Functional Profile Techniques For Claims Reserving

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Motivation and aims

Forecasting costs … a front burner in empirical economics

- **Risk reserving assessment analysis** in non-life insurance amounts to predict stochastically the overall loss reserves to cover possible claims.
- The most common reserving methods are based on different parametric approaches using aggregated **data structured in the run-off triangles**.
- We propose a rather **non-parametric approach**, which handles the underlying loss development triangles as **partially observed functional profiles**.
- **Three competitive** functional-based reserving techniques, each with slightly different scope, are presented.
- The claim reserve distribution is predicted through **permutation bootstrap**.

Run-off or loss development triangles

Accident year i	Development year j: 1	2	$\cdots$	$n-1$	n
1	$Y_{1,1}$	$Y_{1,2}$	$\cdots$	$Y_{1,n-1}$	$Y_{1,n}$
2	$Y_{2,1}$	$Y_{2,2}$	$\cdots$	$Y_{2,n-1}$	
			$\ddots$		
$\vdots$	$\vdots$	$\vdots$	$Y_{i,n+1-i}$		
$n-1$	$Y_{n-1,1}$	$Y_{n-1,2}$			
n	$Y_{n,1}$				

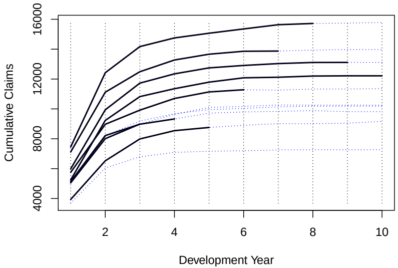
Table: Run-off triangle with the **observed cumulative claim amounts** $Y_{i,j}$ for $i + j \leqslant n + 1$.

Development profiles

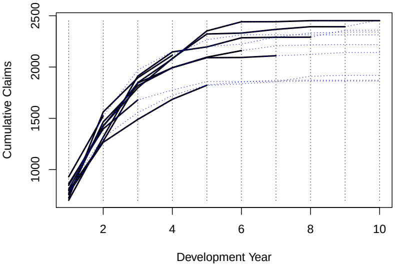
Accid. year i	Development year j: 1	2	3	4	5	6	7	8	9	10
1	5244	9228	10823	11352	11791	12082	12120	12199	12215	12215
2	5984	9939	11725	12346	12746	12909	13034	13109	13113	13115
3	7452	12421	14171	14752	15066	15354	15637	15720	15744	15786
4	7115	11117	12488	13274	13662	13859	13872	13935	13973	13972
5	5753	8969	9917	10697	11135	11282	11255	11331	11332	11354
6	3937	6524	7989	8543	8757	8901	9013	9012	9046	9164
7	5127	8212	8976	9325	9718	9795	9833	9885	9816	9815
8	5046	8006	8984	9633	10102	10166	10261	10252	10252	10252
9	5129	8202	9185	9681	9951	10033	10133	10182	10182	10183
10	3689	6043	6789	7089	7164	7197	7253	7267	7266	7266

Accid. year i	Development year j: 1	2	3	4	5	6	7	8	9	10
1	794	1277	1848	2080	2352	2441	2442	2452	2452	2452
2	847	1427	1796	2084	2322	2331	2367	2393	2393	2459
3	701	1317	1912	2147	2196	2285	2290	2291	2359	2359
4	808	1423	1844	1993	2091	2093	2110	2122	2142	2142
5	756	1465	1819	1993	2096	2160	2206	2216	2219	2217
6	771	1266	1489	1685	1822	1836	1857	1910	1919	1918
7	723	1562	1895	2115	2266	2314	2314	2313	2313	2313
8	862	1397	1679	1775	1858	1858	1859	1863	1863	1863
9	930	1523	1971	2150	2197	2224	2292	2332	2341	2341
10	825	1312	1556	1724	1825	1854	1872	1872	1872	1872

(a) Complete portfolio 1



(b) Complete portfolio 2



The proposed methods

- The building block of all of the proposed methods are so-called **functional development profiles**, “patterns of loss emergence” (Clark, 2003)
- To this end, however, we do not model them parametrically, but rather seek how to do it in **non-parametric ways** (Maciak, Mizera, and Pešta, 2022)

