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Bagging and regression trees in individual claims reserving

Jan Janoušek

October, 31 2025

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This presentation is based on Janoušek and Pešta [2025].

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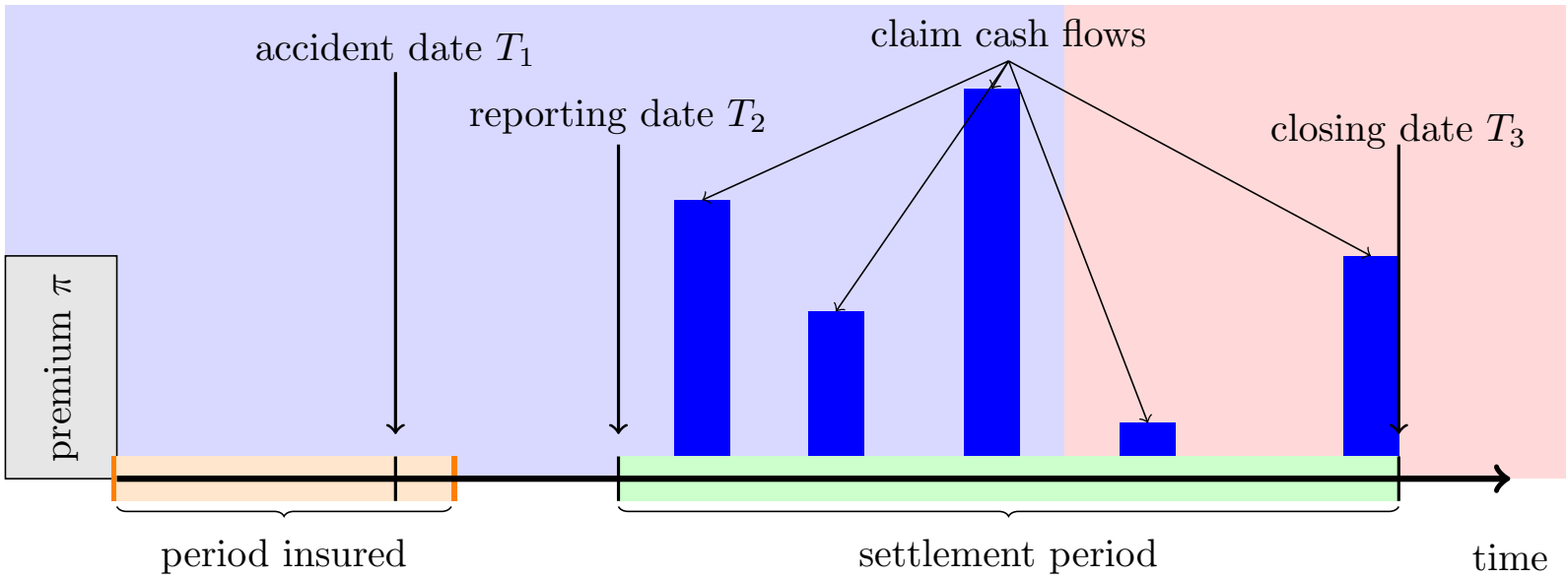

Figure: A non-life insurance claim progress illustration. The blue color in the background represents the past. The red color in the background represents the future that is to be predicted.

Terminology introduction

- U_1 *beginning of insurance period*
- U_2 *end of insurance period* ($U_1 < U2$).
- $[U_1, U_2]$ is called *period insured*
- T_1 *accident date*
- T_2 *reporting date* (T_2 - T_1 *reporting delay*)
- T_3 *settlement time*
- $T_3 - T_2$ *settlement delay*

Terminology introduction - cont.

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- We need to build as homogeneous groups as possible in order to use statistical tools such as LLN
- Claims are grouped by accident years accident year $= i$ if $(T_1 \in [1/1/i, 31/12/i])$

An illustration of the situation

accident year i	development year l: 0	1	$\dots$	l	$\dots$	L
1	${}_0X_1$	${}_1X_1$		$\dots$		${}_LX_1$
$\vdots$	$\vdots$	$\vdots$		$\ddots$		$\vdots$
$I-L$	${}_0X_{I-L}$	${}_1X_{I-L}$		$\dots$		${}_LX_{I-L}$
$\vdots$						
i	$\vdots$	$\vdots$		$\ddots$		$\vdots$
$\vdots$						
$I-1$	${}_0X_{I-1}$	${}_1X_{I-1}$		$\dots$		${}_LX_{I-1}$
I	${}_0X_I$	${}_1X_I$		$\dots$		${}_LX_I$

Figure: An illustration of a run-off triangle.

Notation for claim cash flows

- X the sum of all payments made during the insurance period $[1, I]$.
- X_i the sum of all payments done for claims with accident year i. $(X = \sum_{i=1}^{I} X_i)$
- $X_{i,d}$ the sum of all payments done for claims with accident year i reported in calendar year $i + d$. $(X_i = \sum_{d=0}^{+\infty} X_{i,d})$
- D maximal reporting delay
- $N_{i,d}$ the number of claims with accident year i and reporting delay d.

An illustration of the situation

accident year i	development year l: 0	1	$\dots$	l	$\dots$	L
1	${}_0X_1$	${}_1X_1$		$\dots$		${}_LX_1$
$\vdots$	$\vdots$	$\vdots$		$\ddots$		$\vdots$
$I-L$	${}_0X_{I-L}$	${}_1X_{I-L}$		$\dots$		${}_LX_{I-L}$
$\vdots$						
i	$\vdots$	$\vdots$		$\ddots$		$\vdots$
$\vdots$						
$I-1$	${}_0X_{I-1}$	${}_1X_{I-1}$		$\dots$		${}_LX_{I-1}$
I	${}_0X_I$	${}_1X_I$		$\dots$		${}_LX_I$

Figure: An illustration of a run-off triangle.

Notation for claim cash flows

- $X_{i,d}^{(v)}$ the sum of all payments done for v-th claim with accident year i, reported in calendar year $i + d$.
  $(X_{i,d} = \sum_{v=1}^{N_{i,d}} X_{i,d}^{(v)})$
- (i, d, v) now addresses each claim uniquely.
- $X_{i,d \mid k}^{(v)}$ payments done in calendar year $i + d + k$ for claim (i, d, v). $(X_{i,d}^{(v)} = \sum_{k=0}^{+\infty} X_{i,d \mid k}^{(v)})$
- (i, d, k, v) now addresses each claim payment uniquely.

An illustration of the situation

accident year i	development year l: 0	1	$\dots$	l	$\dots$	L
1	${}_0X_1$	${}_1X_1$		$\dots$		${}_LX_1$
$\vdots$	$\vdots$	$\vdots$		$\ddots$		$\vdots$
$I-L$	${}_0X_{I-L}$	${}_1X_{I-L}$		$\dots$		${}_LX_{I-L}$
$\vdots$						
i	$\vdots$	$\vdots$		$\ddots$		$\vdots$
$\vdots$						
$I-1$	${}_0X_{I-1}$	${}_1X_{I-1}$		$\dots$		${}_LX_{I-1}$
I	${}_0X_I$	${}_1X_I$		$\dots$		${}_LX_I$

Figure: An illustration of a run-off triangle.

Notation for claim cash flows

We want a slightly different approach as we want to predict reserves by years, e.g., next year's reserves. Thus, we look at the claim payments, not as payments in year $i + d + k$ but rather as payments $l = d + k$ years after the accident year i.

- ${}_lX_i$ the sum of all payments for claims with accident year i paid in year $i + l$. $({}_lX_i = \sum_{d=0}^{l} \sum_{v=1}^{N_{i,d}} X_{i,d \mid l-d}^{(v)})$
- X can be then rewritten as:

$$X = \sum_{i=1}^{I} X_i = \sum_{i=1}^{I} \sum_{l=0}^{L} {}_lX_i = \sum_{i=1}^{I} \sum_{l=0}^{L} \sum_{d=0}^{l} \sum_{v=1}^{N_{i,d}} X_{i,d \mid l-d}^{(v)}.$$

An illustration of the situation

accident year i	development year l: 0	1	$\dots$	l	$\dots$	L
1	${}_0X_1$	${}_1X_1$		$\dots$		${}_LX_1$
$\vdots$	$\vdots$	$\vdots$		$\ddots$		$\vdots$
$I-L$	${}_0X_{I-L}$	${}_1X_{I-L}$		$\dots$		${}_LX_{I-L}$
$\vdots$						
i	$\vdots$	$\vdots$		$\ddots$		$\vdots$
$\vdots$						
$I-1$	${}_0X_{I-1}$	${}_1X_{I-1}$		$\dots$		${}_LX_{I-1}$
I	${}_0X_I$	${}_1X_I$		$\dots$		${}_LX_I$

Figure: An illustration of a run-off triangle.

