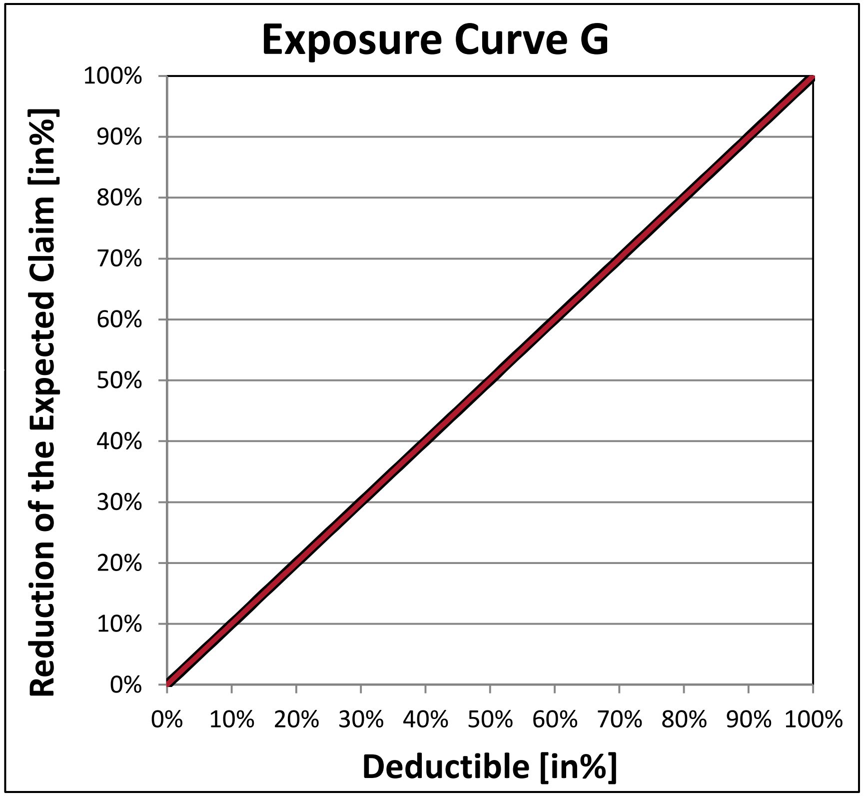
- Because
  - $G'(d) = \frac{1-F(d)}{\mathbb{E}[X]} \geq 0$
  - and $G''(d) = -\frac{F'(d)}{\mathbb{E}[X]} = -\frac{f(d)}{\mathbb{E}[X]} \leq 0$

G is increasing and concave in [0, 1]

Example 1: Total damages only

- Portfolio A produces only total damages
- Then it is obvious that $G(d) = d$





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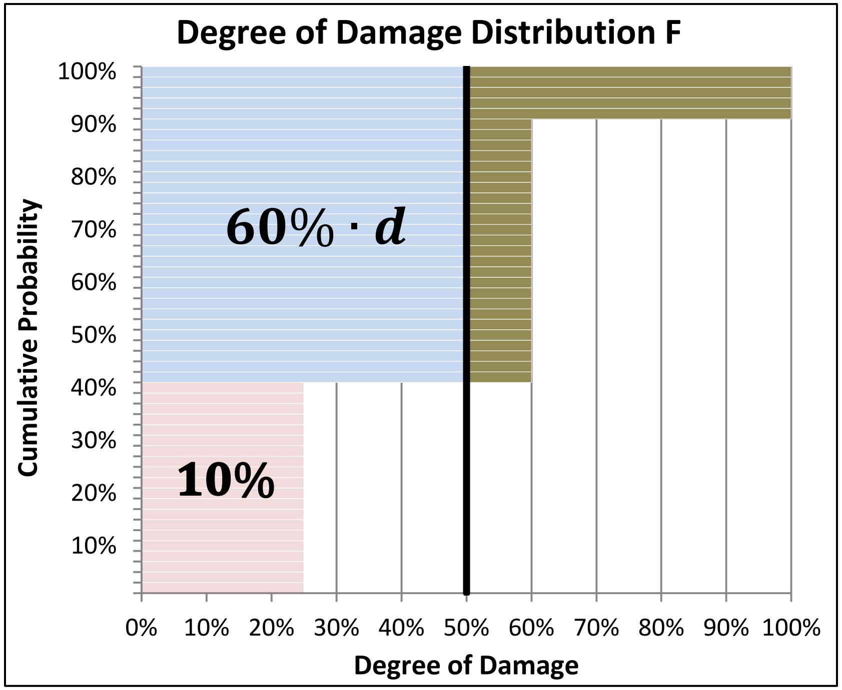
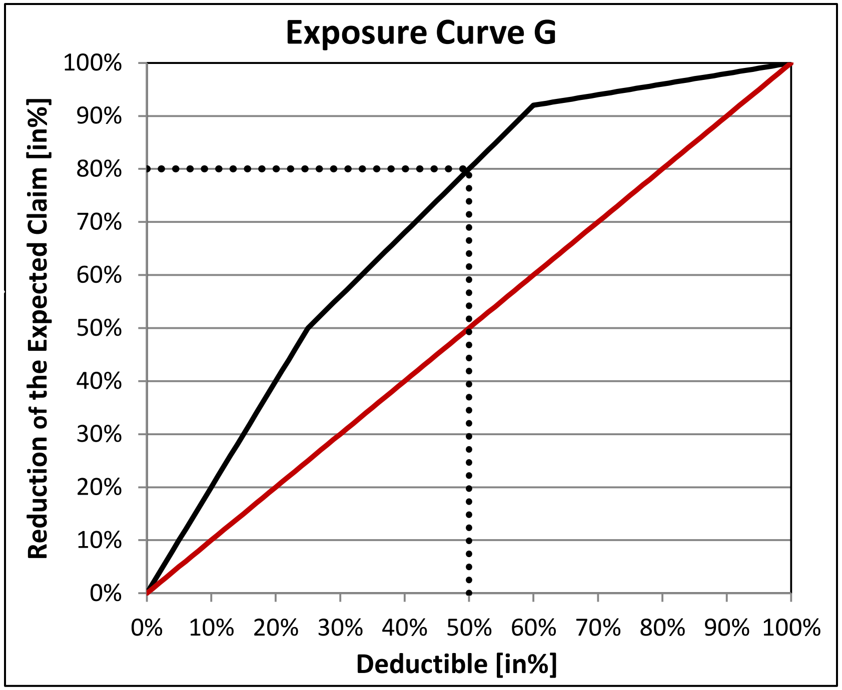
Example 2

- Portfolio B produces
  - 10% of claims that are total damages,
  - 50% of claims are 60% partial damages,
  - and 40% of claims are 25% partial damages
- Then

$$G(d) = \frac{1}{50\%} \cdot \begin{cases} d, & 0\% \leq d \leq 25\% \\ 10\% + 60\% \cdot d, & 25\% \leq d \leq 60\% \\ 10\% + 36\% + 10\% \cdot d, & 60\% \leq d \leq 100\% \end{cases}$$

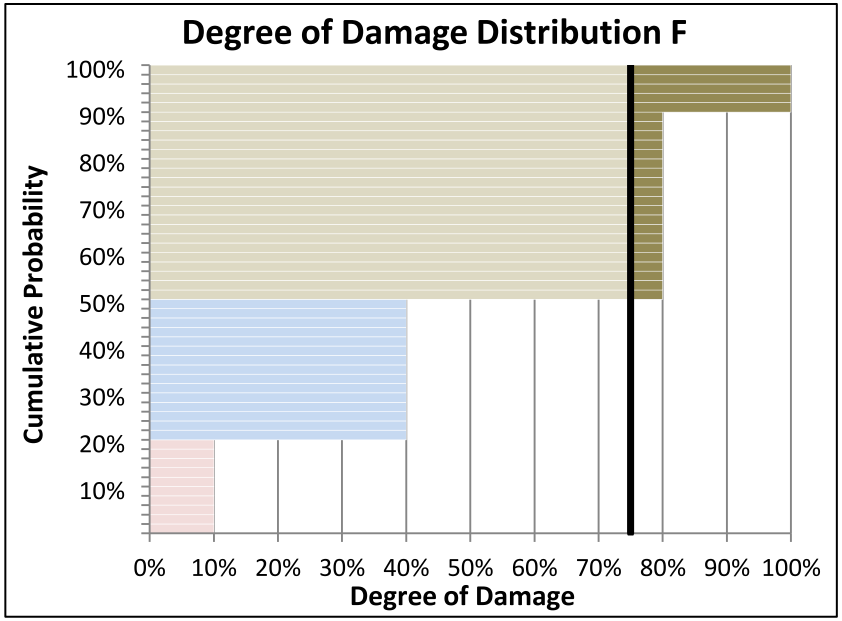


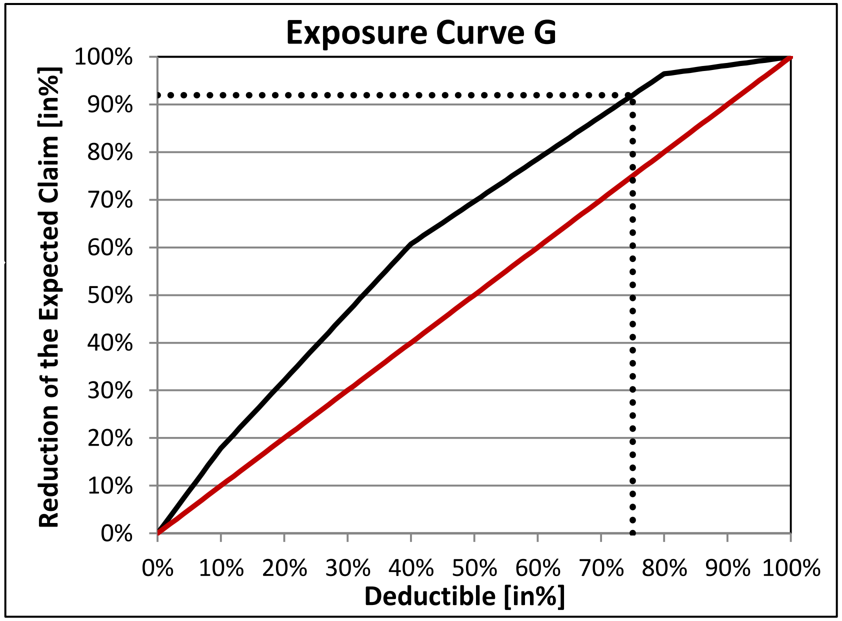


Example 3

- Portfolio C produces
  - 10% of claims that are total damages,
  - 40% of claims are 80% partial damages,
  - 30% of claims are 40% partial damages,
  - and 20% of claims are 10% partial damages
- Then

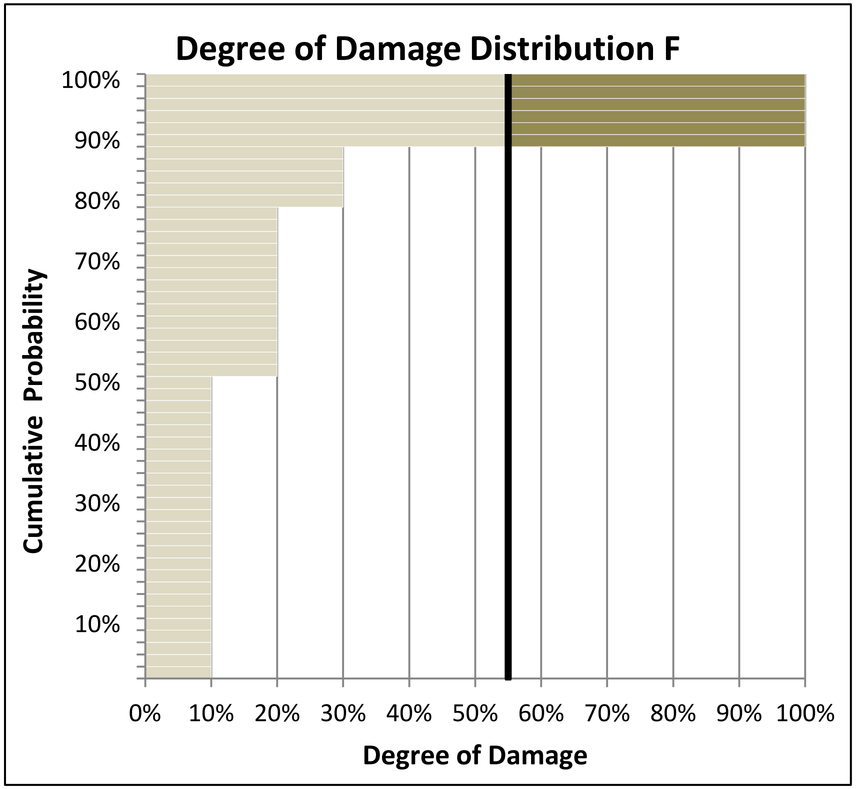
$$G(d) = \frac{1}{56\%} \cdot \begin{cases} d, & 0\% \leq d \leq 10\% \\ 2\% + 80\% \cdot d, & 10\% \leq d \leq 40\% \\ 2\% + 12\% + 50\% \cdot d, & 40\% \leq d \leq 80\% \\ 2\% + 12\% + 32\% + 10\% \cdot d, & 80\% \leq d \leq 100\% \end{cases}$$