PARALLAX: *Parall*el *a*ppro*x*imation of missing fragments

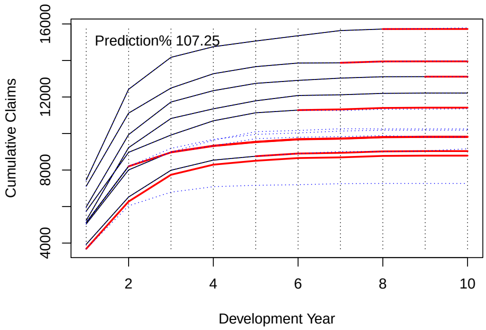
- Set the observed as the predicted $\widehat{Y}_{i,j} = Y_{i,j}$ for $i = 1, \ldots, n$ and $j = 1, \ldots, n + 1 - i$
- Find the most similar development profile

$$\widehat{\ell}_{i,j} = \arg \min_{\ell \in \{1,\ldots,n-j\}} \left| \widehat{Y}_{i,j} - Y_{\ell,j} \right| \tag{1}$$

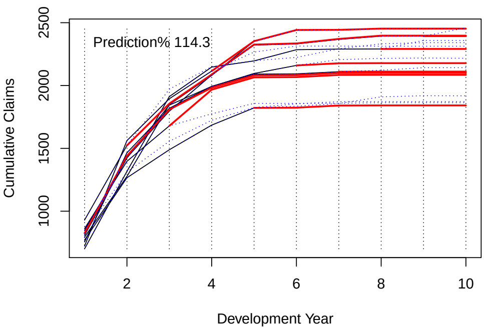
- Predict the unobserved (future) $Y_{i,j+1}$ such that

$$\widehat{Y}_{i,j+1} = \widehat{Y}_{i,j} + \left(Y_{\widehat{\ell}_{i,j},j+1} - Y_{\widehat{\ell}_{i,j},j} \right) \tag{2}$$

PARALLAX: *Parall*el *a*ppro*x*imation of missing fragments



(a) PARALLAX for portfolio 1



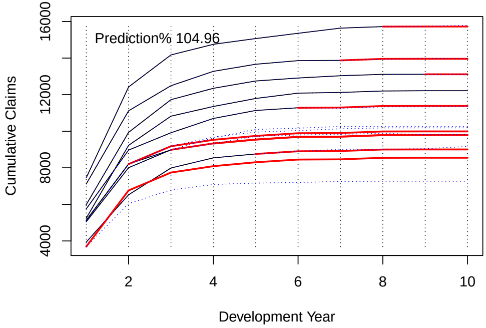
(b) PARALLAX for portfolio 2

REACT: Approximation by the most *re*cent *ac*ciden*t* year

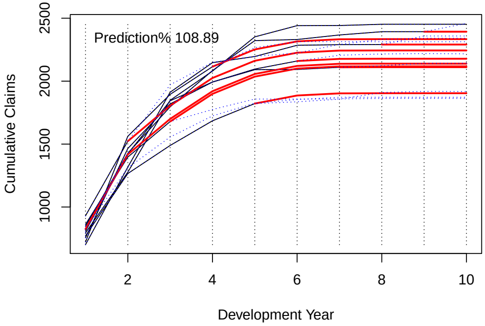
- Set the observed as the predicted $\widehat{Y}_{i,j} = Y_{i,j}$ for $i = 1, \ldots, n$ and $j = 1, \ldots, n + 1 - i$
- Predict the unobserved (future) $Y_{i,j+1}$ such that

$$\widehat{Y}_{i,j+1} = \widehat{Y}_{i,j} + \left(\widehat{Y}_{i-1,j+1} - \widehat{Y}_{i-1,j}\right) \tag{3}$$

REACT: Approximation by the most *re*cent *ac*cident *t* year



(a) REACT for portfolio 1

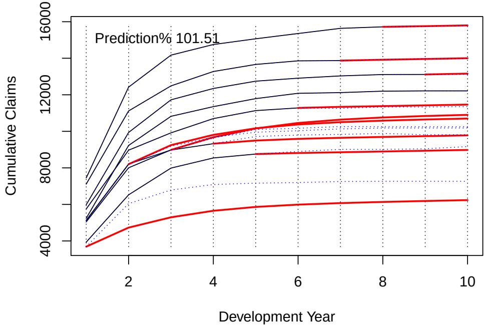


(b) REACT for portfolio 2

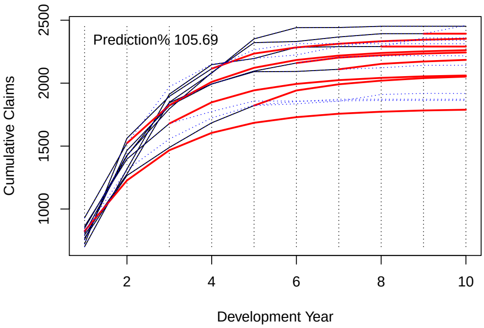
MACRAME: *Ma*rkov *c*hain f*ragme*nt approximation