Notation for claim cash flows

- ${}_{I}X$ the sum of all payments happening in the calendar year I. $\left({}_{I}X = \sum_{i=\max\{1,I-L\}}^{\min\{I,I\}} {}_{I-i}X_i. \right)$
- Altogether, we get:

$$X = \sum_{I=1}^{I+L} {}_{I}X = \sum_{I=1}^{I+L} \sum_{i=\max\{1,I-L\}}^{\min\{I,I\}} {}_{I-i}X_i =$$

$$\sum_{I=1}^{I+L} \sum_{i=\max\{1,I-L\}}^{\min\{I,I\}} \sum_{d=0}^{I-i} \sum_{v=1}^{N_{i,d}} X_{i,d \mid I-i-d}^{(v)}.$$

Our goal

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At time $t > L$ we have the following information

$$\mathcal{D}_t = \{{}_lX_i : i + l \leq t, i \in \{1, 2, \ldots, I\}, l \in \{0, 1, \ldots, L\}\},$$

and our goal is to predict *outstanding loss liabilities at time t*

$$\mathcal{D}_t^c = \{{}_lX_i : i + l > t, i \in \{1, 2, \ldots, I\}, l \in \{0, 1, \ldots, L\}\}.$$

Our goal

This can be translated in the terms of $X^{(v)}_{i,d\,|\,k}$ as

$$\mathcal{F}_t^c = \left\{ X^{(v)}_{i,d\,|\,k};\, i+d+k > t, v \leq N_{i,d} \right\}.$$

and further split into RBNS and IBNR as

$$\mathcal{F}_t^{RBNS} = \left\{ X^{(v)}_{i,d\,|\,k};\ i+d+k > t,\ v \leq N_{i,d},\ i+d \leq t \right\}$$

and

$$\mathcal{F}_t^{IBNR} = \left\{ X^{(v)}_{i,d\,|\,k};\ i+d+k > t,\ v \leq N_{i,d},\ i+d > t \right\}.$$

Regression trees - introduction

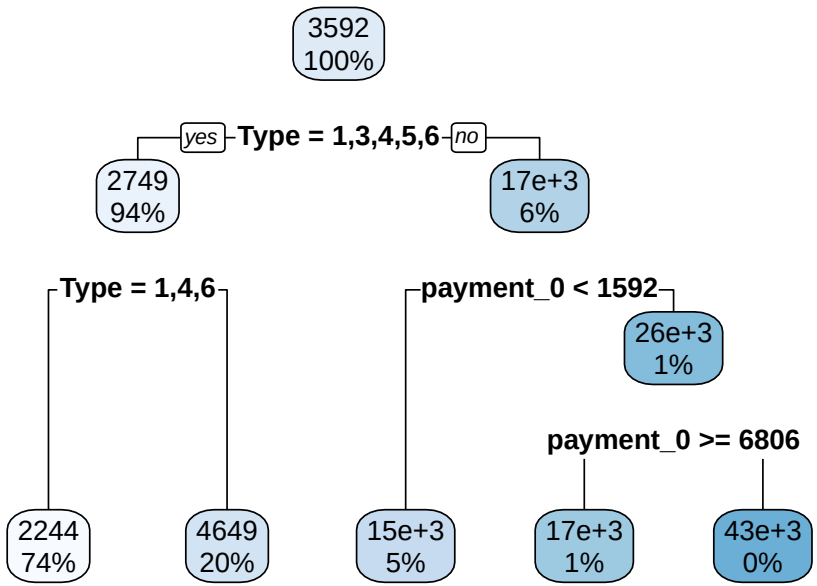

Figure: An example of regression tree from `rpart`.

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Regression trees are a type of decision tree methodology used for predicting continuous variables. Regression trees were originally introduced by (Breiman et al. [1984]), and they are a non-linear and non-parametric technique and are highly effective for capturing complex interactions and dependencies between the predictor variables and the target variable.

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A regression tree is constructed through a process of binary recursive partitioning. Each branch represents the outcome of the decision, and each leaf node represents a predicted value. The objective is to minimize the variance within each subset, thus ensuring that the resulting predictions are as accurate as possible, i.e. our goal is to minimize $\text{SSE}(T) = \sum_{i=1}^{N}(y_i - \hat{y})^2$, where T is the regression tree that for a given pair of covariates and targets $(\mathbf{X}_i, y_i)$ predicts the value $\hat{y}$.
The predictions in leaf nodes are done by simply calculating averages.