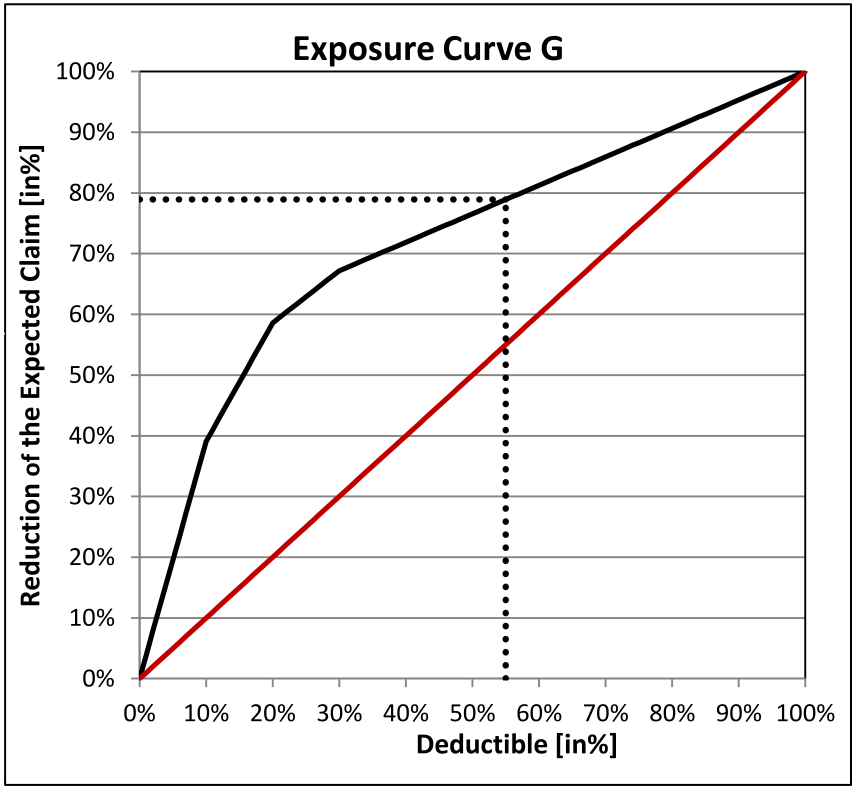




Example 4

- Portfolio D produces
  - 10% of claims that are total damages,
  - 10% of claims are 30% partial damages,
  - 30% of claims are 20% partial damages,
  - and 50% of claims are 10% partial damages



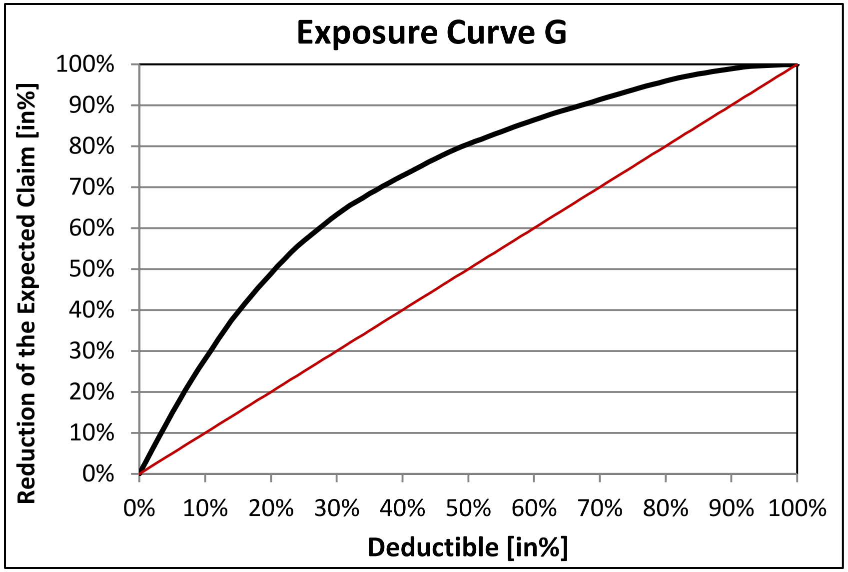


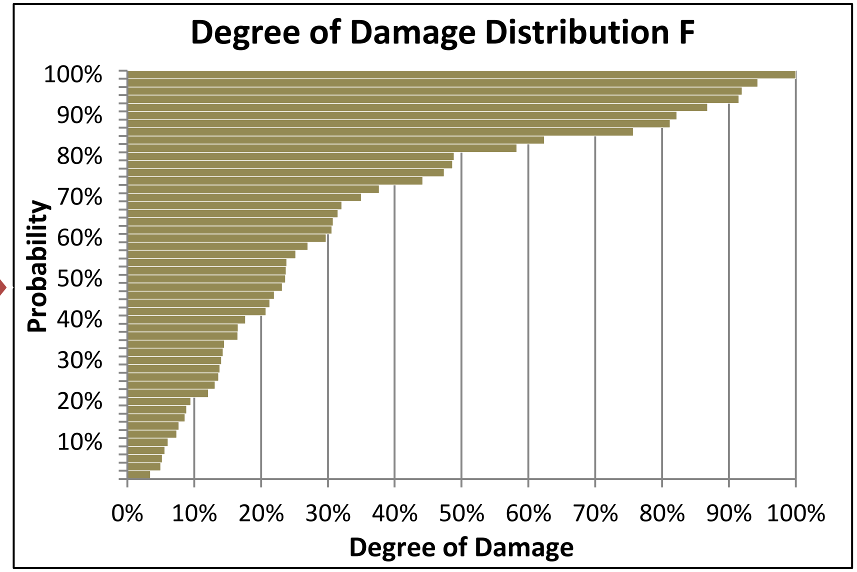
Exposure Models in Reinsurance

Transition Methods

Transition Methods

From Exposure Curve to Degree of Damage Distribution





- Total Damage Probability $= \frac{G'(1)}{G'(0)}$

- It can be derived : $G'(d) = \frac{1-F(d)}{\mathbb{E}[X]}$

$$F(d) = \begin{cases} 1 - \frac{G'(d)}{G'(0)}, & 0 \le d < 1 \\ 1, & d = 1 \end{cases}$$

- to remember: $F(d) = 1 - G'(d) \cdot \mathbb{E}[X]$

$$\mathbb{E}[X] = \frac{1}{G'(0)}$$

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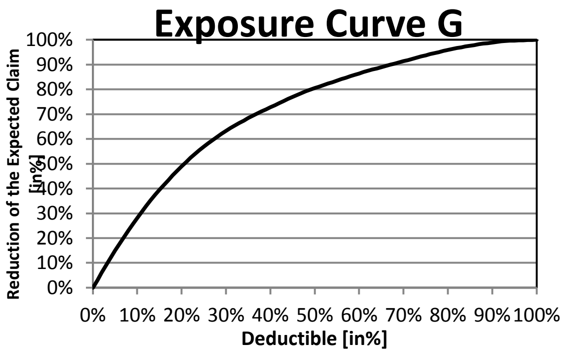
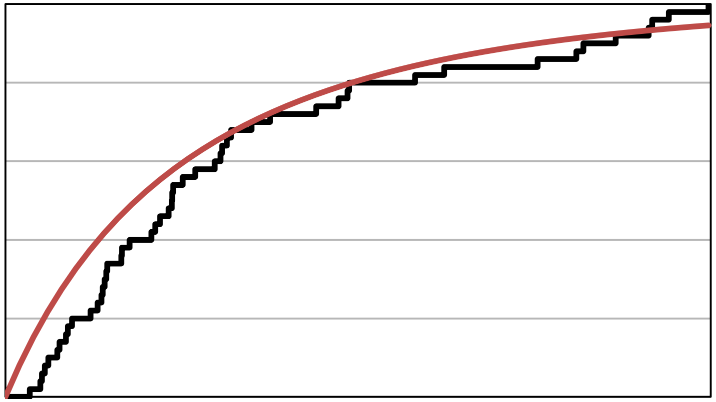
From Exposure Curve to Claim Frequency and Severity Distribution

Sum Insured /PML

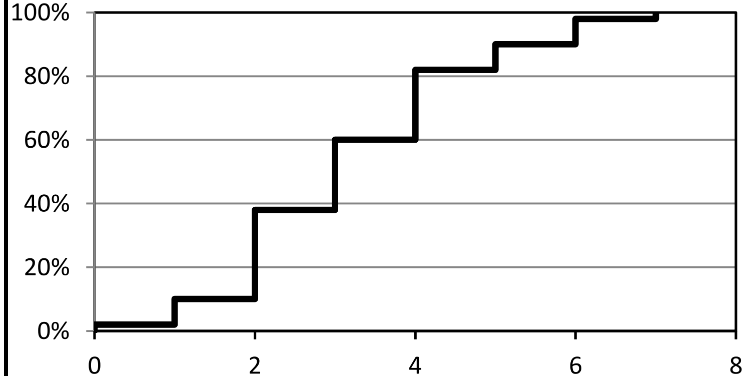


Risk Premium

Claims Severity



Claims Frequency



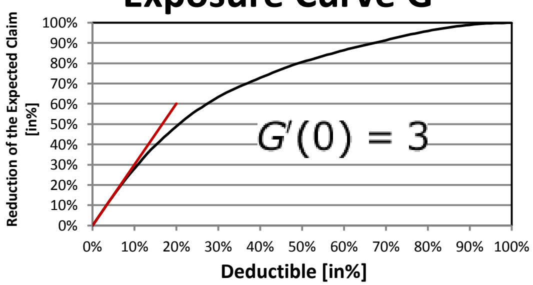
Transition Methods

From Exposure Curve to Claim Frequency

- Example

Sum Insured = 1.5M €

Exposure Curve G



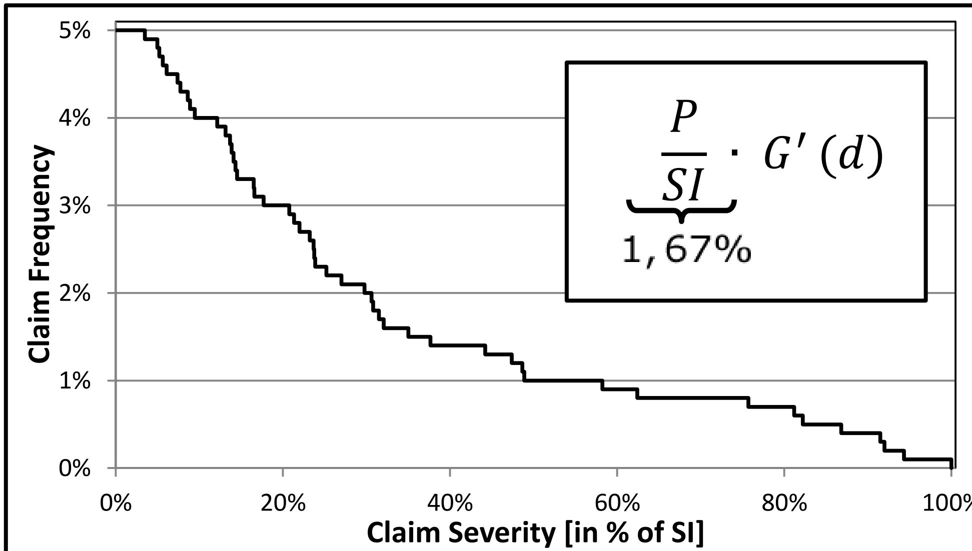
$$\mathbb{E}[X] = \frac{1}{G'(0)} = 33.3\%$$