- Calculate incremental claims $X_{i,j} = Y_{i,j} - Y_{i,j-1}$
- Set $\widehat{X}_{i,j} = X_{i,j}$ and $\widehat{Y}_{i,j} = Y_{i,j}$ for $i = 1, \ldots, n$ and $j = 1, \ldots, n+1-i$
- Use the states $\mathcal{S}$ and transform $\widehat{X}_{i,j}$ into $U_{i,j}$ for $i = 1, \ldots, n$ and $j = 1, \ldots, n+1-i$ such that $U_{i,j} = s$, if $\widehat{X}_{i,j} \in [g_{k-1}, g_k)$ and $\mathcal{S} \ni s \in [g_{k-1}, g_k)$
- Calculate the transition probability estimates $\widetilde{p}(s, s')$
- Predict the unobserved $X_{i,n+1-i+h}$ as $\widehat{X}_{i,n+1-i+h} = \boldsymbol{c}(U_{i,n+1-i})^\top \widetilde{\mathbb{P}}^h \boldsymbol{s}$, where $\boldsymbol{s} = (s_1, \ldots, s_t)^\top$ and $\boldsymbol{c}(U_{i,n+1-i})^\top = (\mathbb{1}\{U_{i,n+1-i} = s_1\}, \ldots, \mathbb{1}\{U_{i,n+1-i} = s_t\})$
- Compute the predicted cumulative amount $\widehat{Y}_{i,n+1-i+h} = \widehat{X}_{i,n+1-i+h} + \widehat{Y}_{i,n-i+h}$
- For missing at random, cf. Delaigle and Hall (2013) and Delaigle and Hall (2016)

MACRAME: *Ma*rkov *c*hain f*ragme*nt approximation

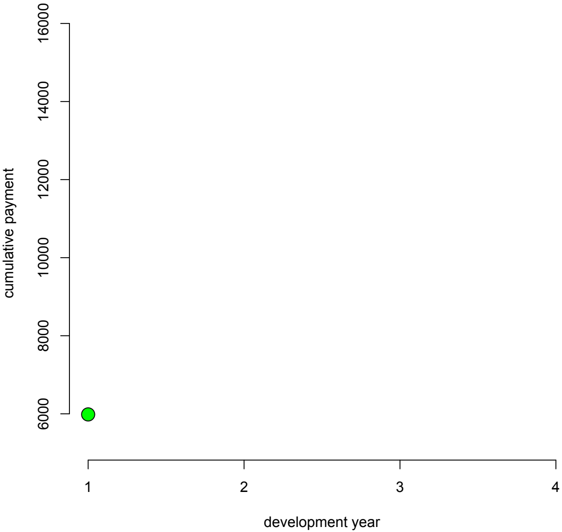


(a) MACRAME for portfolio 1



(b) MACRAME for portfolio 2

So


16000
14000
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6000
cumulative payment
1
2
3
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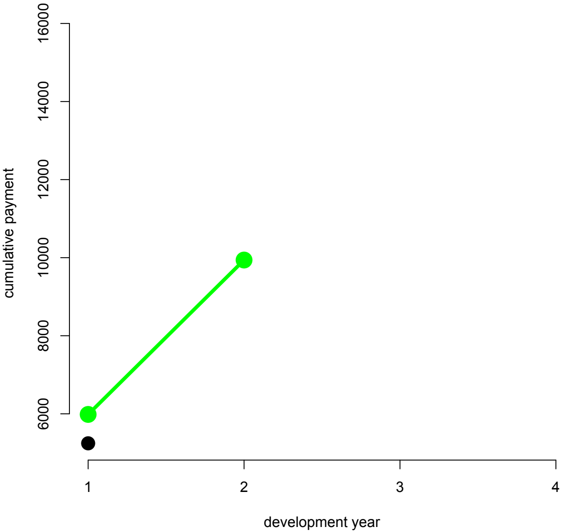
development year