Regression trees - introduction

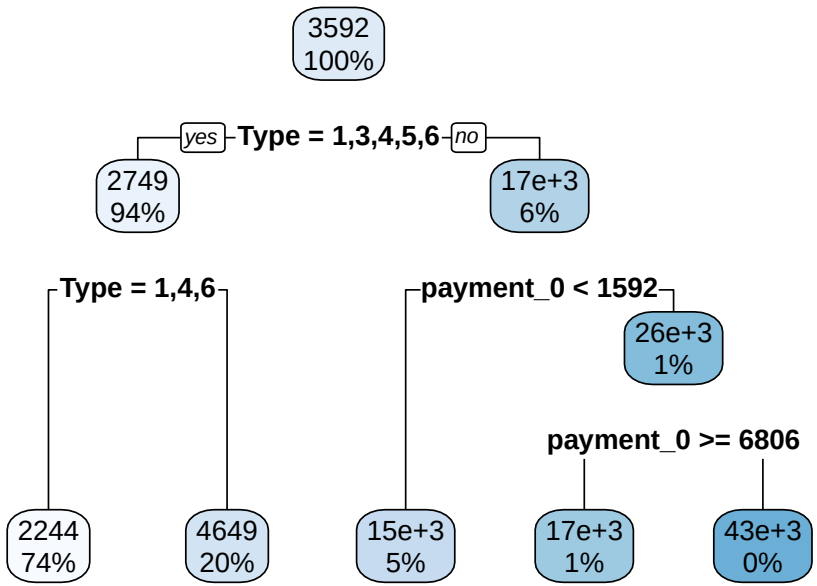


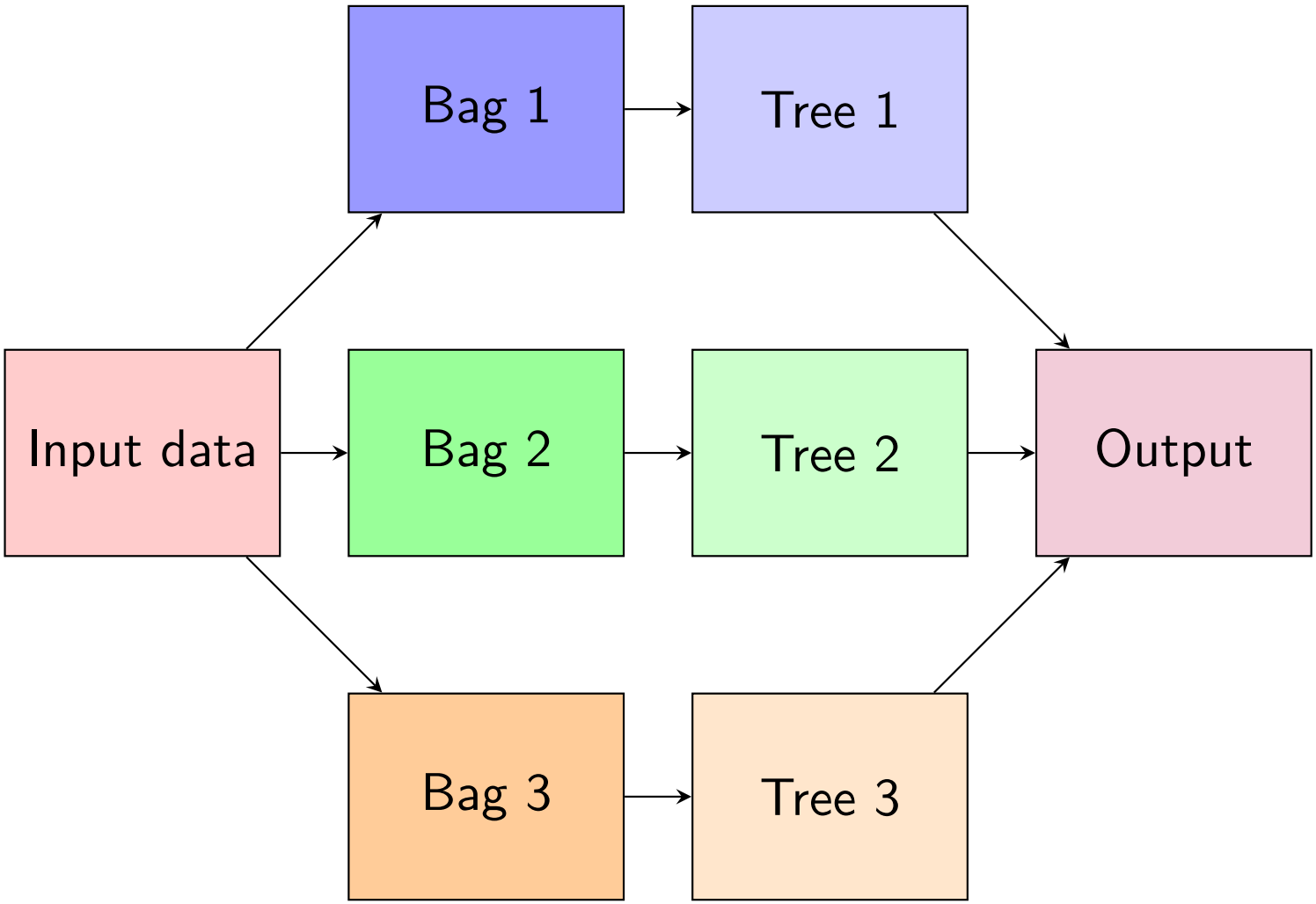
Figure: An example of regression tree from `rpart`.

Bootstrap aggregating

Bootstrap aggregating (bagging), introduced by Breiman (Breiman [1996]) is a simple algorithm based on *averaging* multiple regression trees.

We have B bootstrapped datasets $\mathcal{L}^1, \mathcal{L}^2, \ldots, \mathcal{L}^B$. Upon the datasets we build regression trees $\mathcal{T}_1, \mathcal{T}_2, \ldots, \mathcal{T}_B$. Then for prediction we calculate average prediction among all B trees, i.e., $\frac{1}{B} \sum_{b=1}^{B} \mathcal{T}_b(\mathbf{X})$.

Bagging



Out-of-bag error

For each data point ξ we gather the set of the trees that the data point was not used for training, i.e., $\Xi = \{b \in B; \xi \notin \mathcal{T}_b\}$. And we average the predictions to get an OOB estimate for ξ.

$$\mathrm{OOB}(\xi) = \frac{1}{|\Xi|} \sum_{b \in \Xi} \mathcal{T}_b(\xi).$$

Then

$$\mathrm{OOB}_{\mathrm{error}} = \frac{1}{|X^*|} \sum_{x \in X^*} \left(\mathrm{OOB}(x) - y(x)\right)^2,$$

where $X^* = \{x \in \mathcal{L}; \exists b \in B \text{ s.t. } x \notin \mathcal{L}^b\}$.

Our goal

We build on the work of Wüthrich (Wüthrich [2018]) by not only predicting claim frequency but also extending the framework to include modeling claim severity.
Our goal is to predict:

$$\sum_{d=0}^{\min\{t-i,D\}} \sum_{v=1}^{N_{i,d}} \left(\sum_{k=0}^{\min\{t-i-d,L-d\}} X_{i,d\,|\,k}^{(v)} \right)$$

$$+ \sum_{d=0}^{\min\{t-i,D\}} \sum_{v=1}^{N_{i,d}} \left(\mathbb{I}(L > t-i) \sum_{k=t-i-d+1}^{L-d} \left[X_{i,d\,|\,k}^{(v)} \mid \mathcal{F}_t \right] \right),$$

where $\mathcal{F}_t$ is the information available at time t.

Assumptions

Let $\left(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t=0}^{I+L}, \mathbb{P}\right)$ be a stochastic basis with sufficiently rich and discrete filtration $(\mathcal{F}_t)_{t=0}^{I+L}$. Also let all processes $(N_{i,d})_{i,d}$ be $(\mathcal{F}_t)_{t=0}^{I+L}$-adapted for time $t = i + d$ and $(X^{(v)}_{i,d \mid k})_{i,d,k,v}$ be $(\mathcal{F}_t)_{t=0}^{I+L}$-adapted for time $t = i + d + k$. Furthermore, let us assume the following:

Assumption - cont.;

1. claim counts $(N_{i,d})_{i,d}$ and claim payments $(X^{(v)}_{i,d \mid k})_{i,d,k,v}$ are independent;
2. the random variables $(N_{i,d})_{i,d}$ and $(X^{(v)}_{i,d \mid k})_{i,d,k,v}$ are independent for different accident years $i \in \{1, 2, \ldots, I\}$;
3. the processes $(X^{(v)}_{i,d \mid k})_k$ are independent for different reporting delays $d \in \{1, 2, \ldots, D\}$ and different claims $v \in \{1, 2, \ldots, N_{i,d}\}$;
4. The conditional distribution of $X^{(v)}_{i,d \mid k+1}$ given $\mathcal{F}_{i+d+k}$ is

$$X^{(v)}_{i,d \mid k+1} \mid \mathcal{F}_{i+d+k} \sim p_{d+k}\left(\mathbf{x}^{(v)}_{i,d \mid k}\right),$$

where $\mathbf{x}^{(v)}_{i,d \mid k}$ is a realization of $\mathbf{X}^{(v)}_{i,d \mid k}$, which is a $\mathcal{F}_{i+d+k}$-measurable vector of covariates of the claim (i, d, k, v), and $p_{d+k} : \mathcal{X} \to [0, +\infty)$ is some distribution.

Application on data

$\mathbf{X}^{(v)}_{i,d\mid k}$ is an S-dimensional vector

$$\mathbf{x}^{(v)}_{i,d\mid k} = \left(X^{(v)}_{i,d\mid k}(s)\right)^{\top}_{s\in\{1,2,\ldots,S\}}$$

$$= \left(X^{(v)}_{i,d\mid k}(1), X^{(v)}_{i,d\mid k}(2), \ldots X^{(v)}_{i,d\mid k}(S)\right)^{\top} \in \mathcal{X}.$$

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The dataset was obtained from (Wang and Wüthrich [2022]). It consists of 51,338 closed, 7,596 RBNS, and 2,436 IBNR claims observed in years $i \in \{1, 2, \ldots, 10\}$. The generator also generated claim payments for $t > 10$. Therefore, we can compare our methods with the (generated) reality.
As all claims can be reopened, we decided to consider all claims as RBNS.
We build bagging models to calibrate the probabilities p_l for $l \in \{0, 1, \ldots, L-1\}$, where $l = d + k$ and $L = 9$ is the maximal observed development delay.