Expected Claim Severity = 0.5 M €

Risk Premium
P = 25 000 €

Expected Claim Frequency = 5%

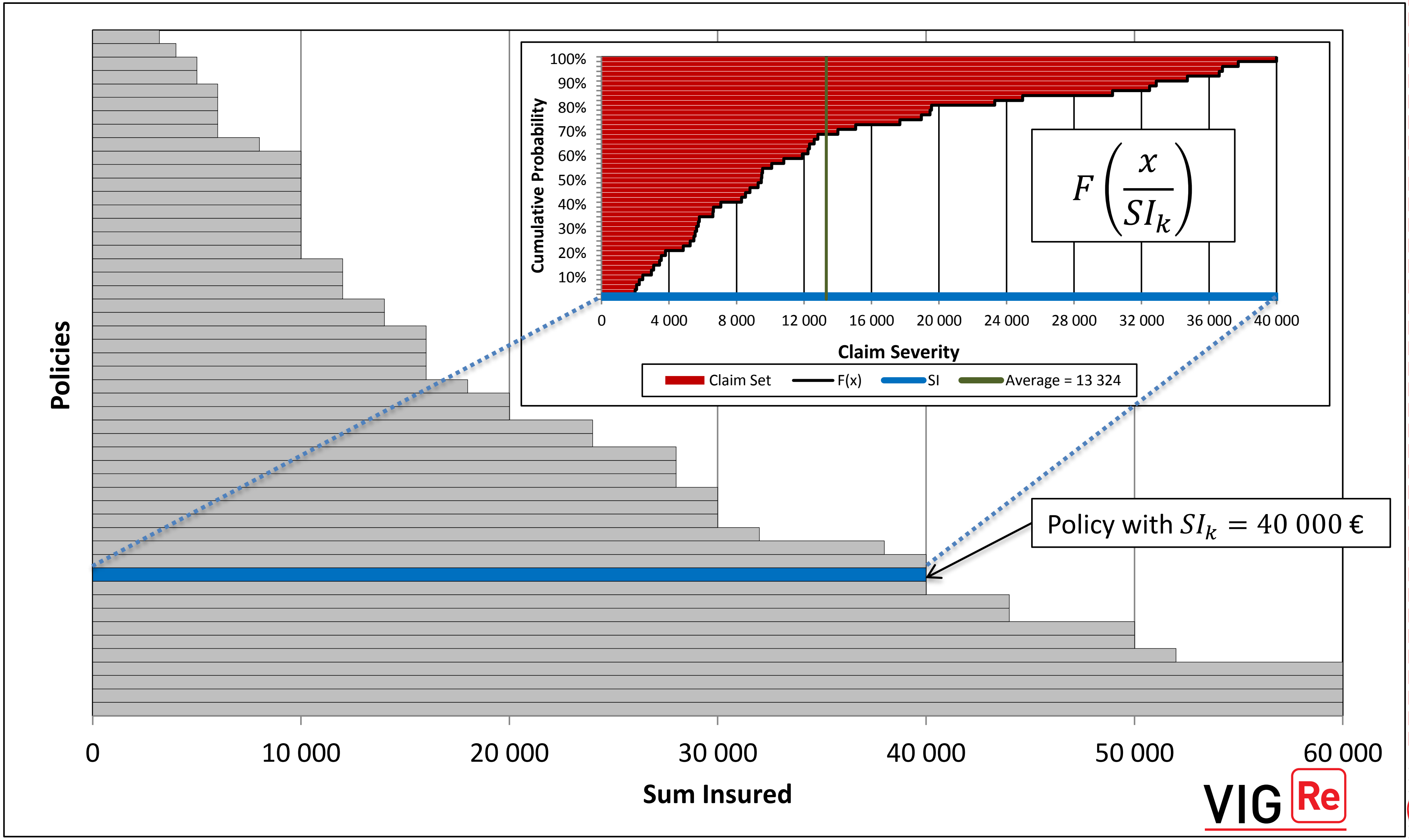
$$= \frac{P}{SI \cdot E(X)}$$



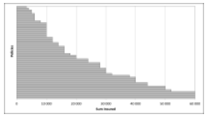
$$\underbrace{\frac{P}{SI}}_{1,67\%} \cdot G'(d)$$

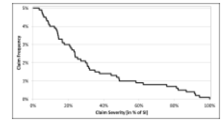
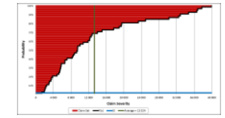
Illustration of the average claims frequency depending on the claims severity [in% of sum insured]

From Exposure Curve to Claim Severity Distribution



Overview - Claim Frequency and Severity Distribution



Policies/ Bands	SI / PML	Premium	Claim Frequency (at x)	Claim Severity
$\#1$	SI_1	P_1	$\lambda_1(x) = \frac{P_1}{SI_1} \cdot G'\left(\frac{x}{SI_1}\right)$	$F\left(\frac{x}{SI_1}\right)$
$\#2$	SI_2	P_2	$\lambda_2(x) = \frac{P_2}{SI_2} \cdot G'\left(\frac{x}{SI_2}\right)$	$F\left(\frac{x}{SI_2}\right)$
…	…	…	…	…
$\#k$	SI_k	P_k	$\lambda_k(x) = \frac{P_k}{SI_k} \cdot G'\left(\frac{x}{SI_k}\right)$	$F\left(\frac{x}{SI_k}\right)$
…	…	…	…	…
$\#n$	SI_n	P_n	$\lambda_n(x) = \frac{P_n}{n} \cdot G'\left(\frac{x}{SI_n}\right)$	$F\left(\frac{x}{SI_n}\right)$
"Collective Model (f.g.u.)" {Poisson($\lambda(0)$), $Y(x)$}			$\lambda(x) = \sum_{k=1}^{n} \lambda_k(x)$	$Y(x) = 1 - \frac{\lambda(x)}{\lambda(0)}$

Exposure Models in Reinsurance

Types of Exposure Curves

Types of Exposure Curves

- **Lloyds Curves**
  - Does not vary by amount of insurance or occupancy class
  - Underlying unknown (marine losses? WWII Fires?)

- **Salzmann (Personal Property)**
  - Based on actual Homeowners data (INA, 1960)
  - Varies by Construction/Protection Class
  - Building losses only and Fire losses only
  - Salzmann recommends not using them, only meant as an example

- **Reinsurer Curves (Munich, Skandia, etc.)**

- **Ludwig Curves (Personal and Commercial)**
  - Based on actual **Homeowners** and Commercial data, (based on relatively small portfolio of Hartford Insurance Group)
  - Includes all property coverages and perils (also 1989 hurricane Hugo losses)
  - Old data: 1984 – 1988

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Types of Exposure Curves

- **ISO's PSOLD (Insurance Services Office)**
  - Recent Data – updated every 2 years
  - Varies by amount of insurance, occupancy class, state, coverage, and peril
  - Continuous Distribution (no need for Interpolation)
  - Based on ISO data only
  - US specific (see White [2005])

- **Swiss Re curves**
  - also called Gasser curves (developed by Peter Gasser)
  - based on the data of "Fire statistics of the Swiss Association of Cantonal Fire Insurance Institutions" for the years 1959-1967.
  - widely used by European reinsurers

- **MBBEFD curves**
  - new parametrisation of all curves above

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Curve Selection

- Whether a lot of total losses occur, or partial and small losses are the rule, depends on various factors
- The decisive factors are (see Guggisberg [2004])
  - **Perils covered**
    - fire causes more damage to an individual building than a windstorm
    - while gas explosion can completely destroy a house, lightning strikes generally causes only partial damage
    - earthquakes cause minor to devastating damage to buildings
  - **Class of risk**
    - gunpowder factories are more likely to suffer total losses than food processing plants

Class of Risk	Average Degree of Damage
Residential Building	1.9%
Administration Building	0.5%
Farm Building	4.9%
Industrial Building	4.4%

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Curve Selection

- **Size of risk**
  - fire often causes only partial damage to a large building, whereas small buildings are more likely to suffer total destruction in the event of fire in terms of Sum Insured or PML
  - the larger a risk, the smaller the PML usually is as a percentage of the SI
- **Fire protection measures**
  - has a considerable influence on the shape of loss distribution function
  - make it possible to stop fires at an earlier stage – total overall loss is smaller and the share of minor losses increases

- **Summary:**

Peril/Type	Curve tends towards the diagonal	Curve runs in the middle area	Curve runs in the outer area
Fire	Risk with poor fire protection	Risks with average fire protection	Risks with above average fire protection
	Personal lines	Commercial lines	Industrial lines
	Farm building	Industrial building	Administrative building
Windstorm		Radio tower	Office building
Hurricane	Radio tower	Office building	

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Exposure Models in Reinsurance

MBBEFD Distributions

Background

- In general **exposure curves** are given in tabular form
- Problems:
  - limited number of curves available
  - piecewise linear functions
  - do not catch slight changes in reinsurance program
  - only conditionally suitable for the calculation of the number of claims
- Aim:
  - replace table values with function
    - For exposure curves this means that piecewise linear function becomes a continuous function

➢ **MBBEFD Distributions**

Background

- the abbreviation stands for
  Maxwell-**B**oltzmann, **B**ose-**E**instein, **F**ermi-**D**irac

- curves from Physics used in the field of Statistical Mechanics

- suitable for damage curve modeling in **property insurance** (see Bernegger [1997])
  - Continuous distributions
  - described by two parameters : $\boldsymbol{b} \geq \mathbf{0}$ and $\boldsymbol{g} \geq \mathbf{1}$
  - Swiss Re Y-Exposure Curves with a single parameter $\boldsymbol{c}$ are the special case of MBBEFD curves

- MBBEFD curves are common in Europe
  - less common in North America

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MBBEFD Exposure Curves

- Exposure Curve for normalized retention $m \in [0;1]$ is defined as

$$G_{b,g}(m) = \begin{cases} m\,, & g = 1 \lor b = 0 \\ \frac{\ln[1+(g-1)m]}{\ln[g]}\,, & b = 1 \land g > 1 \\ \frac{1-b^m}{1-b}\,, & bg = 1 \land g > 1 \\ \frac{\ln\left[\frac{(g-1)b+(1-gb)b^m}{1-b}\right]}{\ln[gb]}\,, & b > 0 \land b \neq 1 \land bg \neq 1 \land g > 1 \end{cases}$$

- case $bg < 1$ corresponds to **MB**, $bg = 1$ to **BE** and $bg > 1$ to **FD** distribution
- Interpretation:
  - $g = \frac{1}{Probability\ of\ Total\ Loss} = \frac{G'(0)}{G'(1)}$
  - b has no direct interpretation

MBBEFD Degree of Damage Distribution Function

- corresponding degree of damage random variable X defined on interval $[0;1]$ has CDF

$$F_{b,g}(x)\begin{cases} 1\,, & x=1 \\ 0, & x<1\ \wedge(g=1\ \vee b=0) \\ 1-\dfrac{1}{1+(g-1)x}\,, & x<1\ \wedge\ b=1\ \wedge g>1 \\ 1-b^{x}\,, & x<1\ \wedge\ bg=1\ \wedge g>1 \\ 1-\dfrac{1-b}{(g-1)\ b^{1-x}+(1\ -gb)}\,, & x<1\ \wedge\ b>0\wedge b\neq 1\wedge bg\neq 1\wedge g>1 \end{cases}$$

MBBEFD Degree of Damage Density Function

- Because of the finite probability $\frac{1}{g}$ for a total loss, the density function $\mathrm{f(x)} = F'(x)$ is defined only on the interval $[0;1)$

$$f_{b,g}(x) = \begin{cases} 0\,, & g = 1 \lor b = 0 \\ \dfrac{(g-1)}{(1+(g-1)x)^2}\,, & b = 1 \land g > 1 \\ -\ln[b]b^x\,, & bg = 1 \land g > 1 \\ \dfrac{(b-1)(g-1)\ln[b]b^{1-x}}{\left((g-1)b^{1-x}+(1-gb)\right)^2}, & b > 0 \land b \neq 1 \land bg \neq 1 \land g > 1 \end{cases}$$

MBBEFD Mean Degree of Damage

$$\mathbb{E}(X) = \begin{cases} 1, & g = 1 \vee b = 0 \\ \dfrac{\ln[g]}{g-1}, & b = 1 \wedge g > 1 \\ \dfrac{b-1}{\ln[b]} = \dfrac{g-1}{\ln[g]g}, & bg = 1 \wedge g > 1 \\ \dfrac{\ln[gb]\,(1-b)}{\ln[b](1-gb)}, & b > 0 \wedge b \neq 1 \wedge bg \neq 1 \wedge g > 1 \end{cases}$$