So



cumulative payment
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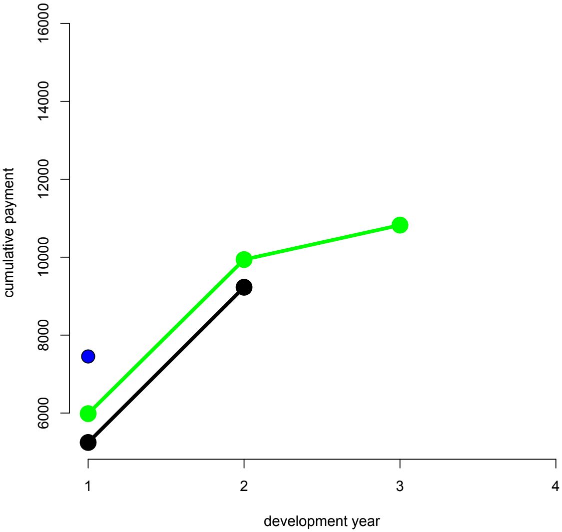

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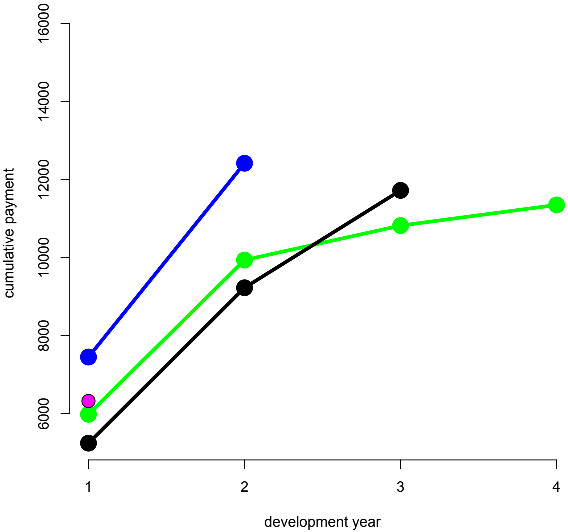
So


16000
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cumulative payment
1
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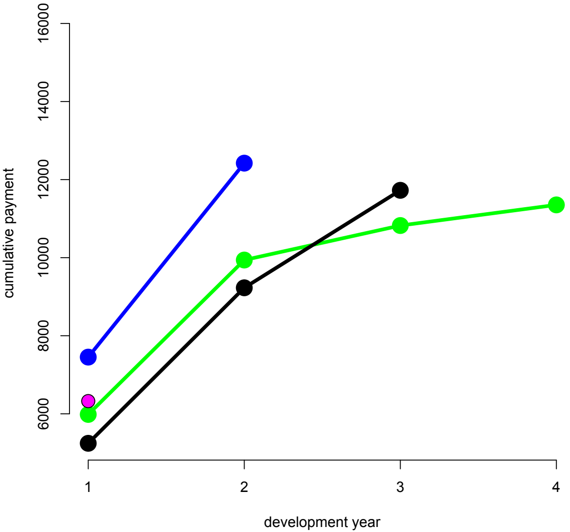
development year

And now: the *second* method (REACT)



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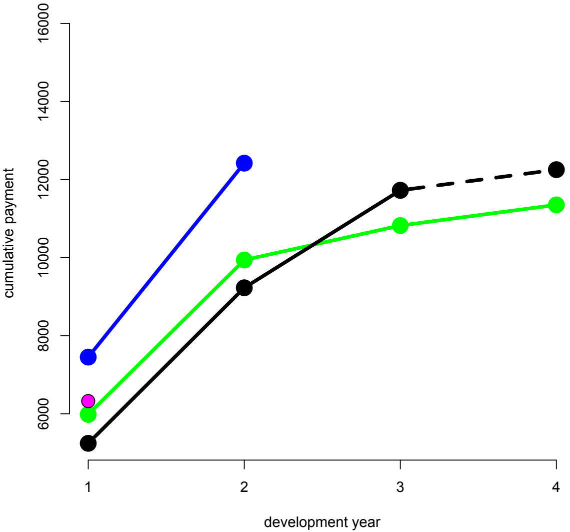
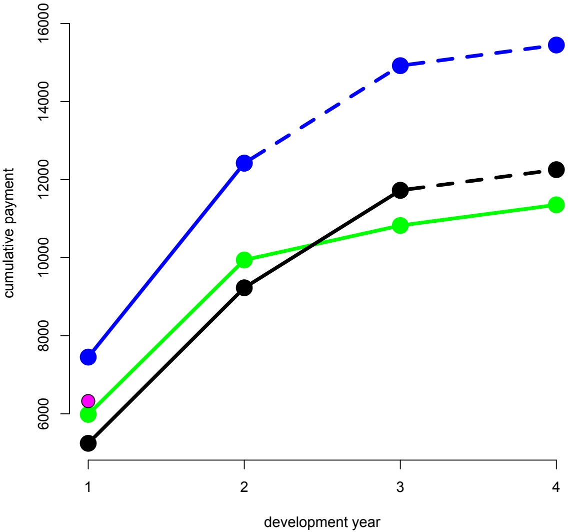
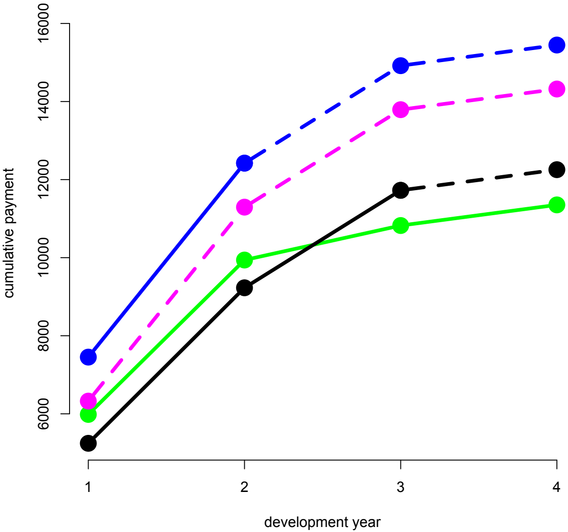