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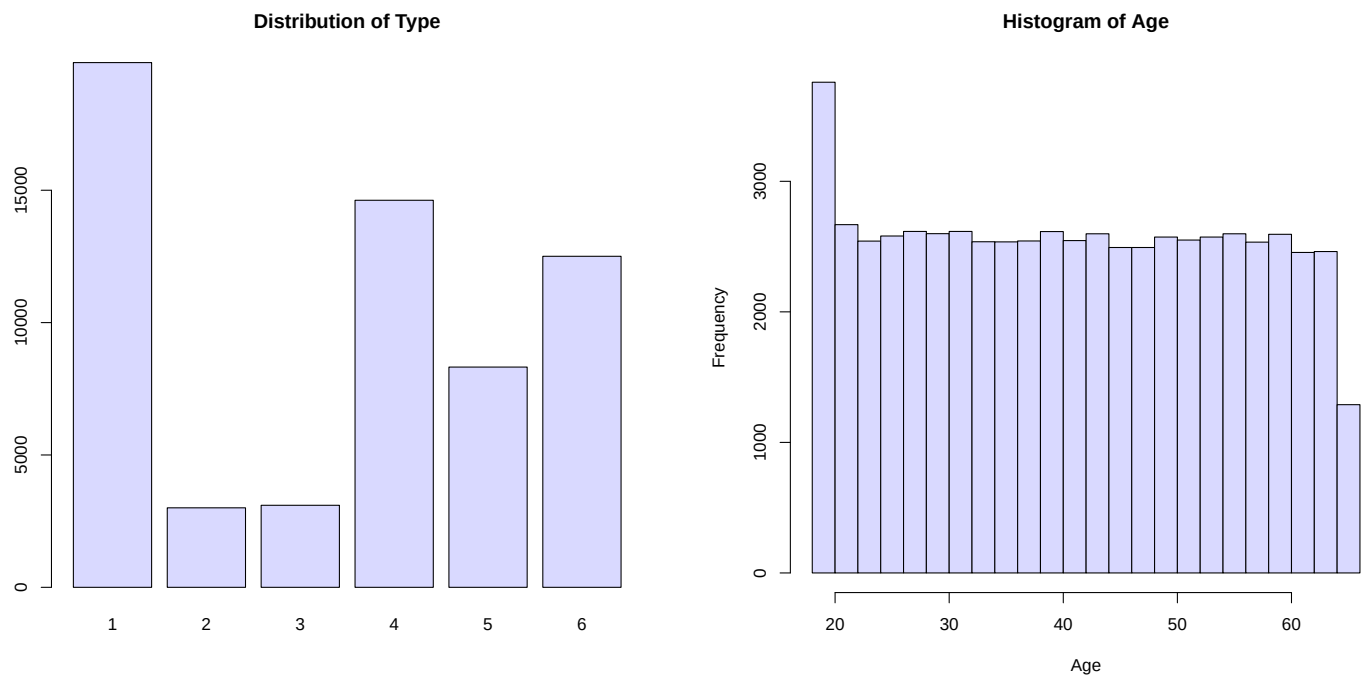
The vector contains the following information:

$x(1)$	type of the insurance claim (`type`)
$x(2)$	age of the person insured (`age`)
$x(3)$	weekday of the accident (`weekday`)
$x(4)$	reporting delay (`d`)
$x(5)$	payment at time i (only present if $d = 0$) ((X
$x(6)$	payment at time $i + 1$ (only present if $d \leq 1$)
$\vdots$	$\vdots$
$x(5 + d)$	payment at time $i + d$ (`Xd+0`)
$x(5 + d + 1)$	payment at time $i + d + 1$ (`Xd+1`)
$\vdots$	$\vdots$
$x(5 + d + k)$	payment at time $i + d + k$ (`Xd+k`)
$x(5 + d + k + 1)$	empty
$\vdots$	empty

Data exploration

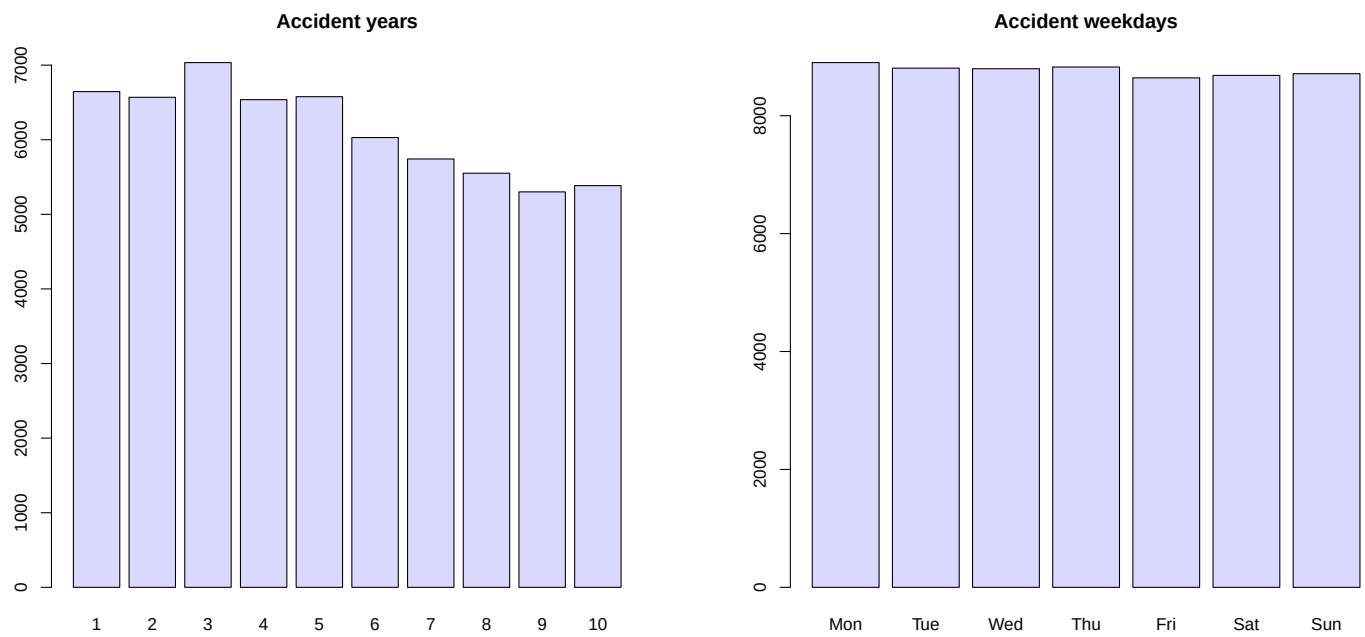


(a) Distribution of type among claims

(b) Distribution of the age of person injured

Figure: Basic data exploration.

Data exploration



(a) Distribution of accident years (b) Distribution of accident weekdays

Figure: Basic data exploration.

Prediction for RBNS

	0	1	2	3	4	5	6	7	8	9
1	${}_0X_1$	${}_1X_1$	${}_2X_1$	${}_3X_1$	${}_4X_1$	${}_5X_1$	${}_6X_1$	${}_7X_1$	${}_8X_1$	${}_9X_1$
2	${}_0X_2$	${}_1X_2$	${}_2X_2$	${}_3X_2$	${}_4X_2$	${}_5X_2$	${}_6X_2$	${}_7X_2$	${}_8X_2$	${}_9\hat{X}_2$
3	${}_0X_3$	${}_1X_3$	${}_2X_3$	${}_3X_3$	${}_4X_3$	${}_5X_3$	${}_6X_3$	${}_7X_3$	${}_8\hat{X}_3$	${}_9\hat{X}_3$
4	${}_0X_4$	${}_1X_4$	${}_2X_4$	${}_3X_4$	${}_4X_4$	${}_5X_4$	${}_6X_4$	${}_7\hat{X}_4$	${}_8\hat{X}_4$	${}_9\hat{X}_4$
5	${}_0X_5$	${}_1X_5$	${}_2X_5$	${}_3X_5$	${}_4X_5$	${}_5X_5$	${}_6\hat{X}_5$	${}_7\hat{X}_5$	${}_8\hat{X}_5$	${}_9\hat{X}_5$
6	${}_0X_6$	${}_1X_6$	${}_2X_6$	${}_3X_6$	${}_4X_6$	${}_5\hat{X}_6$	${}_6\hat{X}_6$	${}_7\hat{X}_6$	${}_8\hat{X}_6$	${}_9\hat{X}_6$
7	${}_0X_7$	${}_1X_7$	${}_2X_7$	${}_3X_7$	${}_4\hat{X}_7$	${}_5\hat{X}_7$	${}_6\hat{X}_7$	${}_7\hat{X}_7$	${}_8\hat{X}_7$	${}_9\hat{X}_7$
8	${}_0X_8$	${}_1X_8$	${}_2X_8$	${}_3\hat{X}_8$	${}_4\hat{X}_8$	${}_5\hat{X}_8$	${}_6\hat{X}_8$	${}_7\hat{X}_8$	${}_8\hat{X}_8$	${}_9\hat{X}_8$
9	${}_0X_9$	${}_1X_9$	${}_2\hat{X}_9$	${}_3\hat{X}_9$	${}_4\hat{X}_9$	${}_5\hat{X}_9$	${}_6\hat{X}_9$	${}_7\hat{X}_9$	${}_8\hat{X}_9$	${}_9\hat{X}_9$
10	${}_0X_{10}$	${}_1\hat{X}_{10}$	${}_2\hat{X}_{10}$	${}_3\hat{X}_{10}$	${}_4\hat{X}_{10}$	${}_5\hat{X}_{10}$	${}_6\hat{X}_{10}$	${}_7\hat{X}_{10}$	${}_8\hat{X}_{10}$	${}_9\hat{X}_{10}$