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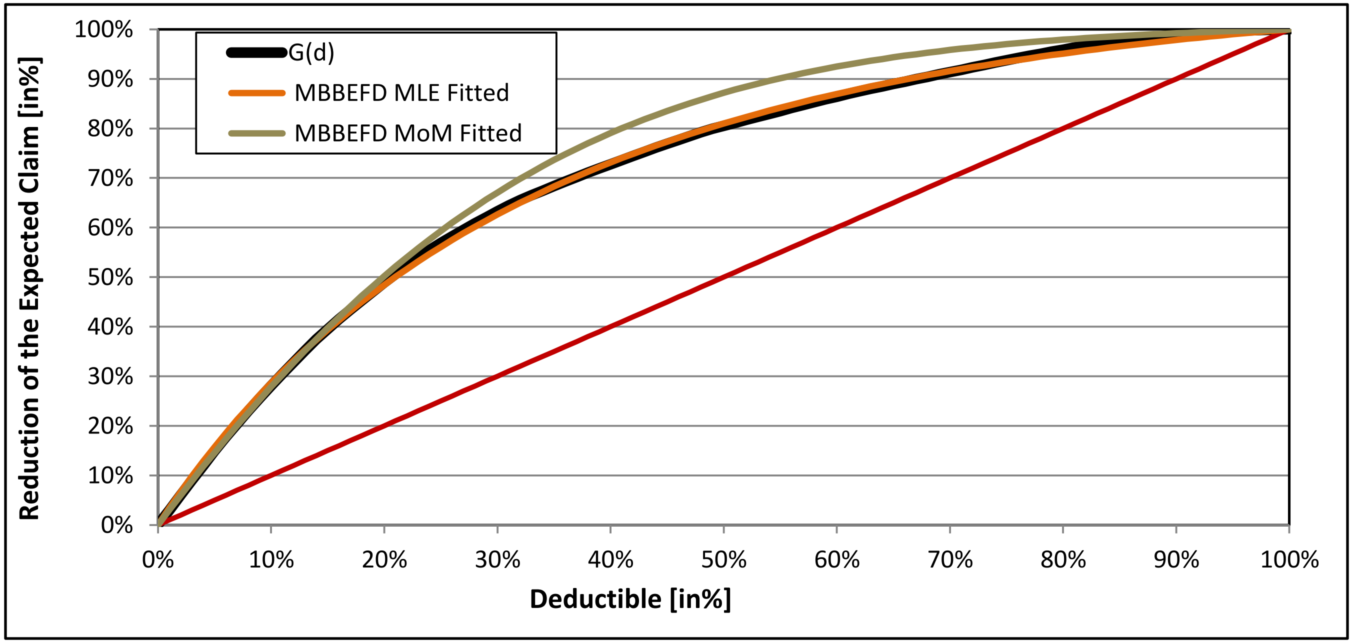
Parameters Estimation

- **Method of Moments**
  - $g = \frac{1}{Probability\ of\ Total\ Loss} = \frac{G'(0)}{G'(1)}$
  - b can be derived iteratively from equation $\mathbb{E}[X] = \frac{\ln(gb)(1-b)}{\ln(b)(1-gb)}$
- **Mean Least Squares** (R package: see Dutang et al. [2016])



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Swiss Re Exposure Curves

- Swiss Re Y_c Exposure Curves are very common among non-proportional underwriters

- parameter $c = 0, 1.5, 2\ , 3, 4$ denotes the concavity of the curve
  - $c = 0$ is the total loss (diagonal)
  - the higher c the curve becomes more concave

- c is a single parameter for defining the MBBEFD parameters b and g:

$$b_c = b(c) = \exp[\,3.1\ - 0.15\,(1 + c)c\,]$$

$$g_c = g(c) = \exp[\,(0.78 + 0.12c)c\,]$$

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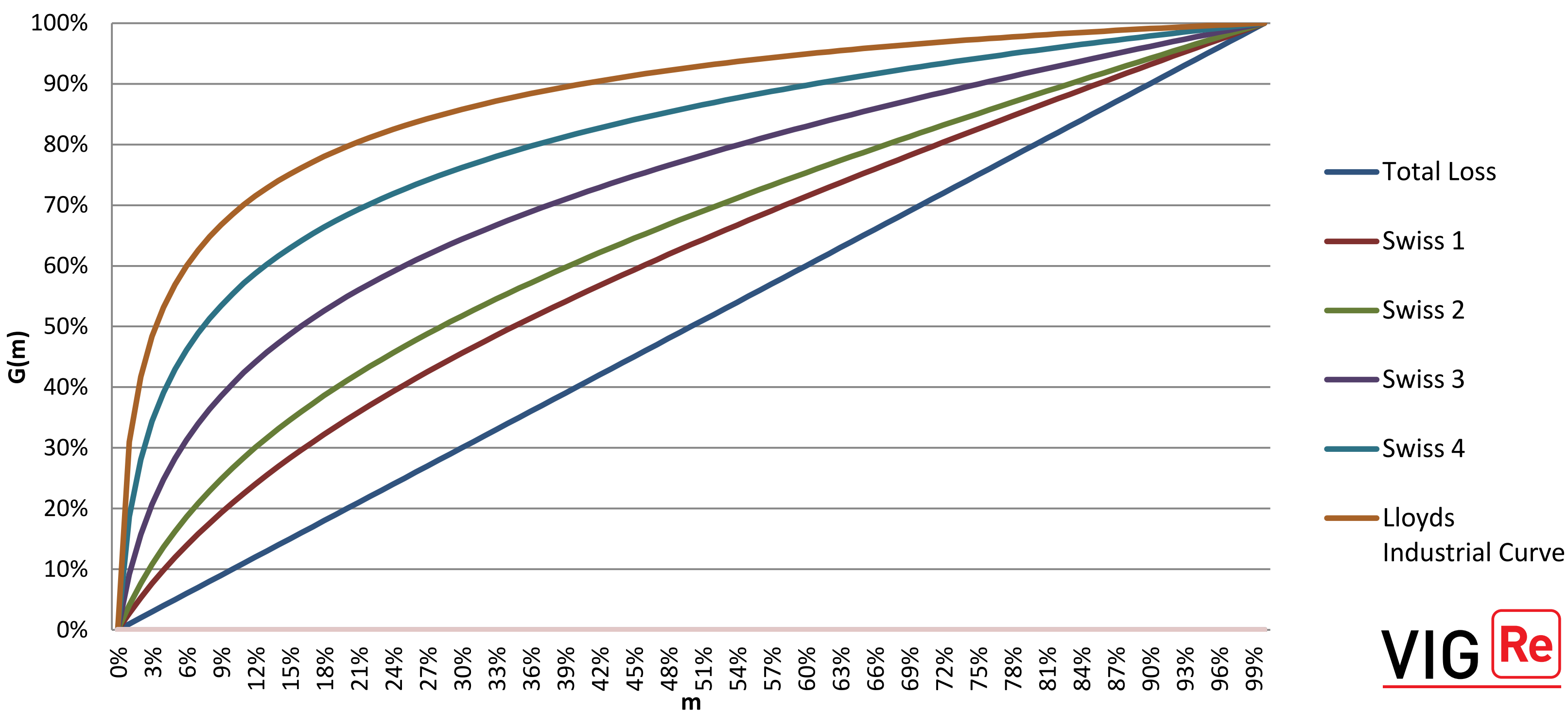
MBBEFD Distributions

Swiss Re Exposure Curves

Risk Group	Building Sum Insured from	Building Sum Insured to
1. Personal lines	200 000 CHF	400 000 CHF
2. Commercial lines (small scale)	400 000 CHF	1 000 000 CHF
3. Commercial lines (medium scale)	1 000 000 CHF	2 000 000 CHF
4. Industrial lines and large commercial	over 2 000 000 CHF	-



Swiss Re Exposure Curves

- big industrial companies insure their risks with **captives**
- many small losses are not longer passed on to the market and so do not appear in the statistics
  - therefore the major and total losses have greater impact
  - Exposure Curves for captive business tend more towards diagonal than those based on the entire claims
- Swiss Re developed three **captive exposure curves**
  - fire
  - business interruption
  - fire and business interruption combined
- can be used on qualitatively comparable portfolios made of policies with high deductibles
- have designation Y_6
  - this naming says nothing about shape
  - curves lie between Gasser curves Y_3 and Y_4

Swiss Re Exposure Curves

- there are three more Exposure Curves for **Oil and Petrochemicals** (OPC)
  - fire – runs in the are of Y_2
  - business interruption – runs between diagonal and Y_1
  - fire and business interruption combined – lies between Y_1 and Y_2

- all three have a high proportion of major losses typical for OPC

- original deductibles in OPC are usually high
  - ➢ major losses are of greater importance
  - ➢ Exposure Curves for OPC business tend more towards diagonal

MBBEFD Distributions

Swiss Re Exposure Curves

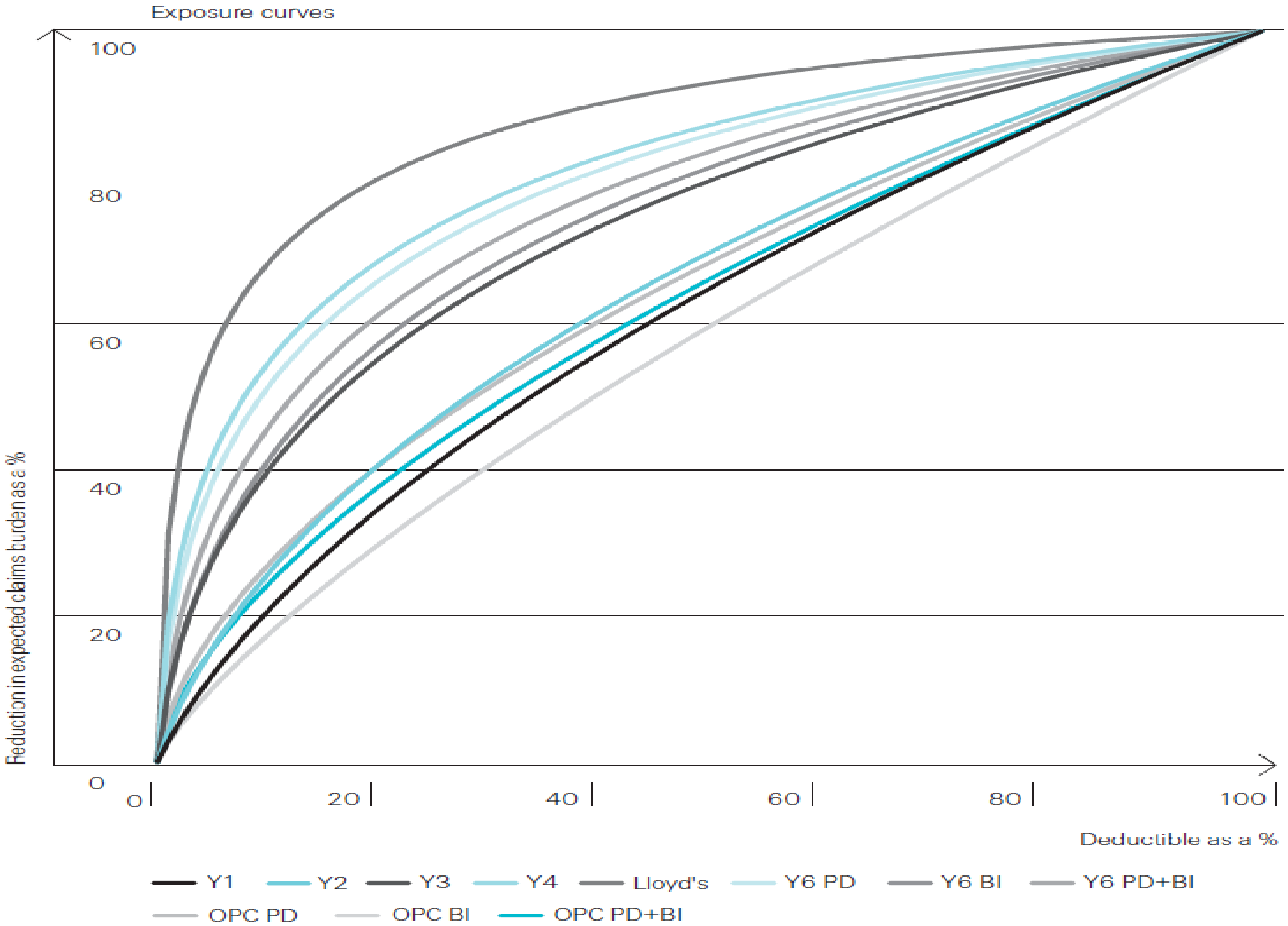
p = Probability of Total Loss

Exposure Curve	Parameter c	b	g	p	$\mathbb{E}[X]$	Scope of application	Basis	Size of Risk
	NA	NA	NA	NA	NA	OPC BI	PML	
Swiss 1	1.5	12.65	4.22	23.69%	34.86%	Personal lines	SI	<400 000 CHF
	NA	NA	NA	NA	NA	OPC Fire & BI combined	PML	
	NA	NA	NA	NA	NA	OPC Fire	PML	
Swiss 2	2.0	9.03	7.69	13%	22.09%	Commercial lines (small scale)	SI	<1 000 000 CHF
Swiss 3	3.0	3.67	30.57	3.27%	8.72%	Commercial lines (medium scale)	SI	<2 000 000 CHF
	3.1	3.29	35.56	2.81%	7.89	Captive BI	PML	
	3.4	2.35	56.78	1.76%	5.84%	Captive Fire & BI combined	PML	
	3.8	1.44	109.6	0.91%	3.89%	Captive Fire	PML	
Swiss 4	4	1.11	154.5	0.65%	3.19%	Industrial lines & large commercial	PML	>2 000 000
Lloyd's	5	0.25	992.3	0.10%	1.22%	Industry	Top location	
	Up to 8	NA	NA	NA	NA	Large scale Industry/ Multinational Companies	PML	

source: Guggisberg [2004]