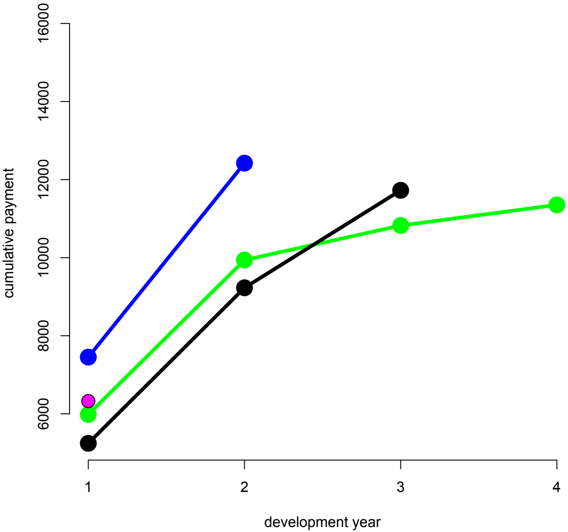
And now: the *second* method (REACT)



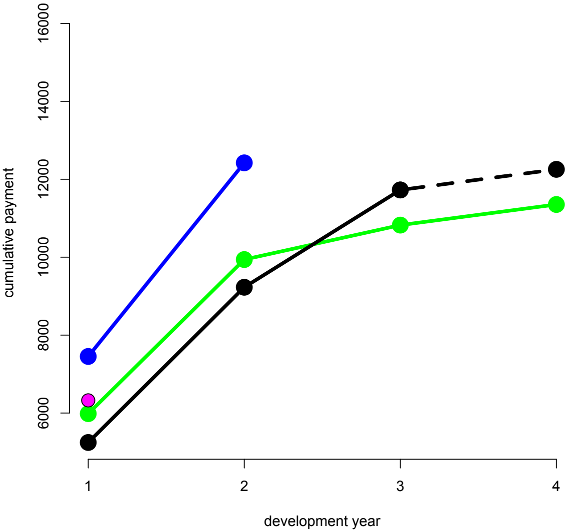
And now: the *second* method (REACT)



The *first* method: (PARALLAX)



The *first* method: (PARALLAX)