The first prediction

	0	1	2	3	4	5	6	7	8	9
1	${}_0X_1$	${}_1X_1$	${}_2X_1$	${}_3X_1$	${}_4X_1$	${}_5X_1$	${}_6X_1$	${}_7X_1$	${}_8X_1$	${}_9X_1$
2	${}_0X_2$	${}_1X_2$	${}_2X_2$	${}_3X_2$	${}_4X_2$	${}_5X_2$	${}_6X_2$	${}_7X_2$	${}_8X_2$	${}_9\hat{X}_2$
3	${}_0X_3$	${}_1X_3$	${}_2X_3$	${}_3X_3$	${}_4X_3$	${}_5X_3$	${}_6X_3$	${}_7X_3$	${}_8\hat{X}_3$	${}_9\hat{X}_3$
4	${}_0X_4$	${}_1X_4$	${}_2X_4$	${}_3X_4$	${}_4X_4$	${}_5X_4$	${}_6X_4$	${}_7\hat{X}_4$	${}_8\hat{X}_4$	${}_9\hat{X}_4$
5	${}_0X_5$	${}_1X_5$	${}_2X_5$	${}_3X_5$	${}_4X_5$	${}_5X_5$	${}_6\hat{X}_5$	${}_7\hat{X}_5$	${}_8\hat{X}_5$	${}_9\hat{X}_5$
6	${}_0X_6$	${}_1X_6$	${}_2X_6$	${}_3X_6$	${}_4X_6$	${}_5\hat{X}_6$	${}_6\hat{X}_6$	${}_7\hat{X}_6$	${}_8\hat{X}_6$	${}_9\hat{X}_6$
7	${}_0X_7$	${}_1X_7$	${}_2X_7$	${}_3X_7$	${}_4\hat{X}_7$	${}_5\hat{X}_7$	${}_6\hat{X}_7$	${}_7\hat{X}_7$	${}_8\hat{X}_7$	${}_9\hat{X}_7$
8	${}_0X_8$	${}_1X_8$	${}_2X_8$	${}_3\hat{X}_8$	${}_4\hat{X}_8$	${}_5\hat{X}_8$	${}_6\hat{X}_8$	${}_7\hat{X}_8$	${}_8\hat{X}_8$	${}_9\hat{X}_8$
9	${}_0X_9$	${}_1X_9$	${}_2\hat{X}_9$	${}_3\hat{X}_9$	${}_4\hat{X}_9$	${}_5\hat{X}_9$	${}_6\hat{X}_9$	${}_7\hat{X}_9$	${}_8\hat{X}_9$	${}_9\hat{X}_9$
10	${}_0X_{10}$	${}_1\hat{X}_{10}$	${}_2\hat{X}_{10}$	${}_3\hat{X}_{10}$	${}_4\hat{X}_{10}$	${}_5\hat{X}_{10}$	${}_6\hat{X}_{10}$	${}_7\hat{X}_{10}$	${}_8\hat{X}_{10}$	${}_9\hat{X}_{10}$

After the first prediction

	0	1	2	3	4	5	6	7	8	9
1	${}_0X_1$	${}_1X_1$	${}_2X_1$	${}_3X_1$	${}_4X_1$	${}_5X_1$	${}_6X_1$	${}_7X_1$	${}_8X_1$	${}_9X_1$
2	${}_0X_2$	${}_1X_2$	${}_2X_2$	${}_3X_2$	${}_4X_2$	${}_5X_2$	${}_6X_2$	${}_7X_2$	${}_8X_2$	${}_9\hat{X}_2$
3	${}_0X_3$	${}_1X_3$	${}_2X_3$	${}_3X_3$	${}_4X_3$	${}_5X_3$	${}_6X_3$	${}_7X_3$	${}_8\hat{X}_3$	${}_9\hat{X}_3$
4	${}_0X_4$	${}_1X_4$	${}_2X_4$	${}_3X_4$	${}_4X_4$	${}_5X_4$	${}_6X_4$	${}_7\hat{X}_4$	${}_8\hat{X}_4$	${}_9\hat{X}_4$
5	${}_0X_5$	${}_1X_5$	${}_2X_5$	${}_3X_5$	${}_4X_5$	${}_5X_5$	${}_6\hat{X}_5$	${}_7\hat{X}_5$	${}_8\hat{X}_5$	${}_9\hat{X}_5$
6	${}_0X_6$	${}_1X_6$	${}_2X_6$	${}_3X_6$	${}_4X_6$	${}_5\hat{X}_6$	${}_6\hat{X}_6$	${}_7\hat{X}_6$	${}_8\hat{X}_6$	${}_9\hat{X}_6$
7	${}_0X_7$	${}_1X_7$	${}_2X_7$	${}_3X_7$	${}_4\hat{X}_7$	${}_5\hat{X}_7$	${}_6\hat{X}_7$	${}_7\hat{X}_7$	${}_8\hat{X}_7$	${}_9\hat{X}_7$
8	${}_0X_8$	${}_1X_8$	${}_2X_8$	${}_3\hat{X}_8$	${}_4\hat{X}_8$	${}_5\hat{X}_8$	${}_6\hat{X}_8$	${}_7\hat{X}_8$	${}_8\hat{X}_8$	${}_9\hat{X}_8$
9	${}_0X_9$	${}_1X_9$	${}_2\hat{X}_9$	${}_3\hat{X}_9$	${}_4\hat{X}_9$	${}_5\hat{X}_9$	${}_6\hat{X}_9$	${}_7\hat{X}_9$	${}_8\hat{X}_9$	${}_9\hat{X}_9$
10	${}_0X_{10}$	${}_1\hat{X}_{10}$	${}_2\hat{X}_{10}$	${}_3\hat{X}_{10}$	${}_4\hat{X}_{10}$	${}_5\hat{X}_{10}$	${}_6\hat{X}_{10}$	${}_7\hat{X}_{10}$	${}_8\hat{X}_{10}$	${}_9\hat{X}_{10}$