MBBEFD Distributions

Swiss Re Exposure Curves



source: Guggisberg [2004]

Notes

- MBBEFD distributions are suitable only for **property insurance**

- These exposure curves are not sensitive to inflation
  - Maximum Loss is assumed to be equal to Sum Insured or to Probable Maximum Loss

- This makes it necessary to check the exposure curves only at relatively long intervals

- Limitation of exposure curves is that these curves were estimated on the market portfolios, so do not have to be accurate and give reasonable results on analyzed portfolio.
  - 1. Validate on loss profile of the company
  - 2. Validation on working layers (amount of losses to the layer implied by the curve)

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Exposure Models in Reinsurance

Pricing of Reinsurance Contracts with Exposure Curves

Components of the Reinsurance Price

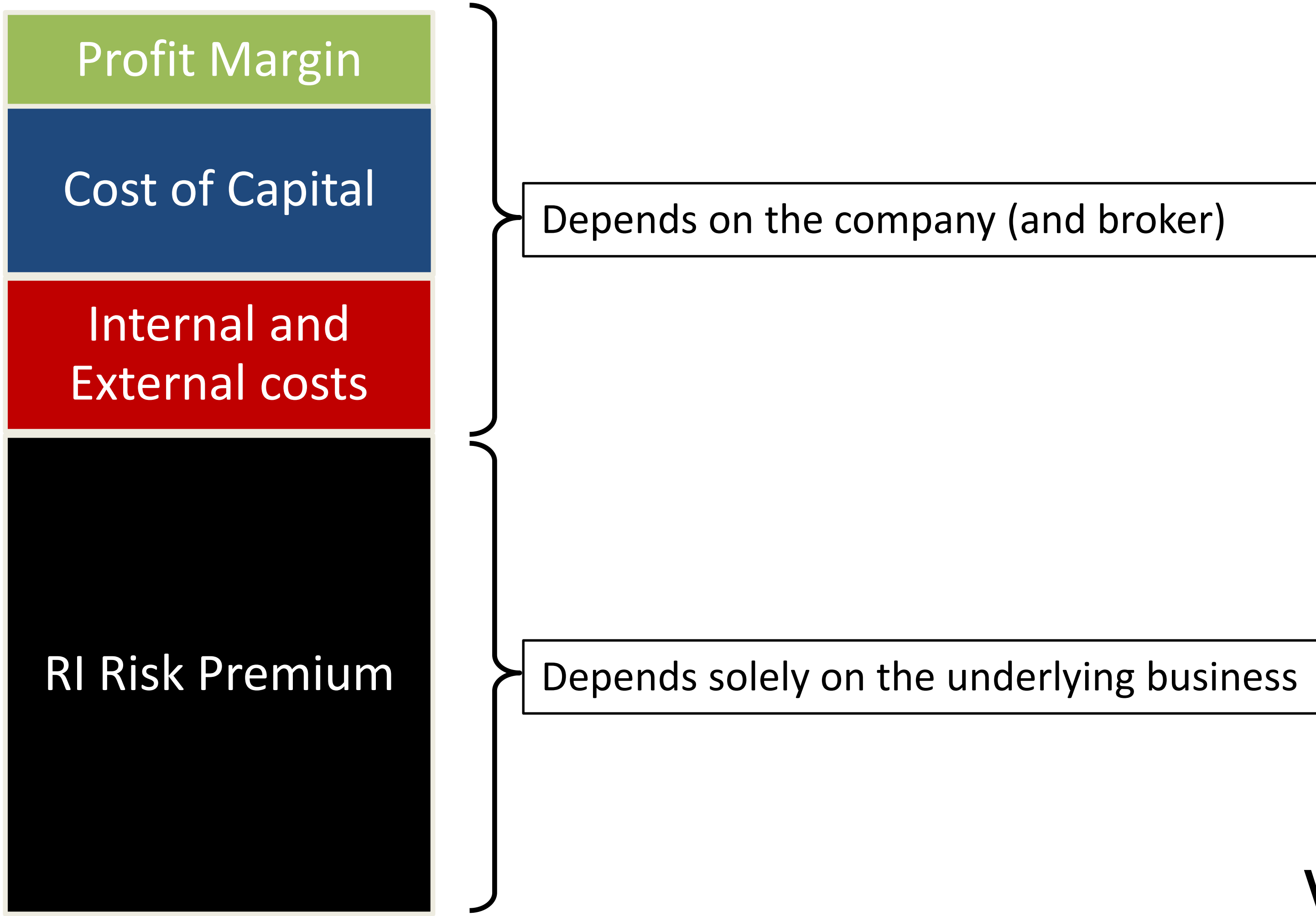
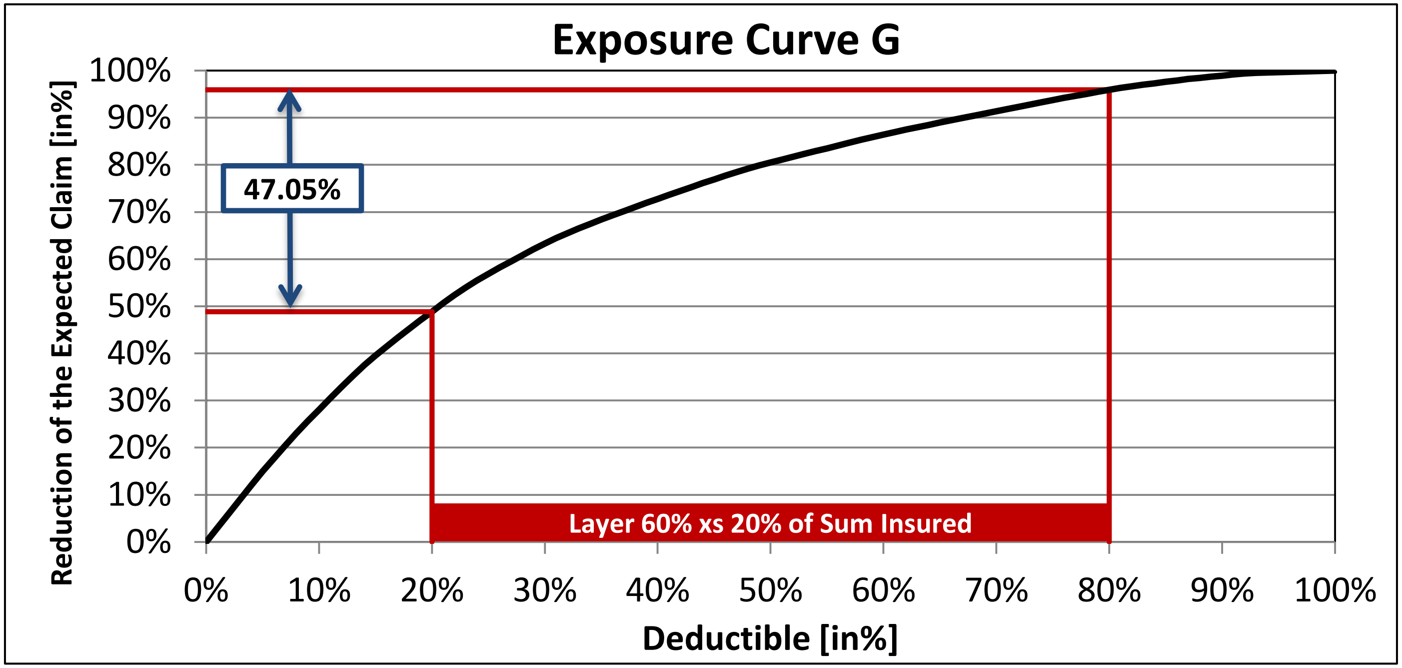


Illustration - premium distribution by Exposure Curve

- original policy with SI = 1.5 M € with risk premium P = 25 000 €
- XL contract 0.9 M € xs 0.3M €
- in terms of SI: XL contract 60% xs 20%



- The non-proportional coverage of the risk costs 47.05% of the original premium, i.e. RI Premium = 11 762.5 €

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Risk Profile

- Each portfolio contains risks of different size and quality with different cover
- Division of portfolio into sub-segments with a homogeneous risk structure

Gross Risk Profile

Risk Profile Name	Band SI/PML		Nr. Of Risks	Total SI/PML	Premium
Fire - Property Small Risks	0	500 000	81 847	4 073 604 954	4 515 515
Fire - Property Small Risks	500 000	1 000 000	1 566	1 118 702 402	984 372
Fire - Propert					
Fire - Propert					
Fire - Propert					
Fire - Propert					
Fire - Propert					

Gross Risk Profile

Risk Profile Name	Band SI/PML		Nr. Of Risks	Total SI/PML	Premium
Fire - Property Large Risks	0	2 000 000	358	407 360 495	451 551
Fire - Property Large Risks	2 000 000	5 000 000	783	2 237 404 803	1 968 744
Fire - Propert					
Fire - Propert					
Fire - Propert					
Fire - Propert					
Fire - Propert					

Gross Risk Profile

Risk Profile Name	Band SI/PML		Nr. Of Risks	Total SI/PML	Premium
Fire - Industrial Risks	0	8 000 000	179	1 018 401 239	903 103
Fire - Industrial Risks	8 000 000	25 000 000	392	4 474 809 607	3 937 488
Fire - Industrial Risks	25 000 000	90 000 000	312	23 703 696 107	16 398 441
Fire - Industrial Risks	90 000 000	80 000 000	73	5 915 249 922	3 721 409
Fire - Industrial Risks	80 000 000	160 000 000	18	2 343 974 273	4 179 388
Fire - Industrial Risks	160 000 000	320 000 000	10	2 111 761 192	2 918 733
Fire - Industrial Risks	320 000 000	480 000 000	4	1 643 798 597	1 506 689

➢ Modelling with different exposure curves

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Modelling Steps

1. Calculation of the **average Sum Insured** per band $k = 1, \dots, n$

$$\overline{SI}^k = \frac{\sum_{i=1}^{N^k} SI_i^k}{N^k},$$

where N^k is number of risks in the k^{th} band

2. Calculation of the **normalized retention** per band as percentage of SI/PML

$$\overline{m}^k = \min\left(\frac{R}{\overline{SI}^k}, 1\right)$$

3. Selection of the appropriate **exposure curve** for each band
4. Calculation of the **value of exposure curve** function $G(\overline{m}^k)$ for each k^{th} band
5. Calculation of **mean gross loss** per band

$$\mathbb{E}[Y^k] = P^k \cdot LR^k = \mathbb{E}\left[Y_{Ced}^k\right] + \mathbb{E}\left[Y_{Re}^k\right],$$

where P^k is the gross premium and LR^k is gross Loss Ratio for k^{th} band

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Modelling Steps

6. Calculation of **reinsurer´s mean ceded loss per band**

$$\mathbb{E}\left[Y_{Re}^{k}\right] = \left(1 - G(\overline{m}^{k})\right) \cdot \mathbb{E}[Y^{k}]$$

7. Calculation of the **mean aggregated loss into layer**

   i. in case of **one layer with unlimited capacity** for all risk profiles it can be expressed as

$$\mathbb{E}[Y_{Re}] = \sum_{k=1}^{n} \mathbb{E}\left[Y_{Re}^{k}\right]$$

   ii. the case of **more (L) layers** with the corresponding retentions denoted as ${}^{(l)}R,\ l = 1, \dots, L$, **reinsurer's mean ceded loss per k^{th} band and l^{th} layer** can be expressed as

$$\mathbb{E}\left[{}^{(l)}Y_{Re}^{k}\right] = \begin{cases} \left(G\left({}^{(l+1)}\overline{m}^{k}\right) - G\left({}^{(l)}\overline{m}^{k}\right)\right) \cdot \mathbb{E}[Y^{k}], & l < L \\ \left(1 - G\left({}^{(l)}\overline{m}^{k}\right)\right) \cdot \mathbb{E}[Y^{k}], & l = L \end{cases}$$

   and **reinsurer's mean ceded loss in l-th layer** as

$$\mathbb{E}\left[{}^{(l)}Y_{Re}\right] = \sum_{k=1}^{n} \mathbb{E}\left[{}^{(l)}Y_{Re}^{k}\right]$$

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Example – Quotation of XL 2M € xs 0.5M €