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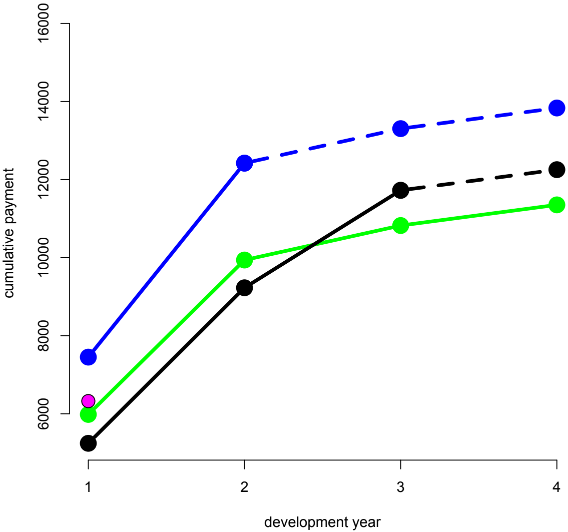
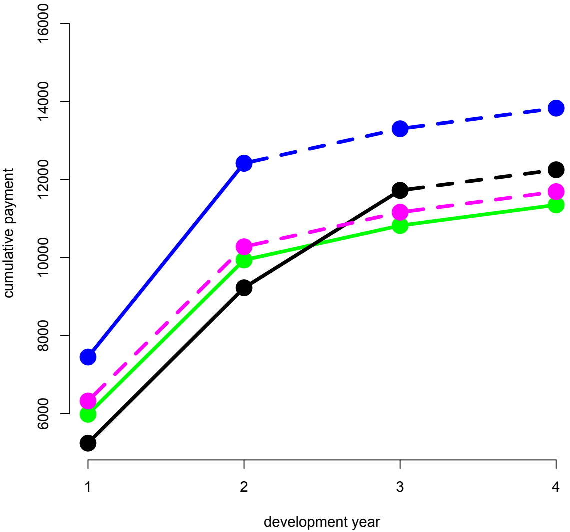


The *first* method: (PARALLAX)



It started in Banff . . .

. . . and ended in Oberwolfach

Rigorous results in theoretical setting – PARALLAX

$$\widehat{f}_{i,j}^{(n)} \xrightarrow[n\to\infty]{\mathsf{L}_2\left(\mathsf{T}_{i,j}^{(n)}\right)} f_j \quad \text{in probability } \mathsf{P}$$

The conditional mean square error of the PARALLAX estimator of f_j is

$$\mathsf{E}\left[\left\{\widehat{f}_{i,j}^{(n)} - f_j\right\}^2 \middle| \mathsf{T}_{i,j}^{(n)}\right] = \frac{\sigma_j^2 Y_{\widehat{\ell}_{i,j},j}}{Y_{n+1-j,j}^2} + (f_j - 1)^2 \left(\frac{Y_{\widehat{\ell}_{i,j},j}}{Y_{n+1-j,j}} - 1\right)^2 \quad [\mathsf{P}]\text{-a.s.},$$

when $\kappa = 1$, and, for $i + j - n = \kappa > 1$, it becomes

$$\mathsf{E}\left[\left\{\widehat{f}_{i,j}^{(n)} - f_j\right\}^2 \middle| \mathsf{T}_{i,j}^{(n)}\right]$$

$$= \frac{\sigma_j^2 Y_{\widehat{\ell}_{i,j},j}}{Y_{i,n+1-i}^2 \prod_{k=n+1-i}^{j-1} \left\{\widehat{f}_{i,k}^{(n)}\right\}^2} + (f_j - 1)^2 \left\{\frac{Y_{\widehat{\ell}_{i,j},j}}{Y_{i,n+1-i} \prod_{k=n+1-i}^{j-1} \widehat{f}_{i,k}^{(n)}} - 1\right\}^2 \quad [\mathsf{P}]\text{-a.s.}$$

Rigorous results in theoretical setting – REACT

$$\widetilde{f}_j^{(n)} \xrightarrow[n\to\infty]{\mathsf{L}_2\left(\mathsf{T}_{n+1-j,j}^{(n)}\right)} f_j \quad \text{in probability } \mathsf{P}$$

The conditional mean square error of the REACT estimator of f_j is

$$\mathsf{E}\left[\left\{\widetilde{f}_j^{(n)} - f_j\right\}^2 \middle| \mathsf{T}_{n+1-j,j}^{(n)}\right] = \frac{\sigma_j^2 Y_{n-j,j}}{Y_{n+1-j,j}^2} + (f_j - 1)^2 \left(\frac{Y_{n-j,j}}{Y_{n+1-j,j}} - 1\right)^2 \quad [\mathsf{P}]\text{-a.s.}$$

Rigorous results in theoretical setting – MACRAME

$$\widetilde{p}(s_1, s_2) = p(s_1, s_2) + O_P(n^{-1/2}),\ n \to \infty,$$

$$\mathsf{E}\left[\widehat{\mathsf{E}}\left\{U_{i,j+1}\right\}\right] = \mathsf{E}\left[U_{i,j+1}\right]$$

Resampling functional development profiles without replacement

- We start with the **predicted lower triangle** $\{\widehat{Y}_{i,j} : i = 2, \ldots, n; j = n + 2 - i, \ldots, n\}$, where the upper triangle predicted elements are kept as the original ones, i.e., $\widehat{Y}_{i,j} = Y_{i,j}$ for $i + j \leqslant n + 1$.
- Consequently, the full predicted square $\{\widehat{Y}_{i,j}\}_{i=1,j=1}^{n,n}$ is **standardized** such that each row value is divided by the first positive value within the row (from the left), i.e.,