Learning to predict for second time

	0	1	2	3	4	5	6	7	8	9
1	${}_0X_1$	${}_1X_1$	${}_2X_1$	${}_3X_1$	${}_4X_1$	${}_5X_1$	${}_6X_1$	${}_7X_1$	${}_8X_1$	${}_9X_1$
2	${}_0X_2$	${}_1X_2$	${}_2X_2$	${}_3X_2$	${}_4X_2$	${}_5X_2$	${}_6X_2$	${}_7X_2$	${}_8X_2$	${}_9\hat{X}_2$
3	${}_0X_3$	${}_1X_3$	${}_2X_3$	${}_3X_3$	${}_4X_3$	${}_5X_3$	${}_6X_3$	${}_7X_3$	${}_8\hat{X}_3$	${}_9\hat{X}_3$
4	${}_0X_4$	${}_1X_4$	${}_2X_4$	${}_3X_4$	${}_4X_4$	${}_5X_4$	${}_6X_4$	${}_7\hat{X}_4$	${}_8\hat{X}_4$	${}_9\hat{X}_4$
5	${}_0X_5$	${}_1X_5$	${}_2X_5$	${}_3X_5$	${}_4X_5$	${}_5X_5$	${}_6\hat{X}_5$	${}_7\hat{X}_5$	${}_8\hat{X}_5$	${}_9\hat{X}_5$
6	${}_0X_6$	${}_1X_6$	${}_2X_6$	${}_3X_6$	${}_4X_6$	${}_5\hat{X}_6$	${}_6\hat{X}_6$	${}_7\hat{X}_6$	${}_8\hat{X}_6$	${}_9\hat{X}_6$
7	${}_0X_7$	${}_1X_7$	${}_2X_7$	${}_3X_7$	${}_4\hat{X}_7$	${}_5\hat{X}_7$	${}_6\hat{X}_7$	${}_7\hat{X}_7$	${}_8\hat{X}_7$	${}_9\hat{X}_7$
8	${}_0X_8$	${}_1X_8$	${}_2X_8$	${}_3\hat{X}_8$	${}_4\hat{X}_8$	${}_5\hat{X}_8$	${}_6\hat{X}_8$	${}_7\hat{X}_8$	${}_8\hat{X}_8$	${}_9\hat{X}_8$
9	${}_0X_9$	${}_1X_9$	${}_2\hat{X}_9$	${}_3\hat{X}_9$	${}_4\hat{X}_9$	${}_5\hat{X}_9$	${}_6\hat{X}_9$	${}_7\hat{X}_9$	${}_8\hat{X}_9$	${}_9\hat{X}_9$
10	${}_0X_{10}$	${}_1\hat{X}_{10}$	${}_2\hat{X}_{10}$	${}_3\hat{X}_{10}$	${}_4\hat{X}_{10}$	${}_5\hat{X}_{10}$	${}_6\hat{X}_{10}$	${}_7\hat{X}_{10}$	${}_8\hat{X}_{10}$	${}_9\hat{X}_{10}$

The second prediction

	0	1	2	3	4	5	6	7	8	9
1	${}_0X_1$	${}_1X_1$	${}_2X_1$	${}_3X_1$	${}_4X_1$	${}_5X_1$	${}_6X_1$	${}_7X_1$	${}_8X_1$	${}_9X_1$
2	${}_0X_2$	${}_1X_2$	${}_2X_2$	${}_3X_2$	${}_4X_2$	${}_5X_2$	${}_6X_2$	${}_7X_2$	${}_8X_2$	${}_9\hat{X}_2$
3	${}_0X_3$	${}_1X_3$	${}_2X_3$	${}_3X_3$	${}_4X_3$	${}_5X_3$	${}_6X_3$	${}_7X_3$	${}_8\hat{X}_3$	${}_9\hat{X}_3$
4	${}_0X_4$	${}_1X_4$	${}_2X_4$	${}_3X_4$	${}_4X_4$	${}_5X_4$	${}_6X_4$	${}_7\hat{X}_4$	${}_8\hat{X}_4$	${}_9\hat{X}_4$
5	${}_0X_5$	${}_1X_5$	${}_2X_5$	${}_3X_5$	${}_4X_5$	${}_5X_5$	${}_6\hat{X}_5$	${}_7\hat{X}_5$	${}_8\hat{X}_5$	${}_9\hat{X}_5$
6	${}_0X_6$	${}_1X_6$	${}_2X_6$	${}_3X_6$	${}_4X_6$	${}_5\hat{X}_6$	${}_6\hat{X}_6$	${}_7\hat{X}_6$	${}_8\hat{X}_6$	${}_9\hat{X}_6$
7	${}_0X_7$	${}_1X_7$	${}_2X_7$	${}_3X_7$	${}_4\hat{X}_7$	${}_5\hat{X}_7$	${}_6\hat{X}_7$	${}_7\hat{X}_7$	${}_8\hat{X}_7$	${}_9\hat{X}_7$
8	${}_0X_8$	${}_1X_8$	${}_2X_8$	${}_3\hat{X}_8$	${}_4\hat{X}_8$	${}_5\hat{X}_8$	${}_6\hat{X}_8$	${}_7\hat{X}_8$	${}_8\hat{X}_8$	${}_9\hat{X}_8$
9	${}_0X_9$	${}_1X_9$	${}_2\hat{X}_9$	${}_3\hat{X}_9$	${}_4\hat{X}_9$	${}_5\hat{X}_9$	${}_6\hat{X}_9$	${}_7\hat{X}_9$	${}_8\hat{X}_9$	${}_9\hat{X}_9$
10	${}_0X_{10}$	${}_1\hat{X}_{10}$	${}_2\hat{X}_{10}$	${}_3\hat{X}_{10}$	${}_4\hat{X}_{10}$	${}_5\hat{X}_{10}$	${}_6\hat{X}_{10}$	${}_7\hat{X}_{10}$	${}_8\hat{X}_{10}$	${}_9\hat{X}_{10}$

Learning to predict for the last time

	0	1	2	3	4	5	6	7	8	9
1	${}_0X_1$	${}_1X_1$	${}_2X_1$	${}_3X_1$	${}_4X_1$	${}_5X_1$	${}_6X_1$	${}_7X_1$	${}_8X_1$	${}_9X_1$
2	${}_0X_2$	${}_1X_2$	${}_2X_2$	${}_3X_2$	${}_4X_2$	${}_5X_2$	${}_6X_2$	${}_7X_2$	${}_8X_2$	${}_9\hat{X}_2$
3	${}_0X_3$	${}_1X_3$	${}_2X_3$	${}_3X_3$	${}_4X_3$	${}_5X_3$	${}_6X_3$	${}_7X_3$	${}_8\hat{X}_3$	${}_9\hat{X}_3$
4	${}_0X_4$	${}_1X_4$	${}_2X_4$	${}_3X_4$	${}_4X_4$	${}_5X_4$	${}_6X_4$	${}_7\hat{X}_4$	${}_8\hat{X}_4$	${}_9\hat{X}_4$
5	${}_0X_5$	${}_1X_5$	${}_2X_5$	${}_3X_5$	${}_4X_5$	${}_5X_5$	${}_6\hat{X}_5$	${}_7\hat{X}_5$	${}_8\hat{X}_5$	${}_9\hat{X}_5$
6	${}_0X_6$	${}_1X_6$	${}_2X_6$	${}_3X_6$	${}_4X_6$	${}_5\hat{X}_6$	${}_6\hat{X}_6$	${}_7\hat{X}_6$	${}_8\hat{X}_6$	${}_9\hat{X}_6$
7	${}_0X_7$	${}_1X_7$	${}_2X_7$	${}_3X_7$	${}_4\hat{X}_7$	${}_5\hat{X}_7$	${}_6\hat{X}_7$	${}_7\hat{X}_7$	${}_8\hat{X}_7$	${}_9\hat{X}_7$
8	${}_0X_8$	${}_1X_8$	${}_2X_8$	${}_3\hat{X}_8$	${}_4\hat{X}_8$	${}_5\hat{X}_8$	${}_6\hat{X}_8$	${}_7\hat{X}_8$	${}_8\hat{X}_8$	${}_9\hat{X}_8$
9	${}_0X_9$	${}_1X_9$	${}_2\hat{X}_9$	${}_3\hat{X}_9$	${}_4\hat{X}_9$	${}_5\hat{X}_9$	${}_6\hat{X}_9$	${}_7\hat{X}_9$	${}_8\hat{X}_9$	${}_9\hat{X}_9$
10	${}_0X_{10}$	${}_1\hat{X}_{10}$	${}_2\hat{X}_{10}$	${}_3\hat{X}_{10}$	${}_4\hat{X}_{10}$	${}_5\hat{X}_{10}$	${}_6\hat{X}_{10}$	${}_7\hat{X}_{10}$	${}_8\hat{X}_{10}$	${}_9\hat{X}_{10}$