Gross Risk Profile					Step 1	Step 2	
Band SI/PML		Total SI/PML	Nr. Of Risks	Premium	Average SI/PML	R in % SI	R+L in % SI
0	100 000	3 895 341 592	46 425	7 502 888	83 906	100.0%	100.0%
100 000	200 000	2 237 330 404	13 994	4 158 031	159 878	100.0%	100.0%
200 000	300 000	1 910 346 260	7 483	3 053 667	255 291	100.0%	100.0%
300 000	500 000	1 316 269 834	4 014	1 150 935	327 920	100.0%	100.0%
500 000	750 000	1 146 935 002	1 599	1 668 885	717 283	69.7%	100.0%
750 000	1 000 000	810 399 944	936	1 280 817	865 812	57.7%	100.0%
1 000 000	1 500 000	697 830 194	563	941 983	1 239 485	40.3%	100.0%
1 500 000	2 500 000	403 707 061	199	523 651	2 028 679	24.6%	100.0%
2 500 000	5 000 000	106 697 299	32	190 575	3 334 291	15.0%	90.0%
5 000 000	10 000 000	40 104 436	8	41 152	5 013 055	10.0%	59.8%
			Total Premium	**20 512 584**			

- Total Gross Loss Ratio is 60%
- for the sake of simplicity we assume that Loss Ratio is equal to 60% for all bands and one exposure curve is appropriate for all bands

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Example – Quotation of XL 2M € xs 0.5M €

Band	Premium	Step 4: $G(d_1)$	Step 4: $G(d_2)$	Step 4: $G(d_2)-G(d_1)$	Step 6: RI Premium
#1	7 502 888	100.0%	100.0%	0.0%	0
#2	4 158 031	100.0%	100.0%	0.0%	0
#3	3 053 667	100.0%	100.0%	0.0%	0
#4	1 150 935	100.0%	100.0%	0.0%	0
#5	1 668 885	91.2%	100.0%	8.8%	146 370
#6	1 280 817	85.2%	100.0%	14.8%	189 866
#7	941 983	73.1%	100.0%	26.9%	253 821
#8	523 651	56.4%	100.0%	43.6%	228 339
#9	190 575	39.5%	98.9%	59.4%	113 236
#10	41 152	25.6%	86.3%	60.7%	24 981
Total Premium	**20 512 584**		**Loss Ratio**	**60%**	**956 614**
RI Rate	**2.798%**				**573 968**

Exposure Models in Reinsurance

Increased Limit Factors

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Introduction

- another type of exposure rating that can be used in **liability non-proportional reinsurance**
- the object of insurance is not known in advance
- the maximum possible loss is hardly to be estimated and can be much higher than sum insured
- helps
  - when not enough historical claims are available
  - if any of the experience rating based approaches is not possible to be applied reasonably (e.g. limited data to develop charges for high limits of liability coverages – these may represent very significant potential loss)
- **parameters based on enough market data** need to be applied

➢ **Increased Limit Factors**

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Definition

- usually available in tabular form
- An **increased limit factor** (*ILF*) at limit L related to basic limit B is defined as:

$$ILF(L) = \frac{\mathbb{E}[Y^L]}{\mathbb{E}[Y^B]},$$

- where
  - $\mathbb{E}[Y^L]$ denotes mean expected aggregate loss at the policy limit L
  - $\mathbb{E}[Y^B]$ denotes the mean aggregate loss at the basic limit B

- Both denote aggregate losses assuming all original policies had limits L or B respectively, i.e.

  - $Y^L = \sum_{i=1}^{N} min(X_i, L)$
  - $Y^B = \sum_{i=1}^{N} min(X_i, B)$

Definition

- it is assumed, that claims frequency is independent of claim severity and the frequencies are equal independently on the purchased limit

- Therefore

$$ILF(L) = \frac{\mathbb{E}[Y^L]}{\mathbb{E}[Y^B]} = \frac{\mathbb{E}[N^L]\mathbb{E}[X^L]}{\mathbb{E}[N^B]\mathbb{E}[X^B]} = \frac{\mathbb{E}[X^L]}{\mathbb{E}[X^B]}$$

- assumptions need to be verified.
  - ILF should be constructed for different classes of liability separately

- in notation as we had for exposure curves:

$$ILF(L) = \frac{LAS(L)}{LAS(B)}$$

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Example

- ILF Table

Claim Severity	Loss at 100 000 Basic Limit	Loss at 250 000 Increased Limit	Loss at 500 000 Increased Limit	Loss at 750 000 Increased Limit	Loss at 1 000 000 Increased Limit	Loss at 1 250 000 Increased Limit	Loss at 1 500 000 Increased Limit
50 000	50 000	50 000	50 000	50 000	50 000	50 000	50 000
60 000	60 000	60 000	60 000	60 000	60 000	60 000	60 000
120 000	100 000	120 000	120 000	120 000	120 000	120 000	120 000
165 000	100 000	165 000	165 000	165 000	165 000	165 000	165 000
270 000	100 000	250 000	270 000	270 000	270 000	270 000	270 000
475 000	100 000	250 000	475 000	475 000	475 000	475 000	475 000
580 000	100 000	250 000	500 000	580 000	580 000	580 000	580 000
780 000	100 000	250 000	500 000	750 000	780 000	780 000	780 000
1 100 000	100 000	250 000	500 000	750 000	1 000 000	1 100 000	1 100 000
2 000 000	100 000	250 000	500 000	750 000	1 000 000	1 250 000	1 500 000
Total	910 000	1 895 000	3 140 000	3 970 000	4 500 000	4 850 000	5 100 000
LAS	91 000	189 500	314 000	397 000	450 000	485 000	510 000
ILF	1	2.08	3.45	4.36	4.95	5.33	5.60

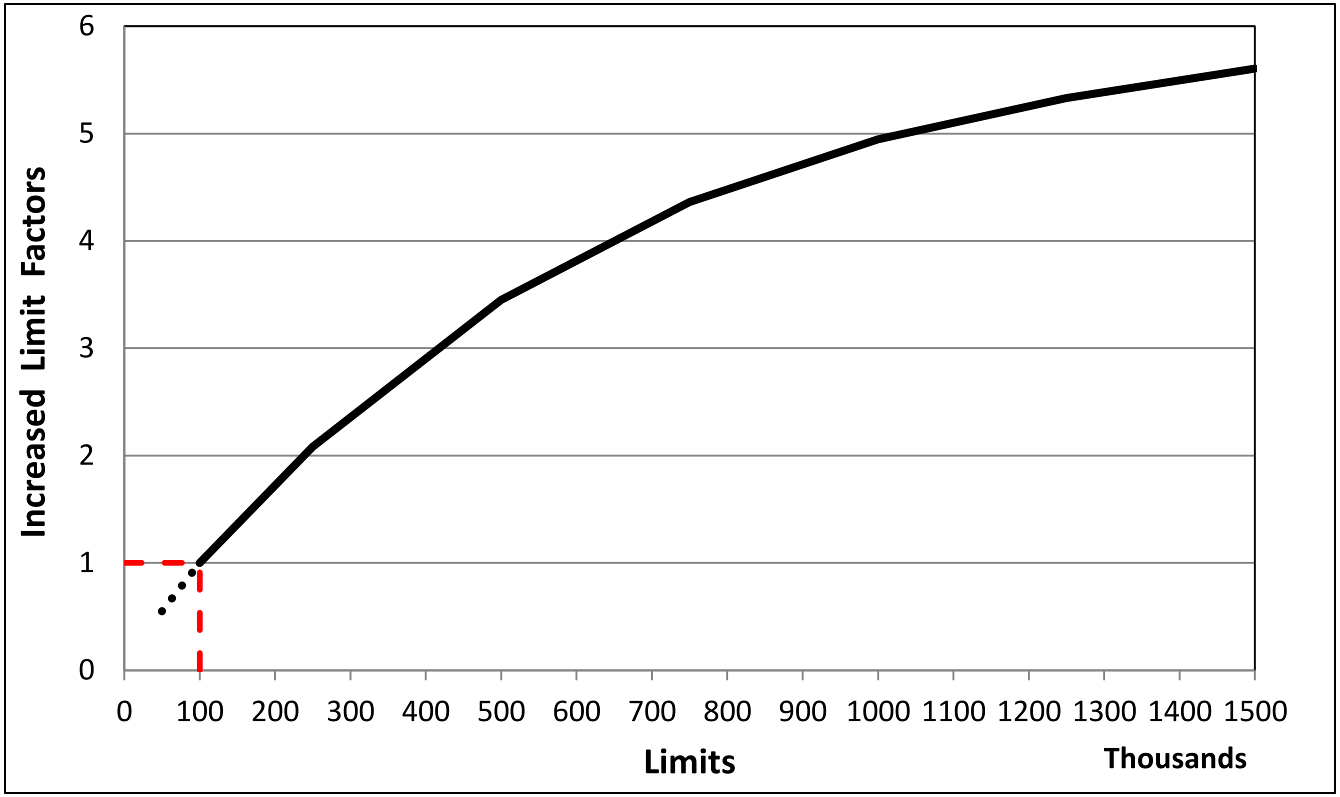
Interpretation:
If the limit increases 10 times, the loss will increase only 4.95 times

Increased Limit Factors

Example

- ILF Curve

Limits	ILF
50 000	0.55
100 000	1
250 000	2.08
500 000	3.45
750 000	4.36
1 000 000	4.95
1 250 000	5.33
1 500 000	5.60



Interpolation between limits is necessary

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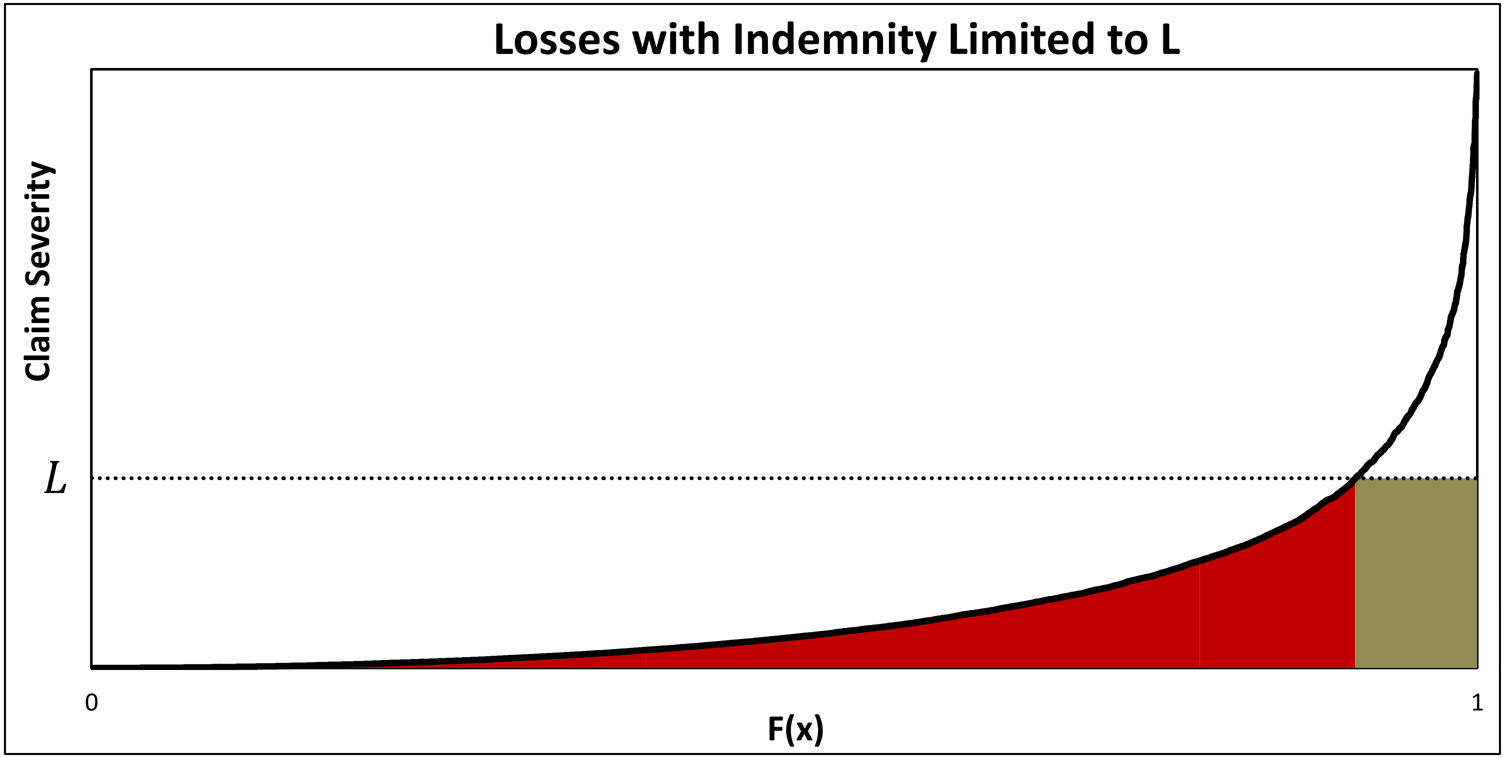
Limited Average Severity