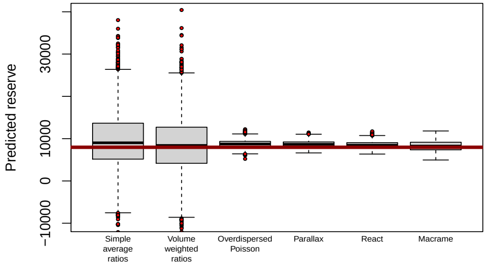
$$\widetilde{Y}_{i,j} := \widehat{Y}_{i,j}/\widehat{Y}_{i,p_i}, \quad p_i = \min\left\{j \in \{1, \ldots, n\} : \widehat{Y}_{i,j} > 0\}\right\}.$$

Permutation bootstrap

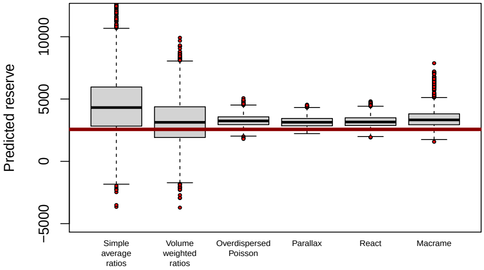
- Next, the standardized square $\{\widetilde{Y}_{i,j}\}_{i=1,j=1}^{n,n}$ is resampled in **a row-wise manner without replacement**. Formally speaking, for every permutation $\pi^{(b)} : (1,\ldots,n) \mapsto (\pi^{(b)}(1),\ldots,\pi^{(b)}(n))$, where $b = 1,\ldots,B$ such that $\pi^{(b)} \neq \pi^{(s)}$ if $b \neq s$, we obtain a permuted square $\{\widetilde{Y}_{\pi^{(b)}(i),j}\}_{i=1,j=1}^{n,n}$.
- We apply this technique to our algorithms proposed by re-running the algorithm in question on the **cut upper triangles** $\{\widetilde{Y}_{\pi^{(b)}(i),j} : \pi^{(b)}(i) + j \leqslant n + 1\}$, obtaining thus the newly predicted standardized cumulative amounts $\left\{\check{Y}_{i,j}^{(b)}\right\}_{i,j}$ for $i = 2,\ldots,n$ and $j = n + 2 - i,\ldots,n$.
- Finally, the predicted standardized $\check{Y}_{i,j}^{(b)}$'s are **back-standardized** yielding the bootstrapped predicted values

$$\left\{\breve{\widetilde{Y}}_{i,j}^{(b)} := \check{Y}_{i,j}^{(b)}\widehat{Y}_{i,p_i}\right\}_{i,j} \quad \text{for} \quad i = 2,\ldots,n \quad \text{and} \quad j = n + 2 - i,\ldots,n.$$

Bootstrapped loss reserve distribution



(a) Portfolio 1



(b) Portfolio 2

Empirical comparisons – real data

- 518 run-off triangles from the National Association of Insurance Commissioners (NAIC) database (Meyers and Shi, 2011)
- We a priori eliminated triangles with only zero observed claim amounts in the last four accident periods and also those triangles having 8 or more development profiles identically equal to zero

(i) 130 run-off triangles that were ODP compliant (with only non-negative increments, but profiles being entirely zero not allowed)

(ii) 299 not ODP compliant triangles (negative increments exists, but still no entirely zero profiles)

(iii) 89 remaining triangles that could be considered "rather atypical", but are still not uncommon in the actuarial practice

Out-of-sample bootstrap performance measures

`Reserve%` gives an absolute relative difference of the predicted reserve and the true reserve defined for each triangle as $100 \times \left|\frac{\text{predicted reserve}}{\text{true reserve}} - 1\right|$ and averaged over all triangles in the given scenario (smaller values are better);

`BootCoV%` expresses a coefficient of variation for the bootstrapped reserve distribution relative to the bootstrap mean $100 \times \frac{\text{Std.Dev}(\text{bootstrapped reserves})}{\text{Avg}(\text{bootstrapped reserves})}$ averaged, again, over all triangles in the given scenario (smaller values are better);

`BootVaR`$_{.995}$ denotes the 99.5% quantile of the bootstrap distribution relative to the bootstrapped mean $\frac{\text{Quantile}_{0.995}(\text{bootstrapped reserves})}{\text{Avg}(\text{bootstrapped reserves})}$ and averaged over all triangles in the given scenario (smaller values are better);

`BootQnt`$_{.950}$ provides a percentage proportion of the triangles in the given scenario for which the true reserve is dominated by the 95% quantile of the bootstrapped distribution (values closest to 95% are preferred).