The last prediction

	0	1	2	3	4	5	6	7	8	9
1	${}_0X_1$	${}_1X_1$	${}_2X_1$	${}_3X_1$	${}_4X_1$	${}_5X_1$	${}_6X_1$	${}_7X_1$	${}_8X_1$	${}_9X_1$
2	${}_0X_2$	${}_1X_2$	${}_2X_2$	${}_3X_2$	${}_4X_2$	${}_5X_2$	${}_6X_2$	${}_7X_2$	${}_8X_2$	${}_9\hat{X}_2$
3	${}_0X_3$	${}_1X_3$	${}_2X_3$	${}_3X_3$	${}_4X_3$	${}_5X_3$	${}_6X_3$	${}_7X_3$	${}_8\hat{X}_3$	${}_9\hat{X}_3$
4	${}_0X_4$	${}_1X_4$	${}_2X_4$	${}_3X_4$	${}_4X_4$	${}_5X_4$	${}_6X_4$	${}_7\hat{X}_4$	${}_8\hat{X}_4$	${}_9\hat{X}_4$
5	${}_0X_5$	${}_1X_5$	${}_2X_5$	${}_3X_5$	${}_4X_5$	${}_5X_5$	${}_6\hat{X}_5$	${}_7\hat{X}_5$	${}_8\hat{X}_5$	${}_9\hat{X}_5$
6	${}_0X_6$	${}_1X_6$	${}_2X_6$	${}_3X_6$	${}_4X_6$	${}_5\hat{X}_6$	${}_6\hat{X}_6$	${}_7\hat{X}_6$	${}_8\hat{X}_6$	${}_9\hat{X}_6$
7	${}_0X_7$	${}_1X_7$	${}_2X_7$	${}_3X_7$	${}_4\hat{X}_7$	${}_5\hat{X}_7$	${}_6\hat{X}_7$	${}_7\hat{X}_7$	${}_8\hat{X}_7$	${}_9\hat{X}_7$
8	${}_0X_8$	${}_1X_8$	${}_2X_8$	${}_3\hat{X}_8$	${}_4\hat{X}_8$	${}_5\hat{X}_8$	${}_6\hat{X}_8$	${}_7\hat{X}_8$	${}_8\hat{X}_8$	${}_9\hat{X}_8$
9	${}_0X_9$	${}_1X_9$	${}_2\hat{X}_9$	${}_3\hat{X}_9$	${}_4\hat{X}_9$	${}_5\hat{X}_9$	${}_6\hat{X}_9$	${}_7\hat{X}_9$	${}_8\hat{X}_9$	${}_9\hat{X}_9$
10	${}_0X_{10}$	${}_1\hat{X}_{10}$	${}_2\hat{X}_{10}$	${}_3\hat{X}_{10}$	${}_4\hat{X}_{10}$	${}_5\hat{X}_{10}$	${}_6\hat{X}_{10}$	${}_7\hat{X}_{10}$	${}_8\hat{X}_{10}$	${}_9\hat{X}_{10}$

Prediction for IBNR claims

Bagging and regression trees in individual claims reserving

Jan Janoušek

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For IBNR claims we use the following estimator:

$$ {}_I\hat{X}_i^{IBNR} = \hat{N}_{i,I}^{(cCL)} \hat{\mathbb{E}}[{}_I X_i^{(1)}], $$

where $\hat{N}_{i,I}^{(cCL)}$ is the chain-ladder estimation of the cumulative number of claims calculated from the reported claims and $\hat{\mathbb{E}}[{}_I X_i^{(1)}]$ is the expected value of the payment $(i, d, k, 1)$ such that $d + k = I$ for a single claim (i, d, v), such that $i + d > I$ with accident year i.

Next year's reserves

	0	1	2	3	4	5	6	7	8	9
1	${}_0X_1$	${}_1X_1$	${}_2X_1$	${}_3X_1$	${}_4X_1$	${}_5X_1$	${}_6X_1$	${}_7X_1$	${}_8X_1$	${}_9X_1$
2	${}_0X_2$	${}_1X_2$	${}_2X_2$	${}_3X_2$	${}_4X_2$	${}_5X_2$	${}_6X_2$	${}_7X_2$	${}_8X_2$	${}_9\hat{X}_2$
3	${}_0X_3$	${}_1X_3$	${}_2X_3$	${}_3X_3$	${}_4X_3$	${}_5X_3$	${}_6X_3$	${}_7X_3$	${}_8\hat{X}_3$	${}_9\hat{X}_3$
4	${}_0X_4$	${}_1X_4$	${}_2X_4$	${}_3X_4$	${}_4X_4$	${}_5X_4$	${}_6X_4$	${}_7\hat{X}_4$	${}_8\hat{X}_4$	${}_9\hat{X}_4$
5	${}_0X_5$	${}_1X_5$	${}_2X_5$	${}_3X_5$	${}_4X_5$	${}_5X_5$	${}_6\hat{X}_5$	${}_7\hat{X}_5$	${}_8\hat{X}_5$	${}_9\hat{X}_5$
6	${}_0X_6$	${}_1X_6$	${}_2X_6$	${}_3X_6$	${}_4X_6$	${}_5\hat{X}_6$	${}_6\hat{X}_6$	${}_7\hat{X}_6$	${}_8\hat{X}_6$	${}_9\hat{X}_6$
7	${}_0X_7$	${}_1X_7$	${}_2X_7$	${}_3X_7$	${}_4\hat{X}_7$	${}_5\hat{X}_7$	${}_6\hat{X}_7$	${}_7\hat{X}_7$	${}_8\hat{X}_7$	${}_9\hat{X}_7$
8	${}_0X_8$	${}_1X_8$	${}_2X_8$	${}_3\hat{X}_8$	${}_4\hat{X}_8$	${}_5\hat{X}_8$	${}_6\hat{X}_8$	${}_7\hat{X}_8$	${}_8\hat{X}_8$	${}_9\hat{X}_8$
9	${}_0X_9$	${}_1X_9$	${}_2\hat{X}_9$	${}_3\hat{X}_9$	${}_4\hat{X}_9$	${}_5\hat{X}_9$	${}_6\hat{X}_9$	${}_7\hat{X}_9$	${}_8\hat{X}_9$	${}_9\hat{X}_9$
10	${}_0X_{10}$	${}_1\hat{X}_{10}$	${}_2\hat{X}_{10}$	${}_3\hat{X}_{10}$	${}_4\hat{X}_{10}$	${}_5\hat{X}_{10}$	${}_6\hat{X}_{10}$	${}_7\hat{X}_{10}$	${}_8\hat{X}_{10}$	${}_9\hat{X}_{10}$

Next year's RBNS reserves

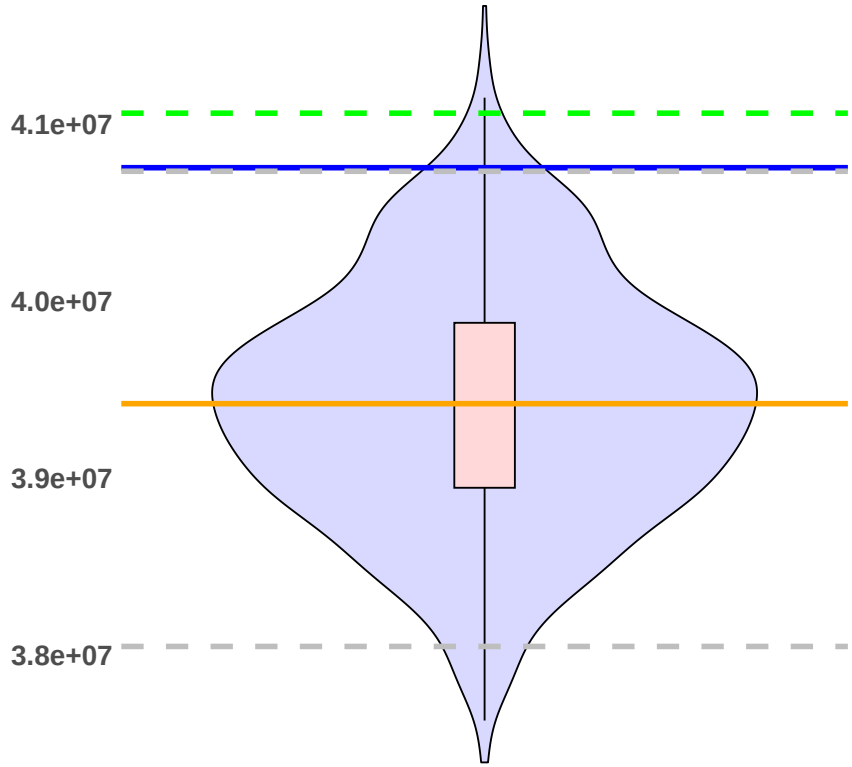

Figure: blue – real value of payments, orange – bagging method, grey dashed lines – 95% confidence interval, green dashed line-99.5% quantile.