- for any limit $L \geq 0$, the limited average severity (LAS), at a limit of L, in the case of a continuous distribution with distribution function F, can be expressed as

$$LAS(L) = \mathbb{E}[X^L] = \int_0^L x\, dF(x) + L \cdot [1 - F(L)] = \int_0^L \big(1 - F(x)\big) dx$$



Properties

- for derivative of ILF it holds:

$$\frac{d}{dL} ILF(L) = \frac{1}{\mathbb{E}[Y^B]} \cdot \frac{d}{dL}\left(\int_0^L x\, dF(x) + L \cdot [1 - F(L)] \right)$$

$$= \frac{1}{\mathbb{E}[Y^B]} \left(L \frac{dF(L)}{dL} + [1 - F(L)] - L \frac{dF(L)}{dL} \right)$$

- because

$$ILF'(L) = \frac{1 - F(L)}{LAS(B)} \geq 0$$

➢ $ILF(L)$ **is an increasing function of** $\boldsymbol{L}$

- as

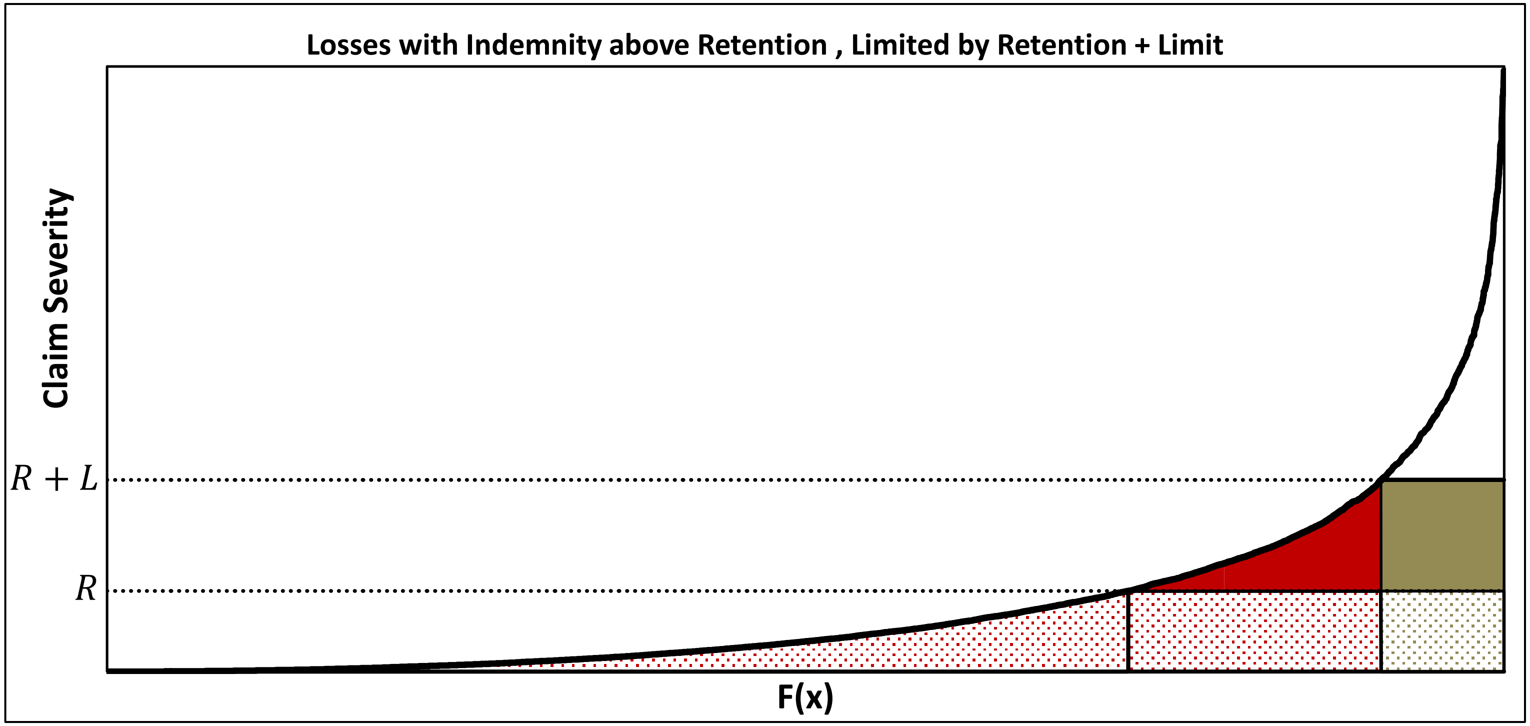
$$ILF''(L) = \frac{-f(L)}{LAS(B)} \leq 0$$

➢ ILF **is concave**

Limited Average Severity

- LAS can be used to express the **loss into XL layer**
- if expected number of losses from ground up is λ then expected loss into XL layer with retention R and limit L is

$$\mathbb{E}[Y_R^{R+L}] := \lambda \cdot \mathbb{E}[X_R^{R+L}] := \lambda \left[\int_R^{R+L} (x-R)\, dF(x) + L \cdot [1 - F(R+L)] \right]$$



Limited Average Severity

- previous can be expressed as

$$\mathbb{E}[Y_R^{R+L}] = \lambda \cdot \left[\int_R^{R+L} (x-R)\,dF(x) + L \cdot [1-F(R+L)] \right]$$

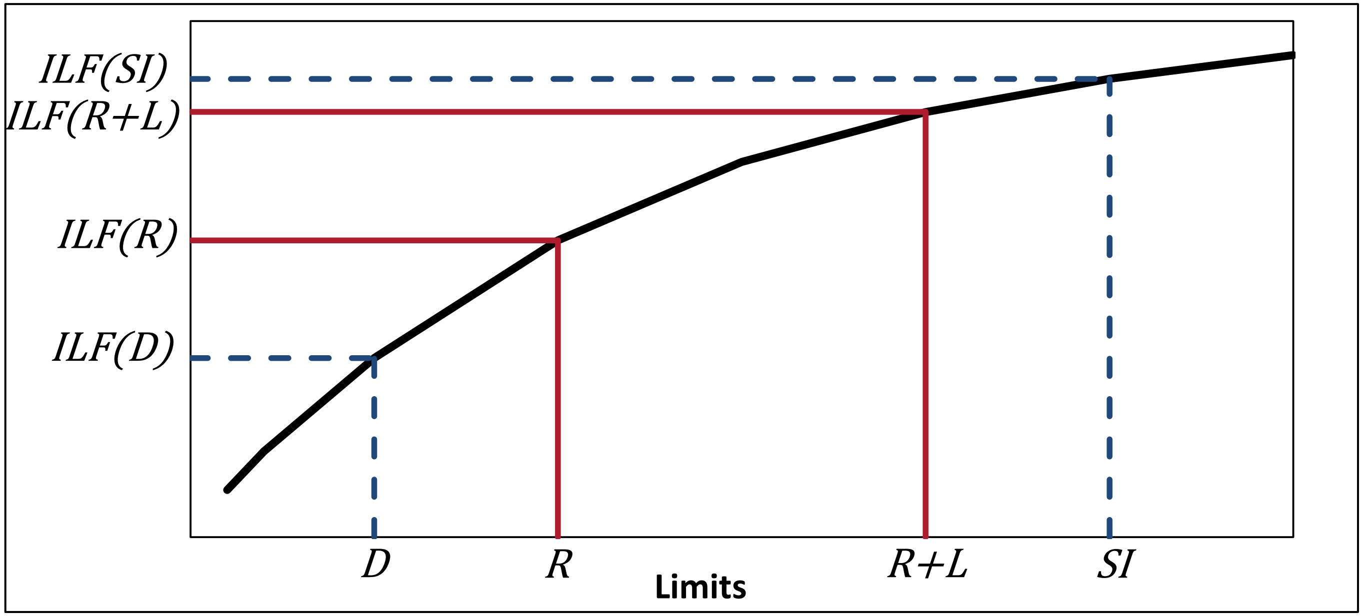
$$= \lambda \cdot \left[\int_R^{R+L} x\,dF(x) + (R+L) \cdot [1-F(R+L)] - R[1-F(R)] \right]$$

$$= \lambda \cdot [LAS(R+L) - LAS(R)]$$

Ceded Share

- ILF determine the ratio in which original loss from policy with sum insured SI and deductible D is divided between reinsurer and cedent
- ceded ratio C can be expressed as

$$C = \frac{\mathbb{E}[Y_R^{R+L}]}{\mathbb{E}[Y_D^{SI}]} = \frac{\lambda \cdot \mathbb{E}[X_R^{R+L}]}{\lambda \cdot \mathbb{E}[X_D^{SI}]} = \frac{LAS(R+L) - LAS(R)}{LAS(SI) - LAS(D)} = \frac{ILF(R+L) - ILF(R)}{ILF(SI) - ILF(D)}$$



- in case $D = 0$

$$C = \frac{ILF(R+L) - ILF(R)}{ILF(SI)}$$

Inflation

- disadvantage of the liability exposure rating method is the sensitivity of LAS and therefore also ILF on inflation

- Assuming a constant inflation applied on all sizes of losses, the basic limit losses ($LAS(B)$) will be inflated by lower rate than losses limited at higher limits of liability ($LAS(L)$)

  - will lead to even higher inflation in excess layers

- This phenomenon is called as "**Leveraged Effect of Inflation**"

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Increased Limit Factors

Inflation Example

Claim Severity	Loss at 100 000 Basic Limit	Loss at 250 000 Increased Limit	Loss at 500 000 Increased Limit	Loss at 750 000 Increased Limit	Loss at 1 000 000 Increased Limit	Loss at 1 250 000 Increased Limit	Loss at 1 500 000 Increased Limit
50 000	50 000	50 000	50 000	50 000	50 000	50 000	50 000
60 000	60 000	60 000	60 000	60 000	60 000	60 000	60 000
120 000	100 000	120 000	120 000	120 000	120 000	120 000	120 000
165 000	100 000	165 000	165 000	165 000	165 000	165 000	165 000
270 000	100 000	250 000	270 000	270 000	270 000	270 000	270 000
475 000	100 000	250 000	475 000	475 000	475 000	475 000	475 000
580 000	100 000	250 000	500 000	580 000	580 000	580 000	580 000
780 000	100 000	250 000	500 000	750 000	780 000	780 000	780 000
1 100 000	100 000	250 000	500 000	750 000	1 000 000	1 100 000	1 100 000
2 000 000	100 000	250 000	500 000	750 000	1 000 000	1 250 000	1 500 000
Total	910 000	1 895 000	3 140 000	3 970 000	4 500 000	4 850 000	5 100 000
LAS	91 000	189 500	314 000	397 000	450 000	485 000	510 000
ILF	1	2.08	3.45	4.36	4.95	5.33	5.60