Claims reserves evaluation I

Method	`Reserve%`	`BootCoV%`	`BootVaR`$_{.995}$	`BootQnt`$_{.950}$
Average	58.79 *(186.00)*	79.46 *(144.67)*	3.67 *(3.57)*	100.00%
Weighted	47.13 *(130.91)*	53.60 *(61.46)*	2.63 *(1.81)*	98.46%
ODP Model	47.10 *(130.89)*	**16.98** *(10.16)*	**1.54** *(0.39)*	86.92%
PARALLAX	57.85 *(125.45)*	**22.34** *(16.13)*	**1.59** *(0.46)*	**96.92%**
REACT	**43.19** *(78.28)*	24.08 *(18.03)*	1.64 *(0.51)*	97.69%
MACRAME	**45.32** *(76.43)*	23.93 *(12.65)*	1.73 *(0.42)*	**95.38%**

Table: Overall empirical performance of six claims reserving techniques when applied to the group (i), 130 **ODP compliant** run-off triangles from Meyers and Shi (2011). The corresponding standard deviations are given in parentheses; two best results are indicated by bold typeface.

Claims reserves evaluation II

Method	`Reserve%`		`BootCoV%`		`BootVaR`$_{.995}$		`BootQnt`$_{.950}$
Average	215.95	*(1128.77)*	4045.61	*(4.0e+04)*	43.14	*(461.41)*	99.67%
Weighted	541.33	*(6135.24)*	−3e+03	*(2.3e+04)*	−7.43	*(132.37)*	97.99%
Chainladder	541.33	*(6135.24)*	**29.78**	*(212.59)*	**1.97**	*(7.58)*	83.28%
PARALLAX	**68.83**	*(132.40)*	**9.53**	*(628.55)*	**1.70**	*(11.04)*	**92.98%**
REACT	97.85	*(334.97)*	66.60	*(182.67)*	2.92	*(4.99)*	**94.31%**
MACRAME	**68.38**	*(93.76)*	51.26	*(36.96)*	2.75	*(1.59)*	91.97%

Table: Empirical performance of six claims reserving techniques applied to the group (ii), 299 "**rather typical**" but ODP non-compliant run-off triangles from Meyers and Shi (2011). The corresponding standard deviations are given in parentheses; two best results are indicated by bold typeface.

Claims reserves evaluation III

Method	Reserve%		BootCoV%		BootVaR$_{.995}$		BootQnt$_{.950}$
Average	255.88	*(654.91)*	−2e+03	*(2.1e+04)*	−47.74	*(505.00)*	**91.01%**
Weighted	181.32	*(526.35)*	4.6e+04	*(4.3e+05)*	167.99	*(1492.61)*	**91.01%**
Chainladder	181.32	*(526.35)*	**177.17**	*(472.73)*	**6.02**	*(11.65)*	79.78%
PARALLAX	142.08	*(567.07)*	**69.77**	*(75.02)*	**3.09**	*(4.63)*	77.53%
REACT	**111.03**	*(256.82)*	240.60	*(1294.93)*	7.94	*(35.34)*	76.40%
MACRAME	**111.02**	*(141.21)*	256.41	*(1175.31)*	10.25	*(52.48)*	69.66%

Table: Empirical performance of six claims reserving techniques applied to the group (iii), 89 "**atypical**" run-off triangles from Meyers and Shi (2011). The corresponding standard deviations are given in parentheses; two best results are indicated by bold typeface.

Conclusions

Three unsupervised loss reserving techniques based on **non-parametric and distribution free approaches** offering the following advantages:

(i) they are **simple, straightforward, and easily applicable**;

(ii) they require **neither distributional nor parametric assumptions** and apply to all kinds of run-off triangles, including those with negative incremental cells or zero cumulative claim amounts over some development periods;

(iii) various stochastic model assumptions can be postulated in order to derive **desirable statistical properties** serving as the methods' **justifications**;

(iv) it is straightforward to obtain also the **overall reserve distributions via bootstrapping** techniques;

(v) the proposed methods are also **robust against outliers**;

(vi) **R package**: ProfileLadder: Functional-Based Chain Ladder for Claims Reserving.

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