Next year's reserves

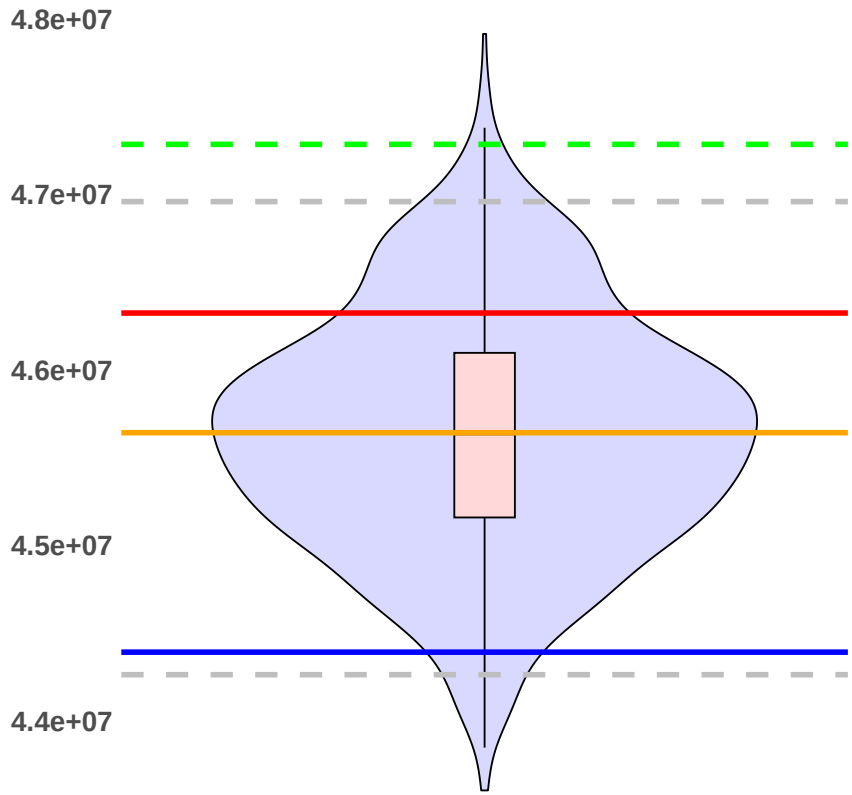

Figure: blue – real value of payments, green – chain-ladder method, red – regression tree method, orange – bagging method, grey dashed lines – 95% confidence interval, green dashed line-99.5% quantile.

Ultimate RBNS reserves

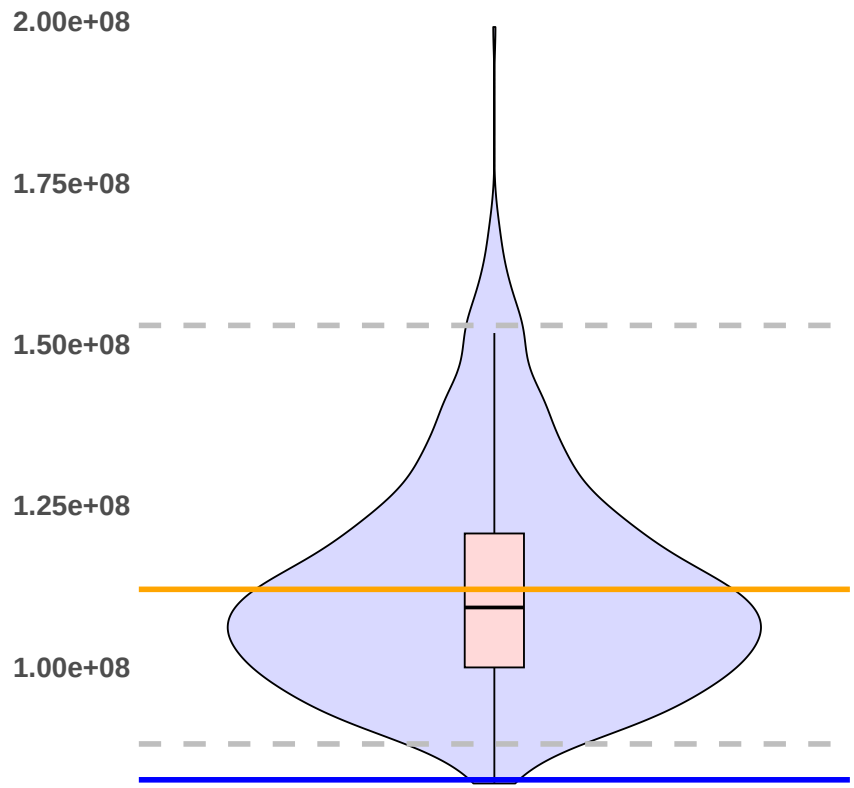

Figure: blue – real value of payments, orange – bagging method, grey dashed lines – 95% confidence interval.

Ultimate reserves

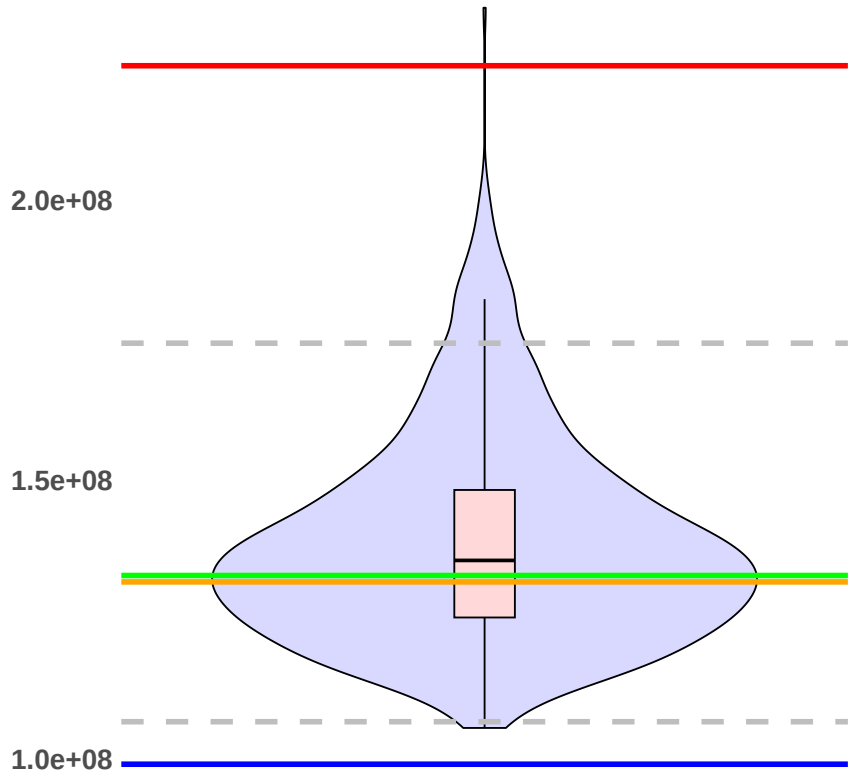

Figure: blue – real value of payments, green – chain-ladder method, red – regression tree method, orange – bagging method, grey dashed lines – 95% confidence interval.

Summary statistics

	next year's	ultimate
real:	44,391,068	99,086,299
mean:	45,609,930	131,146,228
sd:	70,9815	16,920,947
maximum:	48,386,506	232,476,955
99.5% quantile:	47,470,514	188,363,958
97.5% quantile:	47,038,289	171,620,400
75% quantile:	46,083,806	140,646,058
median:	45,599,231	127,951,440
25% quantile:	45,111,226	118,884,735
2.5% quantile:	44,275,804	106,433,734
minimum:	43,024,020	94,354,990

Thank you for your attention

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Thank you for your attention!

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