Inflated by 10 %

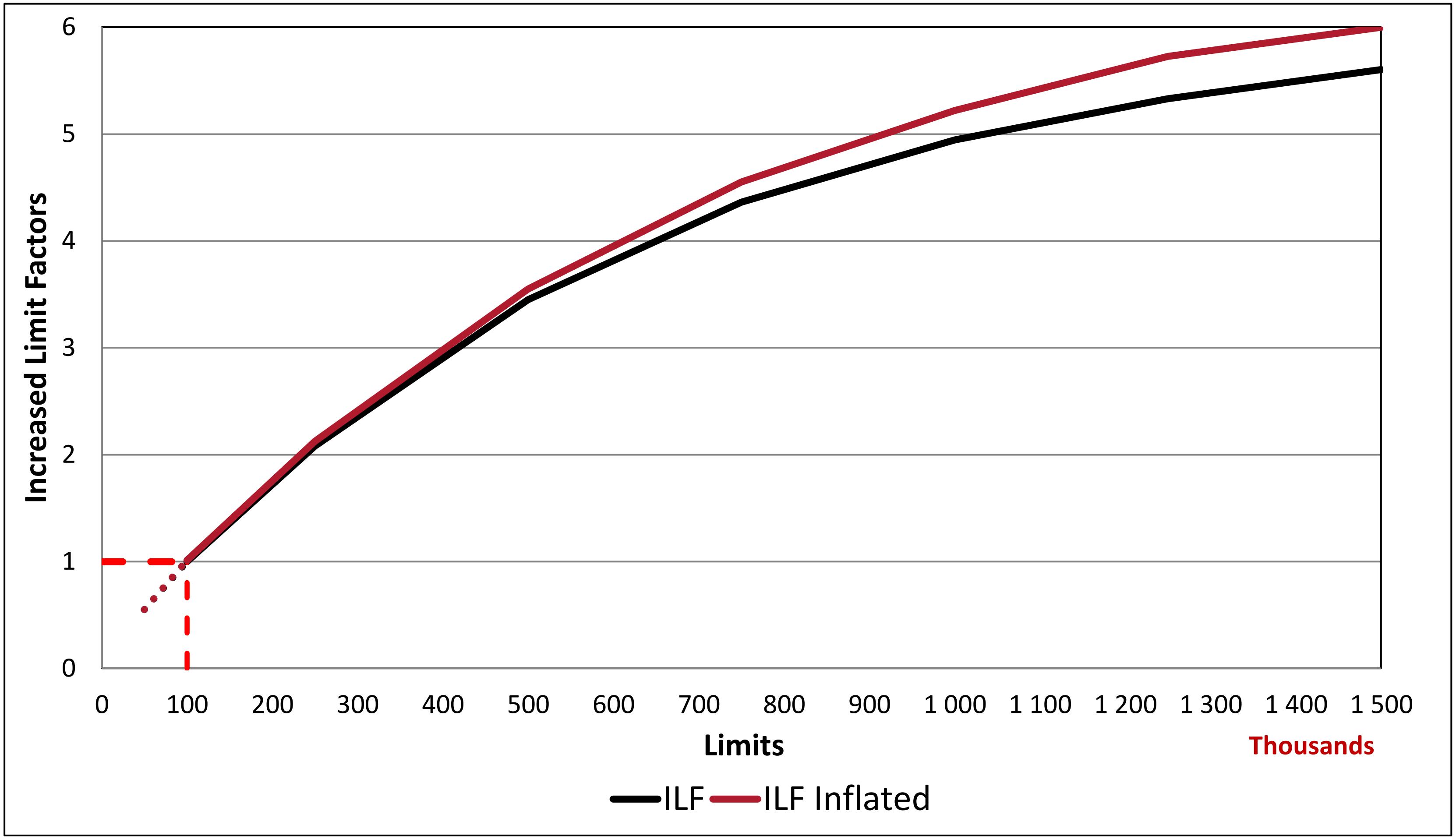
Inflated Claim Severity	Loss at 100 000 Basic Limit	Loss at 250 000 Increased Limit	Loss at 500 000 Increased Limit	Loss at 750 000 Increased Limit	Loss at 1 000 000 Increased Limit	Loss at 1 250 000 Increased Limit	Loss at 1 500 000 Increased Limit
55 000	55 000	55 000	55 000	55 000	55 000	55 000	55 000
66 000	66 000	66 000	66 000	66 000	66 000	66 000	66 000
132 000	100 000	132 000	132 000	132 000	132 000	132 000	132 000
181 500	100 000	181 500	181 500	181 500	181 500	181 500	181 500
297 000	100 000	250 000	297 000	297 000	297 000	297 000	297 000
522 500	100 000	250 000	500 000	522 500	522 500	522 500	522 500
638 000	100 000	250 000	500 000	638 000	638 000	638 000	638 000
858 000	100 000	250 000	500 000	750 000	858 000	858 000	858 000
1 210 000	100 000	250 000	500 000	750 000	1 000 000	1 210 000	1 210 000
2 200 000	100 000	250 000	500 000	750 000	1 000 000	1 250 000	1 500 000
Total	921 000	1 934 500	3 231 500	4 142 000	4 750 000	5 210 000	5 460 000
LAS	92 100	193 450	323 150	414 200	475 000	521 000	546 000
ILF	1	2.13	3.55	4.55	5.22	5.73	6.00

Leveraged Effect of Inflation

LAS Inflation Effect	1.21%	2.08%	2.91%	4.33%	5.56%	7.42%	7.06%
ILF Inflation Effect	1.21%	2.08%	2.91%	4.33%	5.56%	7.42%	7.06%

Increased Limit Factors

Leveraged Effect of Inflation



Riebessel's Parameterization of ILFs

- approach which has been often used by German insurance and reinsurance companies
- it is **inflation resistant**
- **Riebessel's Curves** are based on the assumption that each time $i \in \mathbb{N}$ the sum insured doubles, the risk cost increases by constant factor of $(1+z)$ with $z \in (0,1)$ **,** i.e.

$$P\left(2^i L\right) = P_L \cdot (1+z)^i,$$

  where $P_L = P(L)$ denotes standard risk premium for a limit of L
- here the sum insured acts only as limit of indemnity and not as a measure of the size of the risk like in Property insurance, the premium increases less than the sum insured

Riebessel's Parameterization of ILFs

- z is set according to the type of underlying portfolio
- by using a substitution $a = 2^i$ (i.e. $i = \log_2 a$) we have

$$P(aL) = P_L \cdot (1+z)^{\log_2 a}$$

- can be rewritten more helpful to give the premium for any desired limit in terms of the relativity to the base ($y = aL$)

$$P(y) = P_L \cdot (1+z)^{\log_2\left(\frac{y}{L}\right)} = P_L \cdot \left(\frac{y}{L}\right)^{\log_2(1+z)}$$

- this is called **Riebessel's formula** with $z \in (0,1)$ for net premium $P(y)$ at any sum insured y>0

Riebessel's Parameterization of ILFs

- according to collective model

$$P(y) = \mathbb{E}[N] \cdot \mathbb{E}[\min(X, y)]$$

- Then for ILF we have

$$ILF(L) = \frac{LAS(L)}{LAS(B)} = \frac{P(L)}{P(B)} = \left(\frac{L}{B}\right)^{\log_2(1+z)}$$

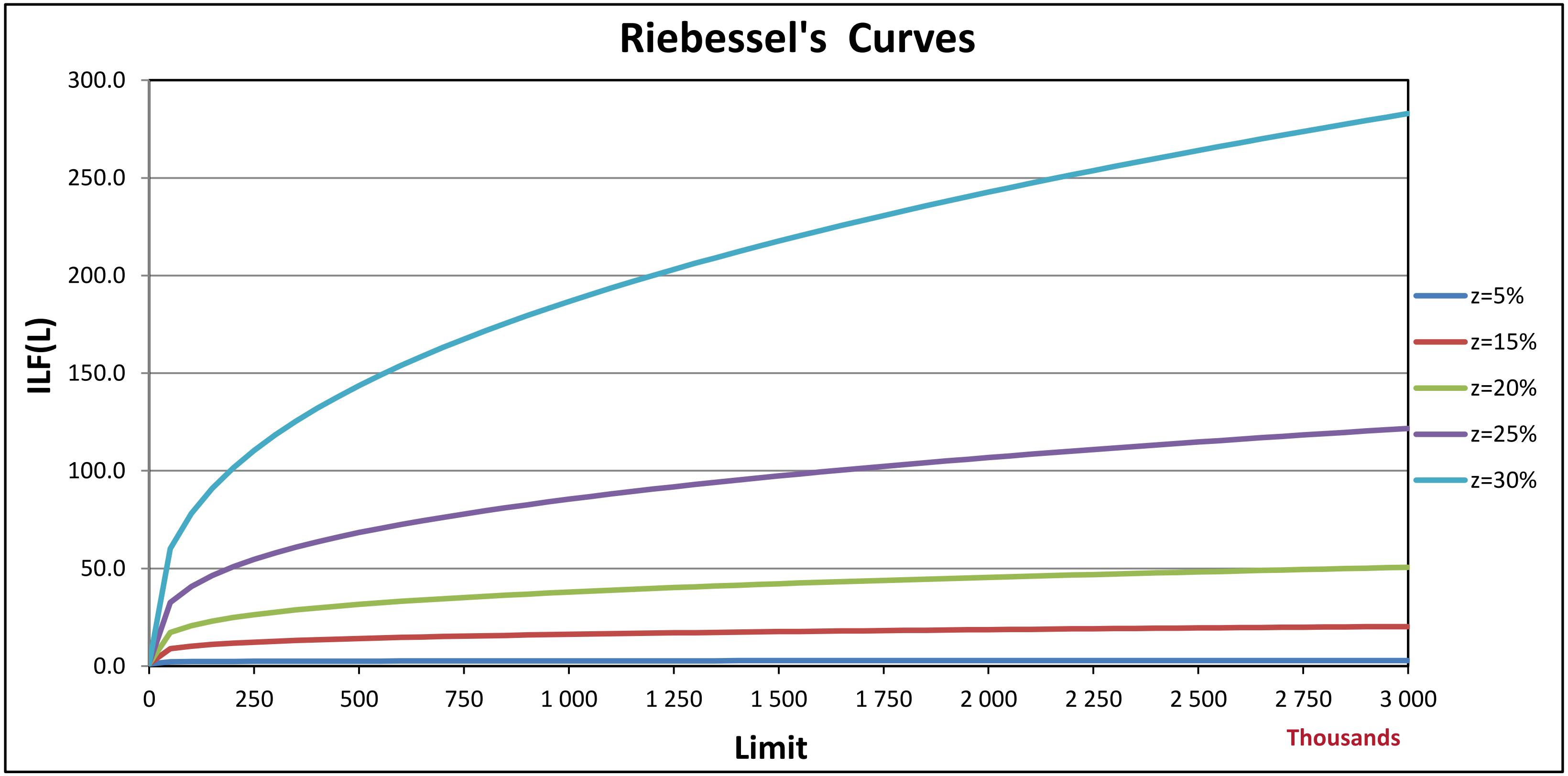
  - not affected by currency changes or inflation

- Mack & Fackler [2003] demonstrated that there exist loss distributions that lead to Riebesell's formula and that the formula is consistent with the assumption that the tail of severity distribution has a Pareto tail above a certain threshold
- generalizations which offer more flexibility regarding the severity distribution can be found in Riegel [2008]

Riebessel Parameterization of ILFs

- with Base Limit =1 we have

$$ILF(SI) = (1+z)^{\log_2 SI}$$



Exposure Models in Reinsurance

Summary & Bibliography

Summary

- Exposure Rating gives us consistent pricing of reinsurance contracts
  - without intensive computational simulation runs
  - it is indispensable in case of insufficient loss history
  - in case of sufficient number of historical losses it can serve for creating second opinion on the final rate after performing experience ratings - Historical experience alone is NOT necessarily the best predictor of future experience

- MBBEFD distributions only depend on two parameters and are suitable for many property insurance branches
  - Not sensitive to inflation
  - One of the weaknesses is the uncertainty about choice of the appropriate exposure curve. The choice of the curve is always subjective and requires an in-depth knowledge of the analyzed portfolio.
  - aggregated risk profile is provided - might be also helpful to use some blended curves for some bands of risk profile which includes mix of various types of risks

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Summary

- ILF curves suitable for casualty branches are always company specific
  - no standard curves
  - sensitive to inflation
  - Riebessel Parameterization of ILFs
    - it is inflation resistant

- Further development can be found in Desmedt et al. [2012]
  - show methods to overcome the different limitations using a combination of experience and exposure rating techniques if historical profile information is available
  - propose an **experience rating** method in which the measure for frequency and the as-if claims are determined **using the evolutions observed in the risk profiles**
  - For pricing unused capacity **exposure rating calibrated on the experience rate** for a working layer is used

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Bibliography

Bernegger Stefan. [1997] '*The Swiss Re Exposure Curves and the MBBEFD Distribution Class*', *ASTIN Bulletin*, 27(1), pp. 99–111. doi: 10.2143/AST.27.1.563208. [back]

Desmedt S., Snoussi M., Chenut X. and Walhin J.F. [2012] '*Experience and Exposure Rating for Property Per Risk Excess of Loss Reinsurance Revisited*', *ASTIN Bulletin*, 42(1), pp. 233–270. doi: 10.2143/AST.42.1.2160742. [back]

Dutang Christophe, Gesmann Markus & Spedicato Giorgio. [2016] '*Exposure rating, destruction rate models and the mbbefd package*', https://cran.r-project.org/web/packages/mbbefd/vignettes/Introduction_to_mbbefd.pdf [back]

Guggisberg Daniel. [2004] '*Exposure rating*', Swiss Re, Zurich 2004. http://media.cgd.swissre.com/documents/pub_exposure_rating_en.pdf [back]

Mack Thomas & Fackler Michael. [2003] '*Exposure Rating in Liability Reinsurance*', Blätter der DGVFM, 26(2), pp. 229-238. doi:10.1007/BF02808374. [back]

Riegel Ulrich. [2008] '*Generalizations of common ILF models*', Blätter der DGVFM, 29(1), pp. 45-71. DOI: 10.1007/s11857-008-0045-3. [back]

White Steve. [2005] '*Property Ratemaking - an Advanced Approach*', Seminar on Reinsurance 6-7 June 2005. https://www.casact.org/education/reinsure/2005/handouts/white.ppt [back]

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